Laplace transform analysis of the coupled climate–carbon system

Nathan Clisby (MASCOS, University of Melbourne)
Ian G. Enting (MASCOS, University of Melbourne)

September 25, 2007

IPCC Fourth Assessment

- Warming of the climate system is unequivocal . . .

IPCC Fourth Assessment

- Warming of the climate system is unequivocal ...
- Global atmospheric concentrations of carbon dioxide, methane and nitrous oxide have increased markedly as a result of human activities since 1950 ...

IPCC Fourth Assessment

- Warming of the climate system is unequivocal . . .
- Global atmospheric concentrations of carbon dioxide, methane and nitrous oxide have increased markedly as a result of human activities since 1950 . . .
- Most of the observed increase in global average temperatures since the mid-20th century is *very likely* due to the observed increase in anthropogenic greenhouse gas concentrations.

IPCC caveats

- The magnitude of the positive feedback between climate change and the carbon cycle is uncertain. (AR4: TS.5.5).

IPCC caveats

- The magnitude of the positive feedback between climate change and the carbon cycle is uncertain. (AR4: TS.5.5).
- Dynamical processes not included in current models but suggested by recent observations could increase the vulnerability of the ice sheets to warming, increasing future sea level rise. (AR4: TS.5.5).

- Climate: complex system. Difficult but important problem.
- Attack with a variety of approaches.
- Want conclusions to be robust: shouldn't depend on a particular model or even kind of model.
- Laplace transform method can help frame appropriate questions.
- Also (potentially) useful numerically to understand feedbacks.

Data

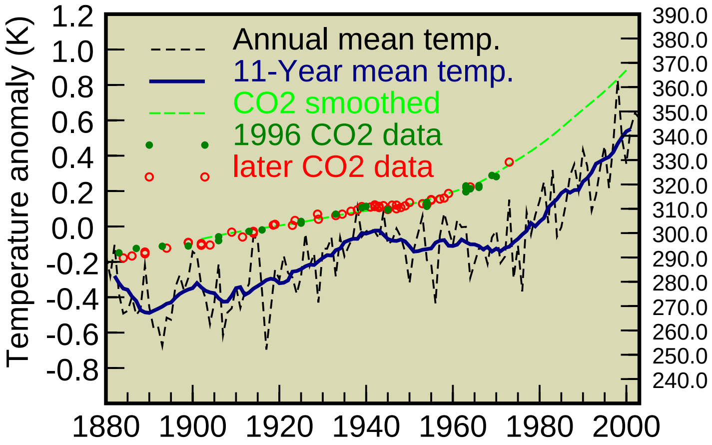
- Direct measurements since 1958.
- Ice cores: air trapped as snow accumulates on glaciers and ice caps.
- High accumulation rates: excellent time resolution (about 11 years for Law Dome), but relatively short records (2000 years).
- Low accumulation rates: poor time resolution, but very long records (up to about 700 000 years).

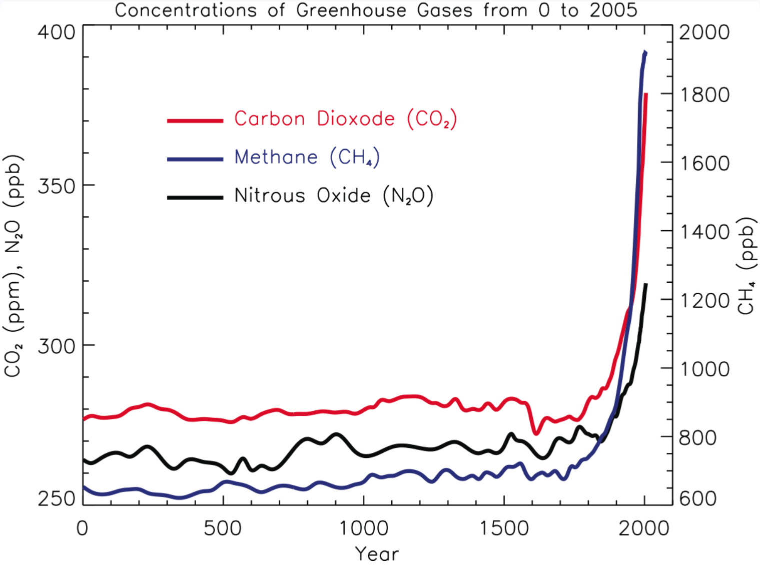
Ice cores give information about

- CO_2 and other gas concentrations.
- Temperature (local to the ice) and global ice volume proxies.

