Off-shell conformal supergravity in 3D

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Based on:

Kuzenko & GTM, JHEP **1303**, 113 (2013), 1212.6852

Butter, Kuzenko, Novak & GTM, JHEP **1309**, 072 (2013), 1305.3132

Butter, Kuzenko, Novak & GTM JHEP **1310**, 073 (2013), 1306.1205

Kuzenko, Novak & GTM *accepted in JHEP*, arXiv:1308.5552

Outline

1. Introduction/Summary
2. Conformal SUGRA in Superspace
3. Superforms-Ectoplasms and actions
4. Conclusion

Introduction: AIM

Construct the off-shell action
for $\mathcal{N}$-extended conformal supergravity
in three space-time dimensions (3D)

Supersymmetry

Supersymmetry (SUSY)

- Supersymmetry is a continuos symmetry between particles generated by fermionic spinorial charges Q that typically anticommute over the momentum P

$$\{Q, Q\} \simeq P$$

- SUSY had a central role in the efforts of the last 40 years to extend the Standard Model of particle physics
- Examples of problems for which Supersymmetry and Supergravity (his extension with gravity) help to formulate possible solutions are:
  – The hierarchy problem;
  – Dark matter;
  – The cosmological constant problem;
  – ingredient for consistency of String Theory, quantum gravity.

Supersymmetry (SUSY)

besides the phenomenological implications, supersymmetry is a framework to understand nontrivial dynamical properties via "toy models":

- SUSY theories proves to have better quantum behaviours (cancellations of loop divergencies...)
- often analytical and exact nonperturbative studies possible (e.g. 4D, $\mathcal{N} = 2$ SUSY and Seiberg-Witten theory, understanding low-energy effective action..., confinement...)
- Quantum mechanically provides consistent finite gravity theories: String Theory..., what about 4D $\mathcal{N} = 8$ Supergravity?

Supersymmetry (SUSY)

new relevant ingredient in connection with mathematics

- SUSY sigma models: there is a direct link between the amount of supersymmetry of a system and the target space geometry of sigma models (e.g. 4D $\mathcal{N} = 1$ SUSY and Kähler geometry)
  Provide a generating and classification framework of geometries
- integrable systems: systems with large amount of supersymmetry often prove to be integrable or solvable using integrability techniques: 2D sCFT...; AdS/CFT and integrability for $\mathcal{N} = 4$ Super-Yang-Mills...; topological models...

an efficient formalism for Supersymmetry? Superspace

A natural question is how to efficiently formulate and study supersymmetric models?

From experience of ordinary symmetries in physics, as gauge theories including gravity, an effective way is by keeping manifest and realize *off-shell* (symmetries close without use of equations of motions).

In the case of supersymmetry, such a covariant technique exist and goes under the name of Superspace introduced in the 1970s

And what is Superspace?

- As supersymmetry extend the Poincaré group to a supergroup with the fermionic charges Q
- Superspace extend the Minkowski space-time to a supermanifold with extra fermionic Grassmannian coordinates θ

$$\{\theta, \theta\} = [x, \theta] = 0$$

- – Minkowski space-time: $\{x\}$ = Poincaré/SO(d-1,1)
  – Flat superspace: $\{x,\ \theta\}$ = Super-Poincaré/SO(d-1,1)
- supersymmetry transformations are generated linearly in superspace as translations of θ coordinates: (γ=gamma-matrices in d-dimensions)

$$\theta' = \theta + \epsilon\ , \quad x' = x - \mathrm{i}(\epsilon\,\gamma\,\theta)$$

Superfields

In formulating supersymmetric field theory and supergravity you then lift fields to Superfields, functions on superspace:

$$\Psi(x,\theta) = C(x) + \theta\xi(x) + \cdots + (\theta)^{\mathcal{N}\times n}B(x) \ ,$$

$\mathcal{N} =$ "number" of supersymmetries, $n =$ dimension spinor representation

- most compact way to organize in simple objects all the component fields of supermultiplets
- keep together physical and auxiliary fields of supermultiplets. auxiliaries (nondynamical) typically needed to have off-shell SUSY

Introduction: AIM

Construct the off-shell action
for $\mathcal{N}$-extended conformal supergravity
in three space-time dimensions (3D)

Introduction: old results in 3D conformal SUGRA

in a series of papers between 1985 and 1993 the on-shell 3D $\mathcal{N}$-extended conformal supergravity actions were constructed:

- $\mathcal{N} = 1$:[van Nieuwenhuizen (1985)];
- $\mathcal{N} = 2$:[Roček, van Nieuwenhuizen (1986)];
- general-$\mathcal{N}$: [Lindström, Roček (1989)] & [Nishino, Gates (1993)].

