# Logarithmic conformal field models based on $sl(2|1)$ quantum group symmetry.

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- Vacuum space of the chiral algebra as an intersection of screening kernels

$$\phi_i(z)\phi_j(w) = \delta_{ij}\log(z-w)$$

$$e^{\mu_j^i \phi_i(z)}$$

$$S_i = \int e^{\alpha_i^j \phi_j(z)} dz$$

$$\bigcap_i \mathrm{Ker} S_i \quad \longrightarrow \quad \text{logarithmic conformal field model}$$

$$S_i \quad \longrightarrow \quad \text{quantum group}$$

- The simplest example

$$p = 2, 3, 4, \ldots$$

$$\phi(z)\phi(w) = \log(z - w)$$

$$e^{\sqrt{2p}n\phi(z)}, \qquad n \in \mathbb{Z}$$

$$S = \int e^{-\sqrt{\frac{2}{p}}\phi(z)} dz$$

$$e = \int e^{\sqrt{2p}\phi(z)} dz$$

$$T = \frac{1}{2}\partial\phi\partial\phi + \frac{p-1}{\sqrt{2p}}\partial^2\phi, \qquad c = 13 - \frac{6}{p} - 6p$$

Ker $S$ is generated by

$$W^+ = e^{\sqrt{2p}n\phi(z)}, \qquad W^0 = [e, W^+], \qquad W^- = [e, W^0]$$

The quantum group is $\overline{\mathfrak{U}}_{\mathfrak{q}} s\ell(2)$ with $q = e^{\frac{\pi i}{p}}$

$$[E, F] = \frac{K - K^{-1}}{q - q^{-1}}, \quad KE = q^2 EK, \quad KF = q^{-2} FK, \qquad K^{2p} = 1, \quad E^p = F^p = 0$$

- Equivalence of representation categories
- Modular properties
- XXZ spin chain related to $\overline{\mathfrak{U}}_{\mathfrak{q}} s\ell(2)$

$$\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \ldots$$

$$H = \Big(\sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \frac{q+q^{-1}}{2} \sigma_i^z \sigma_{i+1}^z\Big) + \frac{q - q^{-1}}{2}(\sigma_1^z - \sigma_N^z)$$

$$\varphi_1(z)\,\varphi_1(w) = \log(z-w), \qquad \varphi_1(z)\,\varphi_2(w) = -\frac{1}{p}\log(z-w),$$
$$\varphi_2(z)\,\varphi_2(w) = \frac{2}{p}\log(z-w).$$

$$B = \oint e^{\varphi_1} \quad \text{and} \quad F = \oint e^{\varphi_2}$$

$$T(z) = -\frac{1}{k}\partial\varphi_1\partial\varphi_1(z) - \frac{1}{k}\partial\varphi_1\partial\varphi_2(z) - \frac{1}{2k(k+2)}\partial\varphi_2\partial\varphi_2(z) - \partial^2\varphi_1(z) - \frac{1}{2(k+2)}\partial^2\varphi_2(z),$$

where we set

$$k := \frac{1}{p} - 2.$$

The central charge is

$$c = \frac{3k}{k+2} - 1.$$

$$\omega_1(z) = \frac{1}{k}(-2\varphi_1(z) - \varphi_2(z)), \quad \omega_2(z) = \frac{1}{k}\big(-\varphi_1(z) - \frac{1}{k+2}\varphi_2(z)\big)$$

$$\mathcal{E} = \oint e^{-\frac{1}{k+2}\varphi_2}$$

$$j^+(z) = e^{\omega_1(z)}$$
$$j^-(z) = -\big(\partial\varphi_1\partial\varphi_1(z) + \partial\varphi_1\partial\varphi_2(z) + (k+1)\partial^2\varphi_1(z)\big)e^{-\omega_1(z)}$$

$$\mathcal{W}^+(z) = e^{2\omega_2(z)},$$
$$\mathcal{W}^0(z) = \mathcal{E}\mathcal{W}^+(z)$$
$$\mathcal{W}^-(z) = \mathcal{E}\mathcal{W}^0(z)$$

- The quantum group from screenings

$$\Psi : F_i \otimes F_k \mapsto q_{i,k} F_k \otimes F_i, \quad 1 \leqslant i,k \leqslant 2$$

$$(q_{ij}) = \begin{pmatrix} -1 & \mathfrak{q}^{-1} \\ \mathfrak{q}^{-1} & \mathfrak{q}^{2} \end{pmatrix}$$

with $\mathfrak{q} = e^{\frac{i\pi}{p}}$

$$F_i = \int_{-\infty}^{\infty} f_i(z)$$

$$\int_{-\infty}^{\infty} f_i(z) \int_{-\infty}^{\infty} f_j(u) = \iint\limits_{-\infty<z<u<\infty} f_i(z) f_j(u) + \iint\limits_{-\infty<z<u<\infty} f_i(u) f_j(z) =$$

$$= (\delta_i^k \delta_j^l + \Psi_{i,j}^{k,l}) \iint\limits_{-\infty<z<u<\infty} f_k(z) f_l(u)$$

