Higher spin fields: Cubic and Quartic interactions

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- Motivation
- BRST Approach
- Solutions for cubic and quartic vertices
- Consistency checks for massless and massive fields
- Conclusions

Based on

- I.L.Buchbinder, P.Dempster, M.T: 1308.5539 (Nucl.Phys.**B 877**, 260, 2013) and also
- P.Dempster, M.T: 1203.5597 (Nucl.Phys.**B 865**, 353, 2012)
- A.Fotopoulos, M.T: 1009.0727 (JHEP **1011**, 086, 2010)

- Massless spin 1 and spin 2 field can propagate on any gravitational background.
- Free higher spin theory is usually formulated on a constant curvature background: flat, de Sitter, Anti- de Sitter - one has a sufficient abelian gauge invariance of a free action for a field with $s \geq 3$.
- Interacting theory of massless higher spin fields: for AdS is consistent: M.A.Vasiliev PLB **285** 225, 1992; One needs an infinite number of fields.
- Coleman-Mandula argument is not applicable: No S-Matrix on AdS.
- Interacting theory for massless and massive higher spin fields on a flat backgrounds. We can build cubic, quartic etc interaction vertices. Is it enough for consistency?

- A higher spin field (either massless or massive) can be described by rank s tensor field $\phi_{\mu_1\mu_2\ldots\mu_s}(x)$.
- On-shell it should obey mass-shell and transversality conditions

$$\Box\phi_{\mu_1\mu_2\ldots\mu_s}(x) = \partial^{\mu_1}\phi_{\mu_1\mu_2\ldots\mu_s}(x) = 0.$$

- For massive fields

$$(\Box - m^2)\phi_{\mu_1\mu_2\ldots\mu_s}(x) = \partial^{\mu_1}\phi_{\mu_1\mu_2\ldots\mu_s}(x) = 0.$$

- Introduce an auxiliary Fock space

$$[\alpha_\mu, \alpha_\nu^+] = \eta_{\mu\nu}, \quad \alpha_\mu|0\rangle = 0.$$

and operators $l_0 = -\Box$, $l = -i\alpha \cdot \partial$ (divergence), $l^+ = -i\alpha^+ \cdot \partial$

$$l_0 = [l, l^+].$$

- Ghost variables c_0, c, c^+, (with the ghost number $+1$) conjugate momenta b_0, b^+, b (with the ghost number -1)

$$\{c_0, b_0\} = \{c^+, b\} = \{c, b^+\} = 1.$$

- Nilpotent BRST charge is

$$Q = c_0 l_0 + c l^+ + c^+ l - c^+ c b_0.$$

- The higher spin field $|\Phi\rangle$ ($|\phi\rangle$ is physical, $|C\rangle$ and $|D\rangle$ are auxiliary):

$$|\Phi\rangle = |\phi\rangle + c^+ b^+ |C\rangle + c_0 b^+ |D\rangle.$$

- "Massive" BRST charge: dimensional reduction from the massless one

$$Q = c_0(l_0 + m^2) + c(l^+ + m\alpha_D^+) + c^+(l + m\alpha_D) - c^+ c b_0.$$

- The free Lagrangian has a form

$$L = \int dc_0 \langle \Phi | Q | | \Phi \rangle.$$

- It is invariant under gauge transformations

$$\delta|\phi\rangle = Q|\Lambda\rangle, \quad |\Lambda\rangle = b^+|\lambda\rangle \quad since \quad Q^2 = 0.$$

since $Q^2 = 0$.

- Equation of motion (massless):

$$l_0|\Phi\rangle = l^+|C\rangle, \quad l_0|D\rangle = l|C\rangle, \quad |C\rangle = l|\phi\rangle - l^+|D\rangle.$$

- Gauge transformations

$$\delta|\phi\rangle = l^+|\lambda\rangle, \quad \delta|C\rangle = l_0|\lambda\rangle, \quad \delta|D\rangle = l|\lambda\rangle.$$

- We can gauge away $|C\rangle$ and $|D\rangle$.

To build quartic interactions we take four copies of Fock spaces introduced above. The Lagrangian

$$
\begin{aligned}
L &= \sum_{i=1}^{4} \int dc_0^i \langle \Phi_i | \, Q_i \, | \Phi_i \rangle + g \left(\int dc_0^1 dc_0^2 dc_0^3 \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | | V_3 \rangle + cyc. + h.c \right) \\
&\quad + g^2 \left(\int dc_0^1 dc_0^2 dc_0^3 dc_0^4 \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | \langle \Phi_4 | | V_4 \rangle + h.c \right),
\end{aligned}
$$

Gauge transformations

$$
\begin{aligned}
\delta | \Phi_i \rangle &= Q_i | \Lambda_i \rangle - g \left(\int dc_0^{i+1} dc_0^{i+2} (\langle \Phi_{i+1} | \langle \Lambda_{i+2} | + \langle \Phi_{i+2} | \langle \Lambda_{i+1} |) | V_3 \rangle + cyc. \right) \\
&\quad + (-1)^i g^2 \int dc_0^{i+1} dc_0^{i+2} dc_0^{i+3} [\langle \Phi_{i+1} | \langle \Phi_{i+2} | \langle \Lambda_{i+3} | | V_4 \rangle + cyc.].
\end{aligned}
$$

Here g is a coupling constant, $|V_3\rangle$ and $|V_4\rangle$ are cubic and quartic vertices.

The invariance of the Lagrangian under the gauge transformations imposes constraints on $|V_3\rangle$ and $|V_4\rangle$.