Introduction: old results in 3D conformal SUGRA, action

- The on-shell action for general $\mathcal{N}$ is a Chern-Simons (CS) type action with Higher Derivatives in gravitini

$$S_{\rm CS} = \frac{1}{4}\int \mathrm{d}^3x\, e\, \varepsilon^{abc}\Big(\omega_a{}^{fg}\mathcal{R}_{bc\,fg} - \frac{2}{3}\omega_{af}{}^{g}\omega_{bg}{}^{h}\omega_{ch}{}^{f} -2\mathcal{R}_{ab}{}^{IJ}V_{c\,IJ} - \frac{4}{3}V_a{}^{IJ}V_{bI}{}^{K}V_{c\,KJ} -\frac{\mathrm{i}}{2}\Psi_{bc}{}_I^{\alpha}(\gamma_d)_\alpha{}^\beta(\gamma_a)_\beta{}^\gamma\varepsilon^{def}\Psi_{ef}{}_\gamma^I\Big)$$

$\omega_a{}^{bc}$, $\mathcal{R}_{ab}{}^{cd}$ Lorentz connection and curvature,
$V_a{}^{IJ}$, $\mathcal{R}_{ab}{}^{IJ}$ SO($\mathcal{N}$) R-symmetry connection and curvature,
$\Psi_{bc}{}_I^{\alpha}$ gravitini field strength.

- Action was constructed in components as a CS action for the $\mathcal{N}$-extended superconformal algebra $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$

How to extend the action off-shell?

Motivations: Why off-shell 3D conformal SUGRA?

An off-shell approach to conformal SUGRA, when available, can be used to generate general supergravity matter couplings:

Example of recent applications in 3D have seen:

- 3D massive gravity and AdS_3/CFT_2. Topological-Massive-Gravity (TMG) [Li-Song-Strominger ('08)]; New-Massive-Gravity (NMG), Generalized-Massive-Gravity (GMG) [Bergshoeff-Hohm-Townsend ('09)]
- Supersymmetric extensions of NMG & GMG:
  $\mathcal{N} = 1$ non-linear case [Andringa-Bergshoeff-deRoo-Hohm-Sezgin-Townsend ('09)], [Bergshoeff-Hohm-Rosseel-Sezgin-Townsend ('10)]
  $\mathcal{N} \geq 2$ understood at the linearized level [Bergshoeff-Hohm-Rosseel-Townsend ('10)]
- Fully non-linear SUSY difficult due to higher-derivative terms
- TMG, GMG includes conformal SUGRA action as building block

Motivations: Why off-shell 3D conformal SUGRA?

- recent renewed interest in constructing off-shell SUSY theories on curved backgrounds in 4D and 3D. This opened a classification problem for SUSY backgrounds of off-shell sugra
  see Kuzenko's talk
- localization techniques for computing Wilson loop, partition functions, indices... see e.g. [Pestun ('07)–('09)]; [Jafferis ('10)]
  Localization needs off-shell SUSY
- in 3D, off-shell curved (Lorentz)-Chern-Simons (conformal sugra actions) have a role in explaining features like contact terms, complexity and F-maximization of the partition function of $\mathcal{N} = 2$ theories [Closset-Dumitrescu-Festuccia-Komargodski-Seiberg ('12)]

Motivations: off-shell 3D superspace SUGRA program

Superspace formulations for 3D $\mathcal{N}$-extended SUGRA help.
Surprisingly this was *not* fully developed in the past

- $\mathcal{N} = 1$: [Howe-Tucker (1977)];[Brown-Gates (1979)]
- $\mathcal{N} \geq 2$ sketched [Howe-Izquierdo-Papadopoulos-Townsend (1995)]
  more for $\mathcal{N} = 8$ [Howe-Sezgin ('04)]

$\Downarrow$

SO($\mathcal{N}$) supergeometry for off-shell Weyl multiplet of conformal SUGRA
for any $\mathcal{N}$ in [Kuzenko-Lindström-GTM ('11)]
and general supergravity-matter systems with $\mathcal{N} \leq 4$

Motivations: off-shell 3D superspace SUGRA program

A missing element in the program was indeed to build the action for $\mathcal{N}$-extended conformal sugra using superspace
(known only for $\mathcal{N} = 1$ [Gates-Grisaru-Roček-Siegel (1981)]; [Zupnik-Pak (1988)])

$\Longrightarrow$ How to build it for general $\mathcal{N}$?