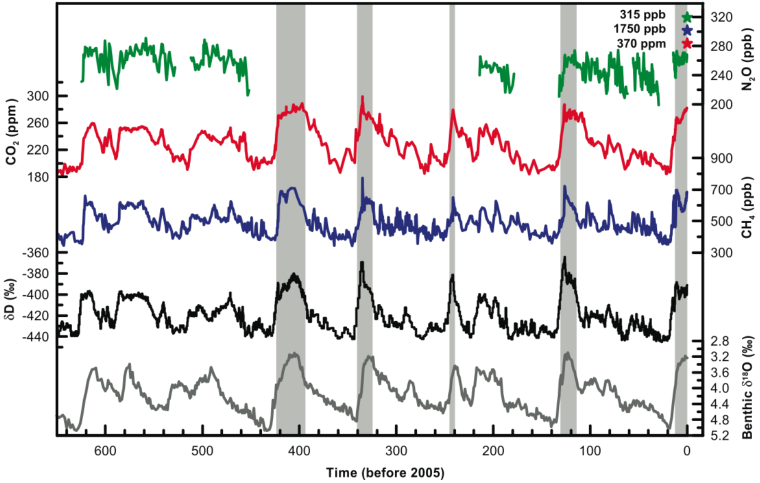
Problem here! Missing image

Law Dome ice core and global temperature data (11-year smoothing of temperature matches smoothing of CO_2 in bubble trapping).



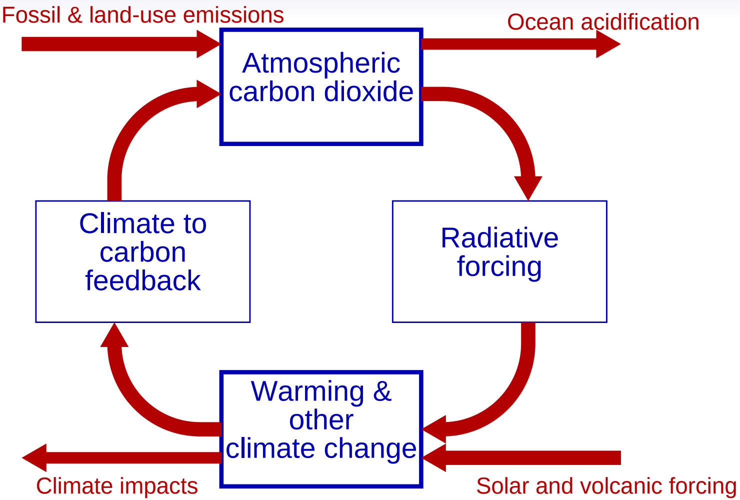

Concentrations of Greenhouse Gases from 0 to 2005
Carbon Dioxode (CO_2)
Methane (CH_4)
Nitrous Oxide (N_2O)
CO_2 (ppm), N_2O (ppb)
CH_4 (ppb)
Year
400
350
300
250
2000
1800
1600
1400
1200
1000
800
600
0
500
1000
1500
2000

Greenhouse gases, temperature and ice volume proxies



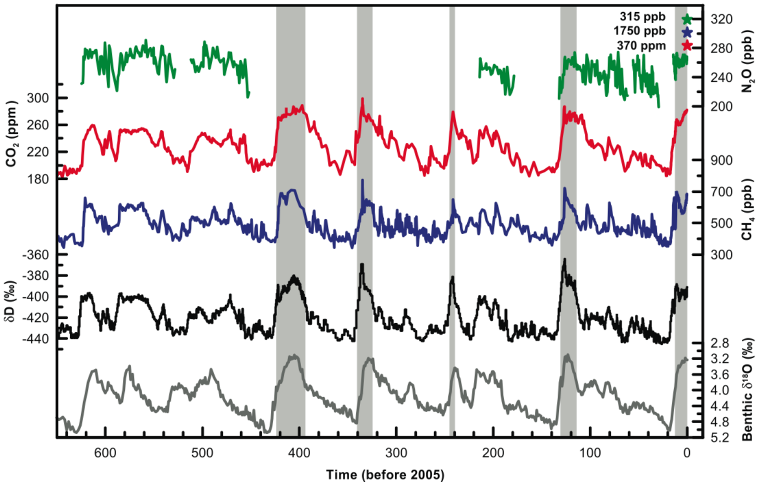
Feedbacks

- The magnitude of the positive feedback between climate change and the carbon cycle is uncertain. (AR4: TS.5.5).
- Increasing temperature may amplify atmospheric CO_2.



From *Twisted: The Distorted Mathematics of Greenhouse Denial.* Ian G. Enting, 2007.

Greenhouse gases, temperature and ice volume proxies



- Forcing from changes in Earth's orbit, northern hemisphere insolation.
- Leads to melting of polar ice.
- Positive feedback as albedo decreased, more radiation absorbed.
- CO_2 concentrations increase due to warming, additional positive feedback.

- Forcing from changes in Earth's orbit, northern hemisphere insolation.
- Leads to melting of polar ice.
- Positive feedback as albedo decreased, more radiation absorbed.
- CO_2 concentrations increase due to warming, additional positive feedback.
- Will there be a corresponding positive climate–carbon feedback under present conditions?