$$\int\limits_{-\infty<z_1<\dots<z_r<\infty}\cdots\int f_{j_1}(z_1)\dots f_{j_r}(z_r),$$

$$\underbrace{F \cdot F \cdot \dots \cdot F}_{p} = (1+\mathfrak{q}^2)(1+\mathfrak{q}^4)\dots(1+\mathfrak{q}^{2p}) \int\limits_{-\infty<z_1<\dots<z_p<\infty}\cdots\int f(z_1)\dots f(z_r)$$

$$F^p = 0$$

$$[F,[F,B]] = B^2 = F^p = 0$$

$$[F,[F,B]] = BF^2 - (\mathfrak{q}+\mathfrak{q}^{-1})FBF + F^2 B = 0$$

Nichols algebra $\mathfrak{B}(X)$ is a braided Hopf algebra

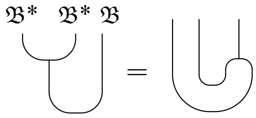
Category of Yetter–Drinfeld modules

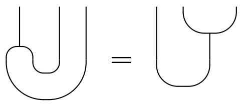
 — braiding

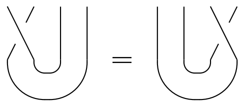
 — inverse braiding

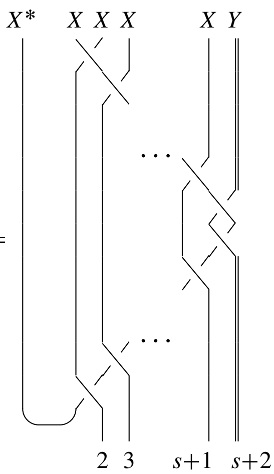
$\mathfrak{B}(X)^*$ the dual Nichols algebra

$\langle\,,\rangle : \mathfrak{B}(X)^* \otimes \mathfrak{B}(X) \to k$ diagrammatically is denoted by 




$\mathscr{K}_2(s+1) =$ 

$$KF = \mathfrak{q}^{-2}FK, \quad EF - FE = \frac{K - K^{-1}}{\mathfrak{q} - \mathfrak{q}^{-1}}, \quad KE = \mathfrak{q}^2 EK,$$
$$F^p = 0, \qquad E^p = 0, \qquad K^{2p} = 1.$$

$$kF = \mathfrak{q}Fk, \quad kE = \mathfrak{q}^{-1}Ek, \quad k^{2p} = 1, \quad kK = Kk.$$

$$KB = \mathfrak{q}BK, \quad kB = -Bk, \quad KC = \mathfrak{q}^{-1}CK, \quad kC = -Ck,$$
$$B^2 = 0, \quad BC - CB = \frac{k - k^{-1}}{\mathfrak{q} - \mathfrak{q}^{-1}}, \quad C^2 = 0,$$
$$FC - CF = 0, \qquad BE - EB = 0,$$
$$FFB - (\mathfrak{q} + \mathfrak{q}^{-1})FBF + BFF = 0, \quad EEC - (\mathfrak{q} + \mathfrak{q}^{-1})ECE + CEE = 0.$$

$$\Delta(F) = F \otimes 1 + K^{-1} \otimes F, \qquad \Delta(E) = E \otimes K + 1 \otimes E,$$
$$\Delta(B) = B \otimes 1 + k^{-1} \otimes B, \qquad \Delta(C) = C \otimes k + 1 \otimes C,$$
$$S(B) = -kB, \quad S(F) = -KF, \quad S(C) = -Ck^{-1}, \quad S(E) = -EK^{-1},$$
$$\varepsilon(B) = 0, \quad \varepsilon(F) = 0, \quad \varepsilon(C) = 0, \quad \varepsilon(E) = 0,$$

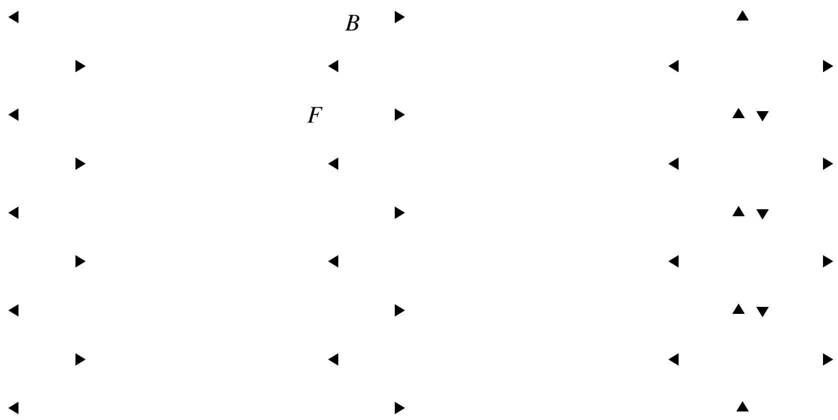



FIGURE 0.1.

t-J model

$$3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \ldots$$

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- Equivalence between representation categories of W-algebra and of the quantum group
- Equivalence of modular properties
- Reconstruction of the W-algebra from the quantum group spin chain