Motivations: How conformal SUGRA action for general $\mathcal{N}$?

- A way is to use superform/ectoplasm techniques to engineer invariant actions from closed super 3-forms.
- "Ectoplasm" in physics has been rediscovered various time: [Hasler (1996)]; [Gates-Grisaru-Knutt–Wehlau-Siegel (1997)]; see also reheonomy approach (book [Castellani-D'Auria-Fre (1991)]);

Successful way: [Butter-Kuzenko-Novak-GTM ('13)]

- manifestly gauge entire $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$ in superspace $\Longrightarrow$ "3D conformal superspace" (turns out to simplify calculations)
- ectoplasm for $\mathcal{N}=1,\cdots,5$ [1306.1205]; then $\mathcal{N}=6$ [1308.5552]

$\mathcal{N}=3,4,5,6$ actions were never constructed before

Step 1:

define a new 3D $\mathcal{N}$-extended conformal superspace

- Supergeometry based on a manifest gauging of the entire superconformal group $\mathrm{OSp}(\mathcal{N}|4,\mathbb{R})$

3D conformal supergravity in conformal superspace

Take an $\mathcal{N}$-extended curved superspace

$$z^M = (x^m, \theta^\mu_{\text{I}}) \ , \qquad m = 0,1,2 \ , \quad \mu = 1,2 \ , \quad \text{I} = \mathbf{1}, \cdots, \boldsymbol{\mathcal{N}}$$

Structure group X is chosen to be:
$\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SO}(\mathcal{N}) \times$(Dilatations)$\times$($S-$susy) $\times$ ($K-$boosts).
The superspace covariant derivatives

$$\nabla_A = E_A{}^M \partial_M - \omega_A{}^{\underline{b}} X_{\underline{b}} = E_A{}^M \partial_M - \frac{1}{2}\Omega_A{}^{ab} M_{ab} - \frac{1}{2}\Phi_A{}^{PQ} N_{PQ} - B_A \mathbb{D} - \mathfrak{F}_A{}^B K_B$$

- $E_A{}^M(z)$ supervielbein, $\partial_M = \partial/\partial z^M$, - $\Omega_A{}^{cd}(z)$ Lorentz connection,
- $\Phi_A(z)$ $\mathrm{SO}(\mathcal{N})$-connection, - B_A dilatation $\mathbb{D}$-connection
- $\mathfrak{F}_A{}^B$ special superconformal connection, $K_A = (K_a, S^I_\alpha)$
- ∇_A gauge super-Poincaré via superdiffeomorphism, $P_A = (P_a, Q^I_\alpha)$.

- The sugra local gauge transformations $\mathrm{OSp}(\mathcal{N}|4,\mathbb{R})$

$$\mathcal{K} := \xi^A \nabla_A + \frac{1}{2}\Lambda^{bc} M_{bc} + \frac{1}{2}\Lambda^{KL} N_{KL} + \sigma\mathbb{D} + \Lambda^A K_A \ , \qquad \delta_\mathcal{K} \nabla_A = [\mathcal{K}, \nabla_A]$$

3D conformal supergravity in conformal superspace

Can prove that the algebra:

$$[\nabla_A, \nabla_B\} = -T_{AB}{}^C \nabla_C - \frac{1}{2} R(M)_{AB}{}^{cd} M_{cd} - \frac{1}{2} R(N)_{AB}{}^{PQ} N_{PQ} - R(\mathbb{D})_{AB}\mathbb{D} - R(S)_{AB}{}^{\gamma}_{K} S^K_{\gamma} - R(K)_{AB}{}^c K_c$$

can be constrained to be for $\mathcal{N} > 3$

$$\{\nabla^I_\alpha, \nabla^J_\beta\} = 2\mathrm{i}\delta^{IJ}\nabla_{\alpha\beta} + \mathrm{i}\varepsilon_{\alpha\beta} W^{IJKL} N_{KL} - \frac{\mathrm{i}}{\mathcal{N}-3}\varepsilon_{\alpha\beta}(\nabla^\gamma_K W^{IJKL}) S_{\gamma L} + \frac{1}{2(\mathcal{N}-2)(\mathcal{N}-3)}\varepsilon_{\alpha\beta}(\gamma^c)^{\gamma\delta}(\nabla_{\gamma K}\nabla_{\delta L} W^{IJKL}) K_c \ ,$$