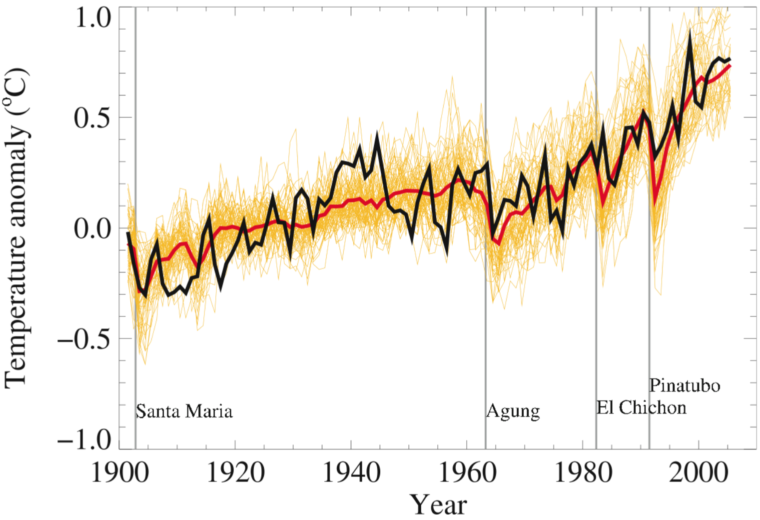
Modelling Climate

- White box: built from basic physical and chemical processes.
- Advantages: can understand what is happening. May have predictive power.
- Disadvantages: hard, difficult to quantify missing processes.
- Black box: purely statistical models.
- Advantages: fewer assumptions.
- Disadvantages: difficult to extrapolate.
- Grey box: include some processes, exclude others.

Modelling Climate

- General Circulation Models: include physical and chemical processes.
  Complex.
  Computationally demanding.
- Simple models - useful for policy decisions, e.g. stabilisation of CO_2.

GCM versus data



C^4MIP

- C^4MIP computer experiment: many coupled climate–carbon GCMS, forced with a standard IPCC scenario.
- Feedback amplification from 11 models range from 1.04 to 1.44, 20ppm to 200ppm.
- Estimate 1.18 ± 0.11. Not a genuine uncertainty - how do we know which model is better?
- Large uncertainty, and possiblity that processes are missing despite ability to reconstruct 20th Century.

Linear model of the Carbon cycle

Linear response function, R, defines how concentrations, C, respond to source, S.

$$Q(t) = C(t) - C(t_0) = \int_{t_0}^{t} R(t - t')\, S(t')\, dt'$$

- Forward modelling: calculate C given model response R and sources, S.
- Inverse problems:
  Deduce $R(t)$ from $C(t)$ and $S(t)$. Model calibration
  Deduce $S(t)$ from $R(t)$ and $C(t)$. Deconvolution

The Laplace transform

Laplace transforms have similar properties to Fourier transforms in analysing linear systems.

- Transforms from time, t to inverse time variable, p.
- Convolution relations transform to products.
- Integration multiples transform by $1/p$.
- More appropriate for one-sided causal relationships than Fourier transforms.
- Notation:

$$f(p) = \mathcal{L}\left[F(t)\right] = \int_0^\infty F(t)\, e^{-pt}\, dt$$

- Carbon relations are $q(p) = r(p)\, s(p)$ whence: $r(p) = q(p)/s(p)$ and $s(p) = q(p)/r(p)$

Information in CO_2 data

For exponentially growing emissions

$$
\begin{aligned}
Q(t) &= \int_{-\infty}^{t} R(t-t')\, A \exp(\beta t')\, dt' \\
&= A \exp(\beta t) \int_{0}^{\infty} R(\tau)\, A \exp(-\beta\tau)\, d\tau
\end{aligned}
$$

and $s(p) = \mathcal{L}\left[A \exp(\beta t)\right] = A/(p-\beta)$, so

$$q(p) = Ar(p)/(p-\beta)$$

Information in CO_2 data

For exponentially growing emissions

$$
\begin{aligned}
Q(t) &= \int_{-\infty}^{t} R(t-t')\, A \exp(\beta t')\, dt' \\
&= A \exp(\beta t) \int_{0}^{\infty} R(\tau)\, A \exp(-\beta \tau)\, d\tau
\end{aligned}
$$

and $s(p) = \mathcal{L}\,[A \exp(\beta t)] = A/(p-\beta)$, so

$$q(p) = Ar(p)/(p-\beta)$$

- For 20th century, $\beta \approx 0.02$.
- Concentrations determined by behaviour of $r(p)$ in vicinity of $p = \beta$.
- Almost no information about $r(p)$ elsewhere.

