Isospectral neutron stars

Arthur Suvorov

University of Melbourne

January 30, 2017

CAN ONE HEAR THE SHAPE OF A DRUM?

MARK KAC, The Rockefeller University, New York

To George Eugene Uhlenbeck on the occasion of his sixty-fifth birthday

> "La Physique ne nous donne pas seulement l'occasion de résoudre des problèmes . . . , elle nous fait presentir la solution." H. POINCARÉ.

Kac asks: is it possible to identify the geometry of a space if you know the spectrum of its (Dirichlet) Laplace operator?

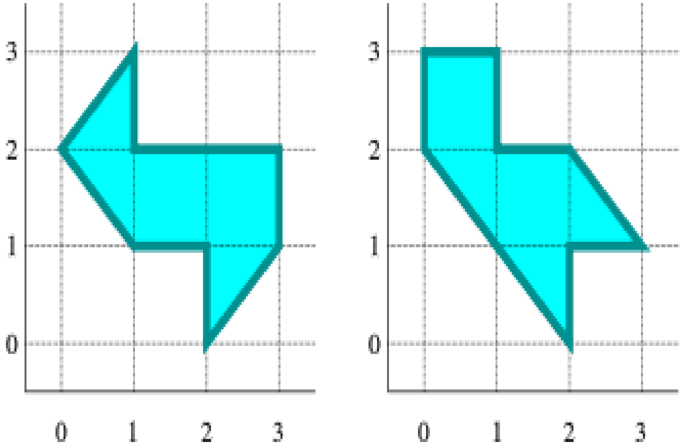
Spectral geometry

Kac's problem generally comes under the heading of the *inverse problem* in Spectral geometry.

Weyl showed in 1911 that the volume of a bounded domain in $\mathbb{R}^n$ can be determined from the asymptotic behavior of the eigenvalues for the Dirichlet boundary value problem of the Laplace operator; can be seen from Poisson-type formulae. Since then, many new results, and many open problems.

Important consequences for physical sciences where one wishes to infer something from waves – medical shape analysis of growths (E. Bernardis et al., 2012).

For Kac's question, the answer is no: several drums may sound the same!

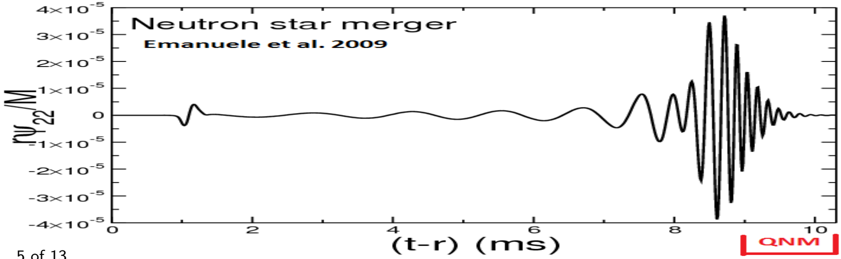


The spectra of these 'drums' are the same: *Isospectral*.

Gravitational waves

In light of the recent and exciting discovery of gravitational waves from black holes and neutron stars, there is a wealth of new astrophysical analysis available to us.

The waves encode properties of the background spacetime from which they originate; some time-dependent disturbance occurs which causes gravitational information to be radiated in the form of gravitational waves.



Perturbations in general relativity

THE UNIVERSITY OF MELBOURNE

Perturbations

Small disturbances in local spacetime structure can be represented as linear perturbations over some background, e.g. two objects in a binary, one with mass much less than the other. This often results in some gravitational radiation.

Depending on the physical nature of the perturbation, the governing mathematical equations can vary – it is important to know which quantity is actually being disturbed.

Ringing neutron stars

General perturbations

A general perturbation can be written as

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}, \quad (1)$$

where $||\mathbf{h}|| \ll ||\mathbf{g}^{(0)}||$.

- Oscillation modes couple to gravitational waves, implying that the associated eigenfrequencies are complex; i.e. $h_{\mu\nu} \sim e^{i\omega t}$ and ω is complex. Real part gives oscillation frequency, and imaginary part gives the inverse of the damping time due to gravitational radiation $\implies$ quasi-normal.

Astroseismology

Fluid or local spacetime perturbations cause a star to ring with a discrete set of oscillation frequencies. We can observe these and attempt to learn something about the star.

Perturbation flavours and Lorentzian isospectrality

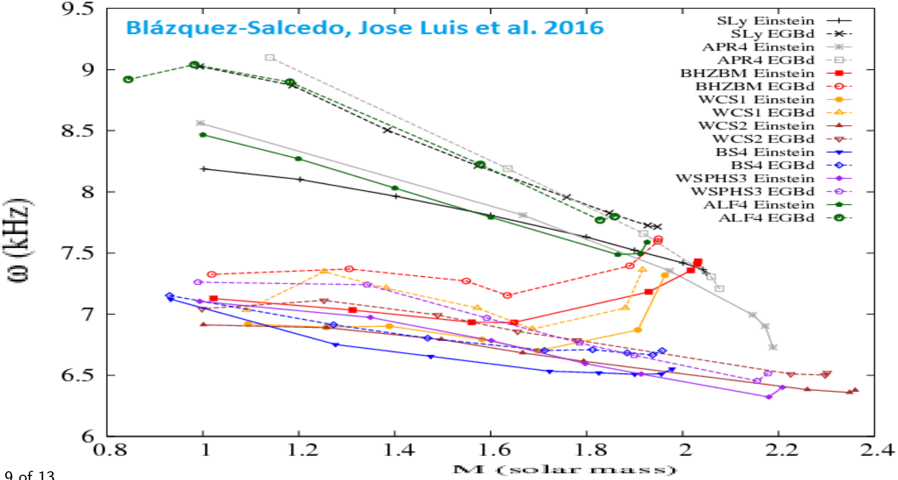
Scalar/test particle

Classical isospectrality; the Klein-Gordon equation due to the perturbation of a scalar field, which may back-react on the spacetime; $\left(\nabla_\mu \nabla^\mu - m^2\right)\varphi = 0$.