$$[\nabla_a, \nabla^J_\beta] = \frac{1}{2(\mathcal{N}-3)}(\gamma_a)_{\beta\gamma}(\nabla^\gamma_K W^{JPQK}) N_{PQ} - \frac{1}{2(\mathcal{N}-2)(\mathcal{N}-3)}(\gamma_a)_{\beta\gamma}(\nabla^\gamma_L \nabla^\delta_P W^{JKLP}) S_{\delta K} - \frac{\mathrm{i}}{4(\mathcal{N}-1)(\mathcal{N}-2)(\mathcal{N}-3)}(\gamma_a)_{\beta\gamma}(\gamma^c)_{\delta\rho}(\nabla^\gamma_K \nabla^\delta_L \nabla^\rho_P W^{JKLP}) K_c \ ,$$

$$[\nabla_a, \nabla_b] = \frac{1}{4(\mathcal{N}-2)(\mathcal{N}-3)}\varepsilon_{abc}(\gamma^c)_{\alpha\beta}\Big(\mathrm{i}(\nabla^\alpha_I \nabla^\beta_J W^{PQIJ}) N_{PQ} + \frac{\mathrm{i}}{\mathcal{N}-1}(\nabla^\alpha_I \nabla^\beta_J \nabla^\gamma_K W^{LIJK}) S_{\gamma L} + \frac{1}{2\mathcal{N}(\mathcal{N}-1)}(\gamma^d)_{\gamma\delta}(\nabla^\alpha_I \nabla^\beta_J \nabla^\gamma_K \nabla^\delta_L W^{IJKL}) K_d\Big) \ ,$$

Algebra parametrized only in terms of one single superfield strength:
$W^{IJKL} = W^{[IJKL]}$: the super-Cotton tensor
describing 3D $\mathcal{N}$-extended Weyl multiplet of conformal Supergravity

3D conformal supergravity in conformal superspace

W^{IJKL} is a dimension-1 primary

$$S_\alpha^P W^{IJKL} = 0 \ , \quad \mathbb{D} W^{IJKL} = W^{IJKL}$$

and satisfies one simple Bianchi identity (for $\mathcal{N} > 4$)

$$\nabla_\alpha^I W^{JKLP} = \nabla_\alpha^{[I} W^{JKLP]} - \frac{4}{\mathcal{N} - 3} \nabla_{\alpha Q} W^{Q[JKL} \delta^{P]I}$$

Step 2:

construct actions with Superforms-Ectoplasms

Ectoplasms: generalities

- closed super p-form $J = E^{\underline{A}_p} \wedge \cdots \wedge E^{\underline{A}_1} J_{\underline{A}_1 \cdots \underline{A}_p}$, $(E^{\underline{A}} = \mathrm{d}z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}})$

$$0 = (\mathrm{d}J)_{\underline{A}_1 \cdots \underline{A}_p \underline{A}_{p+1}} = \frac{1}{p!} \nabla_{[\underline{A}_1} J_{\underline{A}_2 \cdots \underline{A}_{p+1})} - \frac{1}{2((p-1)!)} T_{[\underline{A}_1 \underline{A}_2|}{}^{\underline{B}} J_{\underline{B}|\underline{A}_3 \cdots \underline{A}_{p+1})}$$

- if $p = d$, dimension of the bosonic body of the superspace, then

$$S = \int_{\mathcal{M}^d} J = \int \mathrm{d}^d x \, {}^*J|_{\theta=0} \ , \qquad {}^*J = \frac{1}{d!} \varepsilon^{\underline{m}_1 \cdots \underline{m}_d} J_{\underline{m}_1 \cdots \underline{m}_d}$$

S invariant under superdiffeomorphism $\xi = \xi^{\underline{A}} E_{\underline{A}} = \xi^{\underline{M}} \partial_{\underline{M}}$,

$$\delta_\xi J = \mathcal{L}_\xi J \equiv i_\xi \mathrm{d}J + \mathrm{d} i_\xi J = \mathrm{d} i_\xi J \ .$$