Linearised model of the coupled climate–carbon system

Laplace transformed equations

$q \equiv$ CO_2 concentration,
$r \equiv$ response to emissions,
$s \equiv$ CO_2 emissions,
$h \equiv$ climate to CO_2 feedback,
$w \equiv$ warming,
$f \equiv$ forcings other than CO_2.

$$\begin{aligned}
q(p) &= r(p)[s(p) + h(p)w(p)] \\
w(p) &= u(p)[f(p) + \alpha q(p)] \\
\Rightarrow q(p) &= r(p)[s(p) + h(p)u(p)f(p)]/[1 - \alpha u(p)r(p)h(p)] \\
\Rightarrow w(p) &= u(p)[f(p) + \alpha r(p)s(p)]/[1 - \alpha u(p)r(p)h(p)]
\end{aligned}$$

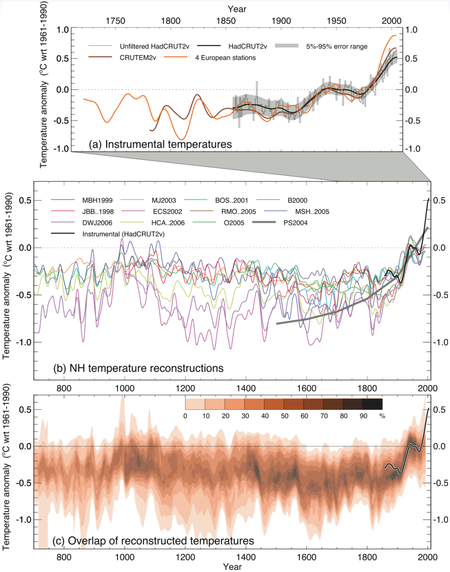
- Atmospheric response $r(p)$ to emissions amplified by $1/(1 - \alpha u(p)r(p)h(p))$.
- Warming response to forcings amplified by $1/(1 - \alpha u(p)r(p)h(p))$.
- Require $\alpha u(p)r(p)h(p) < 1$ for all p for stability.
- Caveat: any predictive power obtained from determination of $h(p)$ depends on dynamics remaining unchanged.

- Atmospheric response $r(p)$ to emissions amplified by $1/(1 - \alpha u(p)r(p)h(p))$.
- Warming response to forcings amplified by $1/(1 - \alpha u(p)r(p)h(p))$.
- Require $\alpha u(p)r(p)h(p) < 1$ for all p for stability.
- Caveat: any predictive power obtained from determination of $h(p)$ depends on dynamics remaining unchanged.
- Is calibration using $C(t)$ and $S(t)$ giving models with $r(p)$ or $r(p)/(1 - \alpha u(p)r(p)h(p))$?
- If yes, need to avoid double counting feedback when making 21st century predictions.

- Scheffer et al., assume a linear model and instantaneous response.

$$\frac{\Delta T_{\text{feedback}}}{\Delta T_0} = \frac{1}{1 - \delta\alpha}$$

- From data of little ice age,
  Moberg gives 1.18, range 1.07-1.25
  Mann and Jones gives 1.78, range 1.28-2.93
- Positive feedback, but very large uncertainties!
- Can improve handling of time dependence with Laplace method, but limiting factor is reconstruction quality.


Year
1750 1800 1850 1900 1950 2000
Temperature anomaly (°C wrt 1961-1990)
Unfiltered HadCRUT2v
HadCRUT2v
5%-95% error range
CRUTEM2v
4 European stations
(a) Instrumental temperatures
MBH1999
MJ2003
BOS..2001
B2000
JBB..1998
ECS2002
RMO..2005
MSH..2005
DWJ2006
HCA..2006
O2005
PS2004
Instrumental (HadCRUT2v)
(b) NH temperature reconstructions
800 1000 1200 1400 1600 1800 2000
0 10 20 30 40 50 60 70 80 90 %
(c) Overlap of reconstructed temperatures
Year

- High quality global instrumental data for temperature available from around 1880.
- CO_2 response to Pinatubo may tell us about feedback $H(t)$ for timescales of years.
- 1940s dip in CO_2, temperature may give information about decadal time scale.
- Little ice age, $H(t)$ for timescales of centuries (large uncertainties due to temperature reconstructions).
- Improved data from high resolution Law Dome ice cores. Will also need improved temperature reconstructions.
- Glacial-interglacial data from Vostok ice core (difficult to quantify relevance for current climate).

Conclusion

- Laplace transform formalism clarifies some issues regarding information content of measurements.
- We do not yet know if feedbacks are typically included via calibration with 20th Century data.
- Use recently obtained high resolution data from Law Dome ice core to obtain quantitative understanding of feedbacks over last 2000 years (work in progress).

Twisted: The Distorted Mathematics of Greenhouse Denial.

Ian G. Enting

October 2007