Geometric structure of percolation clusters

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Collaborators

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Outline

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Fractal structure of percolation clusters

Our results

Conclusion

Percolation

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- Bond percolation on $L \times L$ square lattice

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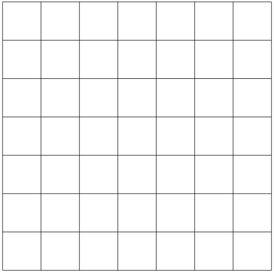

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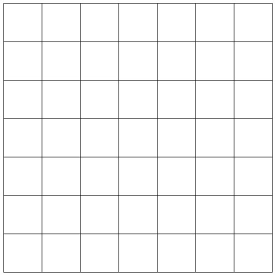

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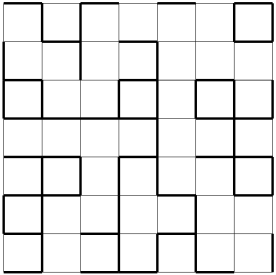

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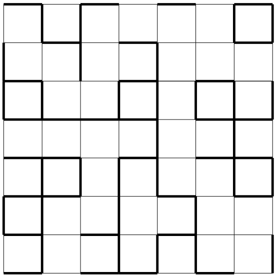

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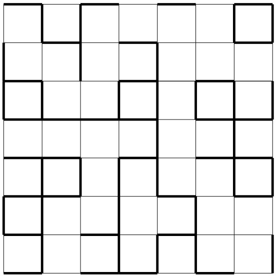

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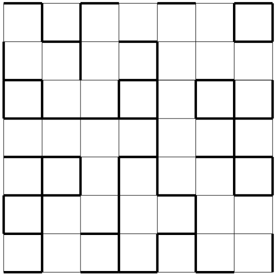

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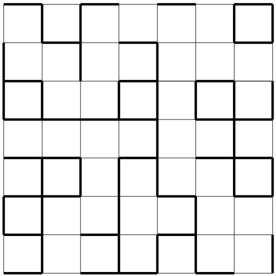

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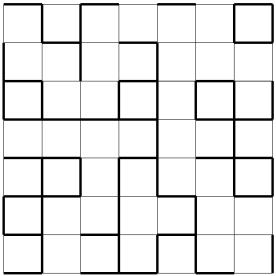

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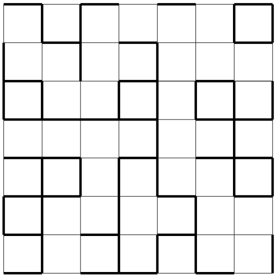

Fractal structure

- Mean size of the largest cluster $\sim L^{d_\mathrm{F}}$, $d_\mathrm{F} = 91/48$
- Backbone
- Mean size of backbone $\sim L^{d_\mathrm{B}}$, $d_\mathrm{B} = 1.643\,36(10)$
- Red bond
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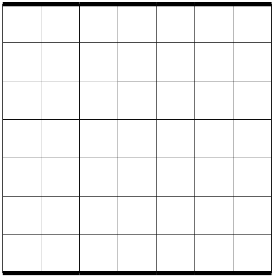

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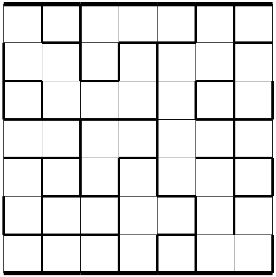

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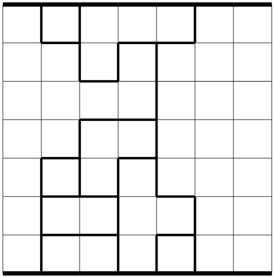

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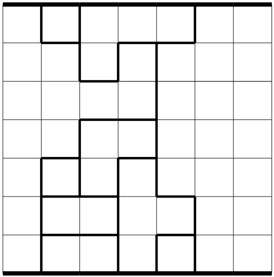

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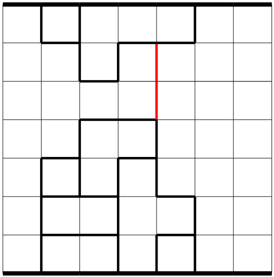

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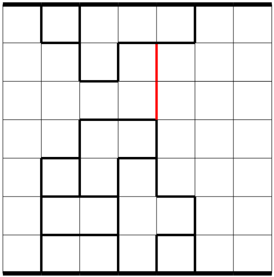