- By moving to flat indices the action is equivalently written as

$$S = \int \mathrm{d}^d x \frac{1}{d!} \varepsilon^{\underline{m}_1 \cdots \underline{m}_d} E_{\underline{m}_d}{}^{\underline{A}_d} \cdots E_{\underline{m}_1}{}^{\underline{A}_1} J_{\underline{A}_1 \cdots \underline{A}_d}|_{\theta=0} \ ,$$

$$S = \int \mathrm{d}^d x \, \mathrm{e} \, \varepsilon^{\underline{a}_1 \cdots \underline{a}_d} \Big[J_{\underline{a}_1 \cdots \underline{a}_d} + c_2 \Psi_{\underline{a}_1}{}^{\underline{\alpha}_1} J_{\underline{\alpha}_1 \underline{a}_2 \cdots \underline{a}_d} + c_3 \Psi_{\underline{a}_1}{}^{\underline{\alpha}_1} \Psi_{\underline{a}_2}{}^{\underline{\alpha}_2} J_{\underline{\alpha}_1 \underline{\alpha}_2 \underline{a}_3 \cdots \underline{a}_d} + \cdots + c_d \Psi_{\underline{a}_1}{}^{\underline{\alpha}_1} \cdots \Psi_{\underline{a}_d}{}^{\underline{\alpha}_d} J_{\underline{\alpha}_1 \cdots \underline{\alpha}_d} \Big] |_{\theta=0} \ ,$$

Ectoplasms: generalities

- To have an action invariant under the full supergravity gauge group one needs also invariance under the tangent space structure group X
- we need closed forms such that

$$\delta_X J = \mathrm{d} Y(X) \ , \qquad \text{for some } (d-1) - \text{form} \quad Y(X)$$

$$\Downarrow$$

the ectoplasm action S is invariant under X-transformations
and then full local supergravity gauge transformation $\mathcal{K}$

3D Conformal SUGRA actions: $\mathcal{N}$-extended CS term

- From idea of the component in 80s, the first natural object to start with is the Chern-Simon 3-form for $\mathrm{OSp}(\mathcal{N}|4,\mathbb{R})$
- denote the $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$ generators collectively by $X_{\tilde{a}}$, $f_{\tilde{a}\tilde{b}}{}^{\tilde{c}}$ are the structure constants

$$[X_{\tilde{a}}, X_{\tilde{b}}\} = -f_{\tilde{a}\tilde{b}}{}^{\tilde{c}} X_{\tilde{c}} \ , \quad f_{\tilde{a}\tilde{b}}{}^{\tilde{c}} = -(-1)^{\varepsilon_{\tilde{a}}\varepsilon_{\tilde{b}}} f_{\tilde{b}\tilde{a}}{}^{\tilde{c}} \ .$$

- Cartan-Killing metric:

$$\Gamma_{\tilde{a}\tilde{b}} = f_{\tilde{a}\tilde{d}}{}^{\tilde{c}} f_{\tilde{b}\tilde{c}}{}^{\tilde{d}} (-1)^{\varepsilon_{\tilde{c}}} \ , \qquad \Gamma_{\tilde{a}\tilde{b}} = (-1)^{\varepsilon_{\tilde{a}}\varepsilon_{\tilde{b}}} \Gamma_{\tilde{b}\tilde{a}}$$

- With Cartan-Killing construct a gauge invariant closed four-form:

$$\langle R^2 \rangle := R^{\tilde{b}} \wedge R^{\tilde{a}} \Gamma_{\tilde{a}\tilde{b}} \ , \quad \mathrm{d}\langle R^2 \rangle = 0 \ .$$

- Chern-Simons three-form for $\mathfrak{osp}(\mathcal{N}|4,\mathbb{R})$

$$\Sigma_{\rm CS} = R^{\tilde{b}} \wedge \omega^{\tilde{a}} \Gamma_{\tilde{a}\tilde{b}} + \frac{1}{6} \omega^{\tilde{c}} \wedge \omega^{\tilde{b}} \wedge \omega^{\tilde{a}} f_{\tilde{a}\tilde{b}\tilde{c}} \ , \quad \mathrm{d}\Sigma_{\rm CS} = \langle R^2 \rangle \ .$$

in general not closed

Conformal SUGRA actions: $\mathcal{N}$-extended CS term and Σ_R

- Philosophy is then to search for a "curvature induced" 3-form Σ_R

$$\mathrm{d}\Sigma_R = \langle R^2 \rangle$$

  constructed only out of the covariant fields of conformal superspace

- If Σ_R exists, then we have found a closed 3-form $\mathfrak{J}$

$$\mathfrak{J} = \Sigma_{\mathrm{CS}} - \Sigma_R \ , \quad \mathrm{d}\mathfrak{J} = 0$$

  that gives the action of $\mathcal{N}$-extended conformal supergravity.