Fluid

More physically interesting case where the metric tensor is perturbed due to some fluid perturbation, $g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$, due to some fluid perturbation $T_{\mu\nu} \to T_{\mu\nu}^{(0)} + \delta T_{\mu\nu}$. Separates into several classes (axial or polar).

w-modes of neutron stars: these are spacetime modes; perturbations which do not couple to the hydroynamical variables.



By writing out the perturbed Einstein equations, and the perturbed hydrodynamic equations, it's possible to show that different equations of state can lead to the same spectrum for the arising wave equations where variables go like $\sim e^{i\omega t}$; end up with Schrödinger-like equations:

$$\frac{d^2 Z_\ell^-(r)}{dr_\star^2} + \left[\omega^2 - V_\ell^-(r)\right] Z_\ell^- = 0, \tag{2}$$

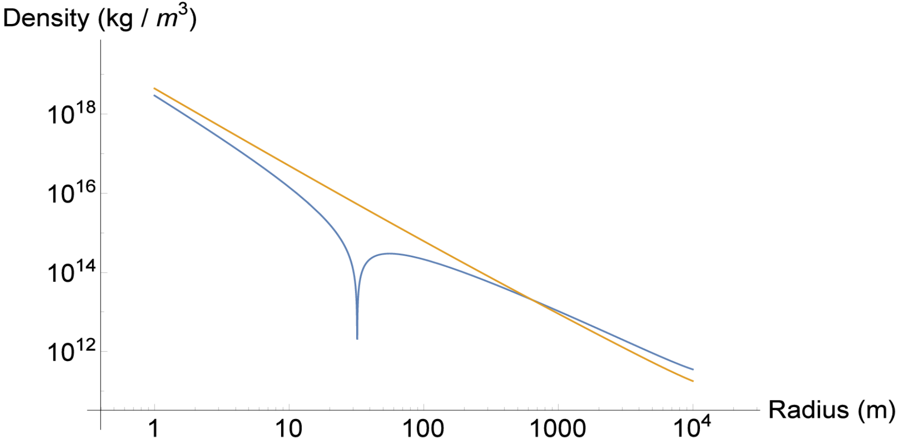
As an example, for the classical isospectral case:

$$V_\ell^-(r) = -g_{tt}\frac{\ell(\ell+1)}{r^2} - \frac{1}{2r}\frac{g_{tt}}{g_{rr}}\left(\frac{g_{tt,r}}{g_{tt}} - \frac{g_{rr,r}}{g_{rr}}\right) \tag{3}$$

where for $r < R_\star$ we have the stellar coefficients and for $r > R_\star$ the spacetime is Schwarzschild.

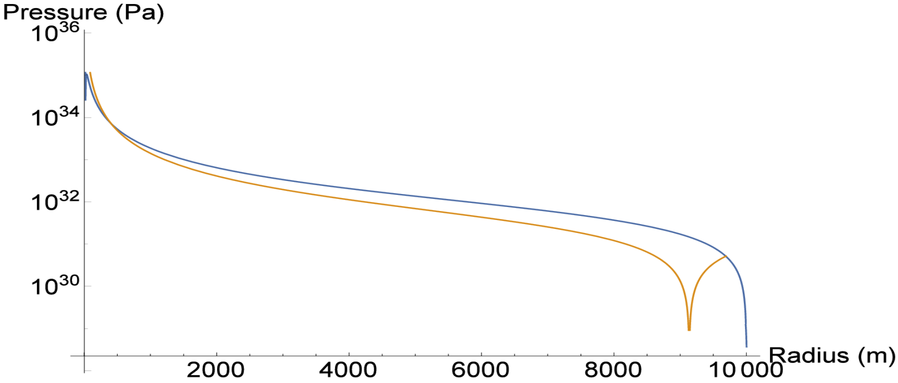
– The implication is that there may be a possible degeneracy between observed spectra and stellar properties if two V_ℓ match.

Classically isospectral stars: Density



These two stars are 'clasically-isospectral': same scalar perturbations.

Classically isospectral stars: Pressure



It appears that both solutions exhibit unphysical behaviour with pressure or density valleys; maybe no realistic stars?

Summary

- Astroseismology of neutron stars can teach us a lot about matter in extreme, astrophysical environments
- However, the exact amount of 'information' they convey can be unclear; it is possible that two stars can ring in the same way.
- Whether any 'realistic' and isospectral EOS exist requires some further investigation.
- 'Nearly-isospectral' stars also interesting; error limits on instruments could make stars effectively isospectral even if not exactly.