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Partition all bonds into branches, junctions and non-bridges

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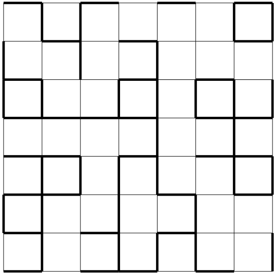

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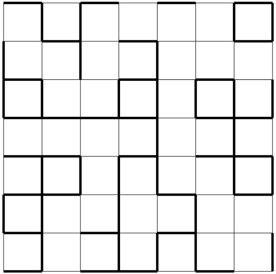

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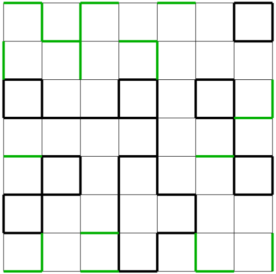

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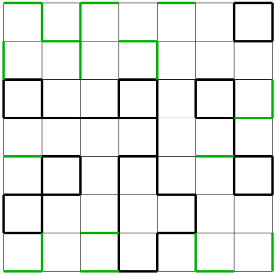

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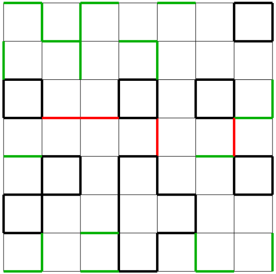

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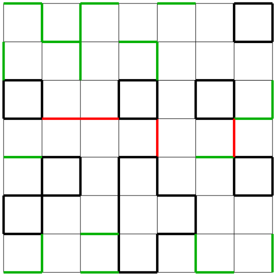

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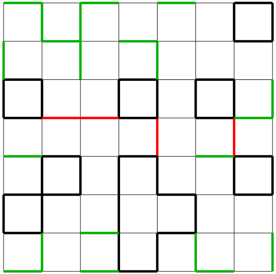

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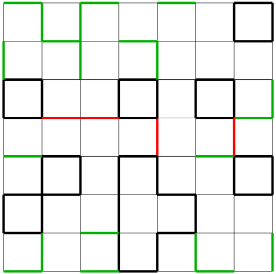

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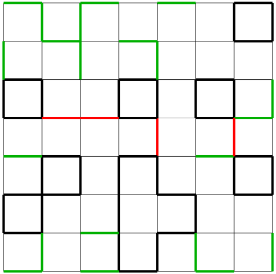

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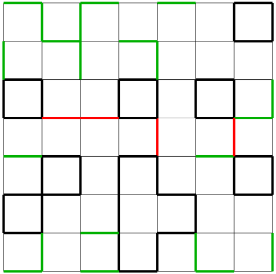

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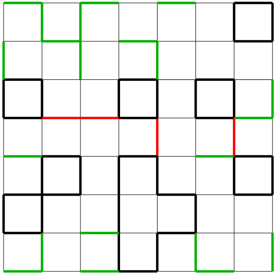

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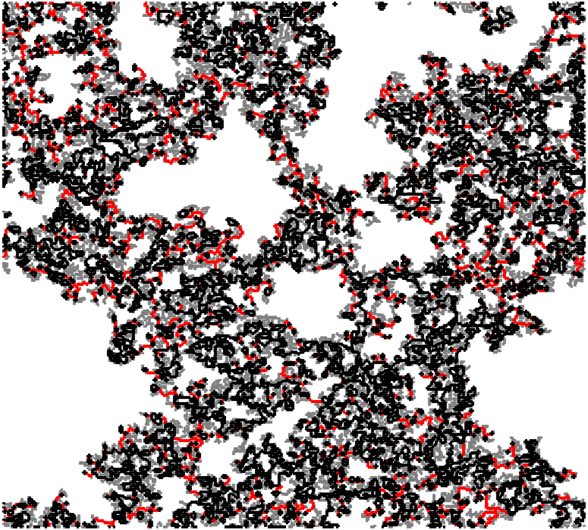

Loop configuration

- Bond configuration $\to$ loop configuration
- Loops drawn on medial graph
- Mean length of the largest loop $\sim L^{d_{\mathrm{H}}}$, with $d_{\mathrm{H}} = 7/4$
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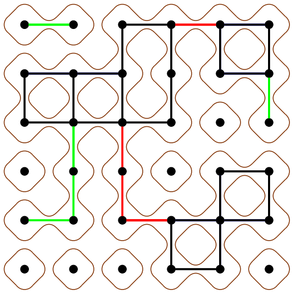