- Invariance in the zeroth order in g guaranteed by $Q_i^2 = 0$.
- Invariance in the first order in g requires

$$(Q_i + Q_j + Q_k)|V_3\rangle_{i,j,k} = 0, \quad i \neq j \neq k.$$

- Invariance in the second order in g requires

$$\begin{aligned} \frac{1}{3}\sum_{i=1}^{4} Q_i|V_4\rangle_{a,b,\alpha,\beta} &= (\mathcal{V}(a,b;\alpha,\beta) + \mathcal{V}(b,\alpha;a,\beta) - (a \leftrightarrow b)) \\ &\quad +(\mathcal{V}(\alpha,a;b,\beta) - (a \leftrightarrow \alpha)). \end{aligned} \tag{1}$$

where

$$\int dc_0^i \;{}_{a,b,i}\langle V_3||V_3\rangle_{\alpha,\beta,i} = \mathcal{V}(a,b;\alpha,\beta),$$

- Massless flat (the ghost completion is omitted) cubic vertices

$$V_3 = F(\Delta_1)R(\Delta_2)Q(\Delta_3)$$

where

$$\Delta_1 = a_1(\alpha^{(1)+}\cdot(\partial^{(2)}-\partial^{(3)})+cyc), \quad \Delta_2 = a_2(\alpha^{(1)+}\cdot\alpha^{(1)+}+cyc)$$

$$\Delta_3 = a_3((\alpha^{(2)+}\cdot\alpha^{(3)+})(\alpha^{(1)+}\cdot(\partial^{(2)}-\partial^{(3)}))+cyc)$$

- Massless flat quartic vertices (the simplest ones, s-channel)

$$\begin{aligned}|V_4\rangle_s \quad = \quad & -\frac{1}{s}\sum_{k,m,n=0}^{\infty}\frac{F^{(k+m)}(0)F^{(k+n)}(0)}{k!m!n!}\left(a_1^2 p_{12}\cdot p_{34}\right)^k \\ & \times\left[a_1\left(2\alpha^{1+}\cdot p_2-2\alpha^{2+}\cdot p_1-c^{1+}b_0^2+c^{2+}b_0^1\right)\right]^m \\ & \times\left[a_1\left(2\alpha^{3+}\cdot p_4-2\alpha^{4+}\cdot p_3-c^{3+}b_0^4+c^{4+}b_0^3\right)\right]^n|0\rangle_{1234}.\end{aligned}$$

- Massive case R. Metsaev: Phys. Lett. **B 720** 237, 2013.
- Let us consider a system of $s - s - 0$ when all fields have the same mass m.

  Cubic vertices are functions of (ghosts are omitted again)

$$L^{(i)} = a_1 \left(\alpha^{(i)} \cdot (p^{(i+1)} - p^{(i+2)}) \right),$$

  and

$$Q^{(i,i+1)} = a_2 \left(\alpha^{(i)} \cdot \alpha^{(i+1)} + \frac{\alpha_D^{(i)}}{2a_1 m} L^{i+1} - \frac{\alpha_D^{(i+1)}}{2a_1 m} L^{i} - \frac{1}{2} \alpha_D^{(i)} \alpha_D^{(i+1)} \right),$$

- We have undetermined coupling constants a_i both for massive and massless cases.
- We have guaranteed the gauge invariance, but is it enough?

- Consider four point amplitude with scalars as external fields.
- It is a simplest possible one and is a direct analog of Veneziano amplitude.
- One can check different types of coupling constants a_i and use different methods: direct analysis via Feynman diagrams or BCFW (R.Britto, F.Cachazo, B.Feng, E.Wittem, Phys.Rev.Lett **94**, 181602, 2005) relations.
- The consistency checks indicate that the theory which contains only massless higher spin point particles on Minkowski background should have a trivial S–matrix unless some nonlocal/composite objects are added into the theory.
- An example of such nonlocal composite object is Stringy Pomeron. (first introduced by R.Brower, J. Polchinski, M.Strassler, C.Tan JHEP **0712**, 005, 2007.)

- Massive case: consider a cubic vertex for $s-s-0$ system, where the scalar is considered as a background field.
- Potentially two problems:
- Nonlinear terms violate transversality condition, thus nonphysical polarizations appear. Causal propagation is violated (the Velo-Zwanziger problem)
- For $3-3-0$ the first condition requires

$$0 = g\frac{a_1^4 a_2}{4m}\left[8(\partial_{\mu_1\mu_2}\phi_{\nu_1\nu_2\nu_3})(\partial_{\nu_1\nu_2\nu_3}\phi) - 16(\partial_{\mu_1}\phi_{\nu_1\nu_2\nu_3})(\partial_{\mu_2\nu_1\nu_2\nu_3}\phi)\right.$$
$$\left.+8\phi_{\nu_1\nu_2\nu_3}(\partial_{\mu_1\mu_2\nu_1\nu_2\nu_3}\phi)\right].$$

- The second condition requires

$$a_2 = -\frac{2}{\mathcal{D}+2}a_1^2 m^2,$$

I.e., a very large m is required.

- The method of construction of cubic and quartic interaction verices is valid also for an $AdS_{\mathcal{D}}$ background. The BRST charge on $AdS_{\mathcal{D}}$: A.Sagnotti, M.T. : Nucl.Phys.**B 682**, 83, 2004.
- On the Minkowski background the gauge invariance is necessary but not a sufficient condition for a consistent higher order interactions.
- The theory of point particles + composite/extended objects can be very interesting. One example of such kind of theories is the String Theory itself.
- The theory of massive higher spin particles (again apart from the String Theory), what can we say about it?
- Many other open questions and problems.