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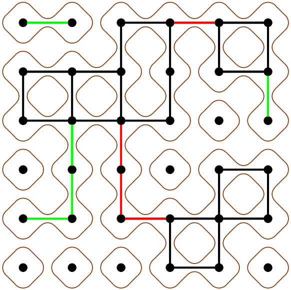

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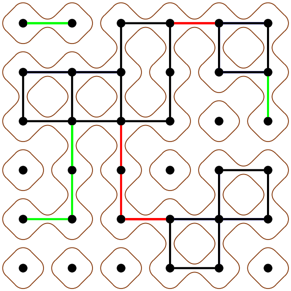

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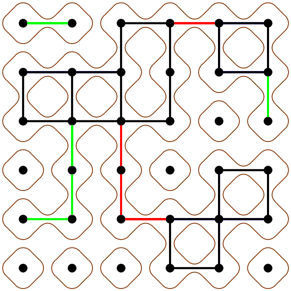

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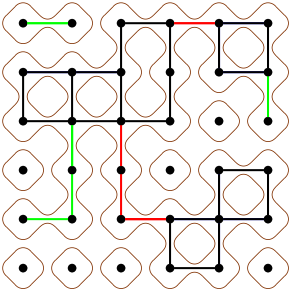

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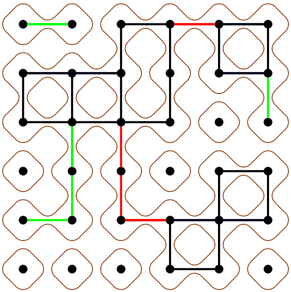

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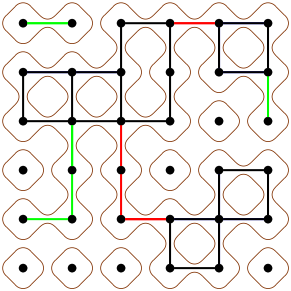

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- $\rho_1 = \rho_g + \rho_j$
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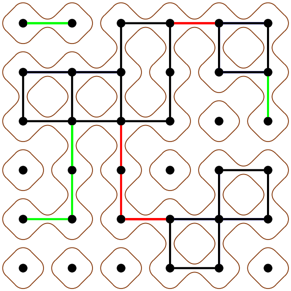

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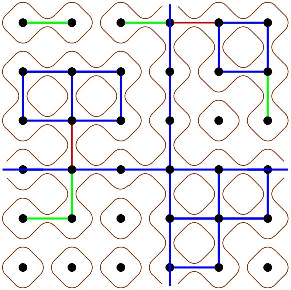

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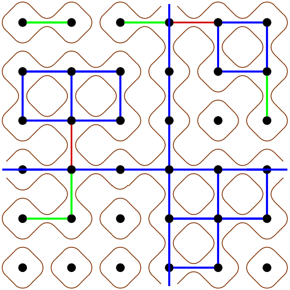

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For non-trivial topology

- pseudo-bridge

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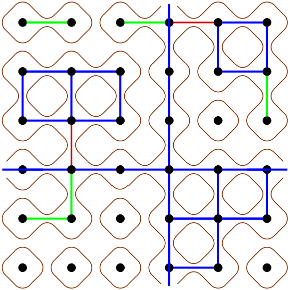
- Mean length of the largest loop $\sim L^{d_{\mathrm{H}}}$, with $d_{\mathrm{H}} = 7/4$
- Accessible external perimeter $\sim L^{d_{\mathrm{E}}}$, with $d_{\mathrm{E}} = 4/3$
- Bonds bounded by the same loop, density ρ_1
- Bonds bounded by distinct loops, density ρ_2



For trivial topology

- $\rho_1 = \rho_g + \rho_j$
- $\rho_2 = \rho_n$

For non-trivial topology

- pseudo-bridge
- $\rho_1 > \rho_g + \rho_j$
- $\rho_2 < \rho_n$

Findings

- Denote the original graph and the dual graph as G and G^*
- $m = |E(G)| = |E(G^*)|$
- Let $A \subset E(G)$ and define $A^* \subset E(G^*)$ via $e^* \in A^*$ iff $e \notin A$
- Let $\ell_1(e)$ be the event that e is bounded by the same loop
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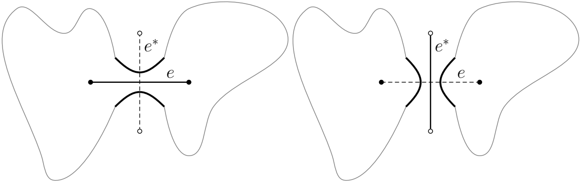

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- This gives

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- On the torus, we expect:

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- For finite L, $\rho_{\mathrm{n}} = \rho_{\mathrm{n},0} + b_1 L^{y_1}$.

- $\rho_{\mathrm{n},0} = 0.250\,000\,1(2)$
- $y_1 = -1.250(1)$
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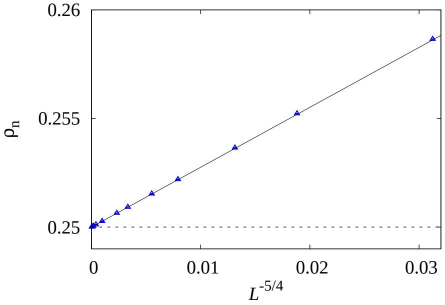

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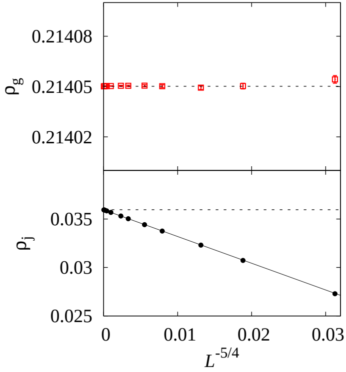

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- For leaf-free clusters
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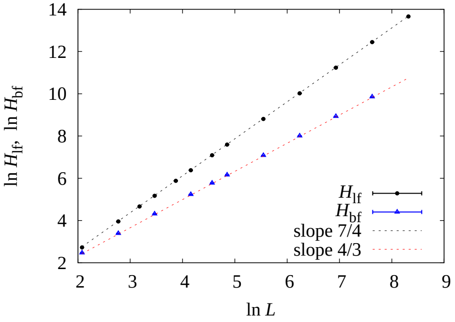

Outline

Fractal structure of percolation clusters

Our results

Conclusion

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- Partition bonds into branches, junctions and non-bridges
- $\rho_1 = \rho_2$
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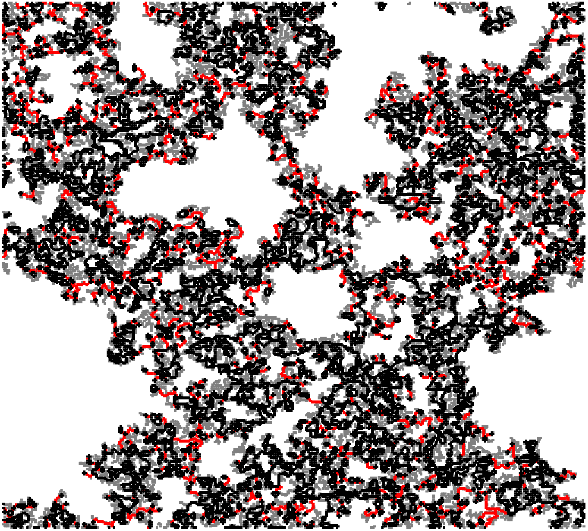

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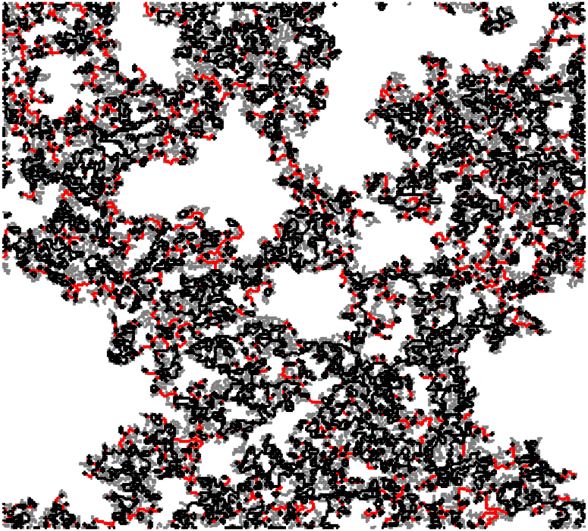

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