$\mathcal{N}$-extended curvature induced form

- want Σ_R only constructed in terms of W^{IJKL} and its derivatives

the only possible ansatz for the lowest components of Σ_R is

$$\Sigma_{\alpha}^{I}{}_{\beta}^{J}{}_{\gamma}^{K} = 0 \ , \quad \Sigma_{a}{}_{\beta}^{J}{}_{\gamma}^{K} = \mathrm{i}\,(\gamma_a)_{\beta\gamma}\Big(A\,\delta^{JK}\,W^{ILPQ}\,W_{ILPQ} + B\,W^{LPQJ}\,W_{LPQ}{}^{K}\Big) \ ,$$

Plug in $\mathrm{d}\Sigma_R =< R^2 >$ at lowest mass dimension you get

$$0 = E_L^\delta E_K^\gamma E_J^\beta E_I^\alpha\,\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta}\Big(-W^{PQIJ}\,W^{KL}{}_{PQ} + A\,W^{PQRS}\,W_{PQRS}\,\delta^{J[K}\,\delta^{L]I} + B\,W^{PQRJ}\,W_{PQR}{}^{[K}\,\delta^{L]I}\Big)$$

first term contains a double traceless contribution of the form

$$\Big(\delta^R_{[K}\delta^{[I}_{|S|} - \frac{1}{\mathcal{N}}\delta^R_S\delta^{[I}_{[K}\Big)\Big(\delta^{|T|}_{L]}\delta^{J]}_U - \frac{1}{\mathcal{N}}\delta^{J]}_{L]}\delta^T_U\Big)\,W^{SUPQ}\,W_{RTPQ}$$

which cannot be cancelled by the Ansatz for $\mathcal{N} > 5$...
but $\mathcal{N} = 3, 4, 5$ work easily

$\mathcal{N} = 5$ curvature induced form

rewrite super Cotton tensor

$$W^{IJKL} = \varepsilon^{IJKLP} W_P \ , \qquad W^2 := W^I W_I \ .$$

then $\boldsymbol{\Sigma}_R$ for $\mathcal{N} = 5$ has components

$$\boldsymbol{\Sigma}_{\alpha}^{I}{}_{\beta}^{J}{}_{\gamma}^{K} = 0 \ ,$$

$$\boldsymbol{\Sigma}_{a}{}_{\beta}^{J}{}_{\gamma}^{K} = -\mathrm{i}(\gamma_a)_{\beta\gamma}\Big(4\delta^{JK} W^2 - 8W^J W^K\Big) \ ,$$

$$\boldsymbol{\Sigma}_{ab}{}_{\delta}^{L} = -4\varepsilon_{abc}(\gamma^c)_\delta{}^\rho\Big((\nabla_\rho^{[L} W^{S]})W_S - \frac{1}{5}(\nabla_\rho^P W_P)W^L\Big) \ ,$$

$$\boldsymbol{\Sigma}_{abc} = -\mathrm{i}\varepsilon_{abc}\Big(\frac{2}{25}(\nabla^{\rho P} W_P)(\nabla_\rho^Q W_Q) - (\nabla^\rho_{[P} W_{Q]})(\nabla_\rho^{[P} W^{Q]}) - \frac{2}{3}(\nabla^\rho_P \nabla_\rho^P W^S)W_S\Big) \ .$$

$\mathcal{N} = 5$ conformal SUGRA action

- use superform to get a invariant action;
- The 3-form $\mathfrak{J} = \mathbf{\Sigma}_{\rm CS} - \mathbf{\Sigma}_R$, gives exactly the action for $\mathcal{N} = 5$ off-shell conformal supergravity
- perform standard components reduction;
- For $\mathcal{N} = 5$ we have defined our auxiliary fields as

$$w_I := \frac{1}{4!}\varepsilon_{IJKLP}w^{JKLP} = W_I| \ , \qquad y_I := \frac{1}{4!}\varepsilon_{IJKLP}y^{JKLP} = -\frac{\rm i}{3}\nabla^\gamma_P\nabla^P_\gamma W_I|$$

$$w_\alpha{}^{IJ} := \frac{1}{3!}\varepsilon^{IJKLP}w_{\alpha KLP} = -\frac{\rm i}{2}\nabla^{[I}_\alpha W^{J]}| \ ,$$

$$X_\alpha := \frac{1}{5!}\varepsilon_{IJKLP}X_\alpha{}^{IJKLP} = \frac{\rm i}{5}\nabla^I_\alpha W_I| \ .$$

$\mathcal{N} = 5$ conformal SUGRA action

The off-shell $\mathcal{N} = 5$ action is

$$
\begin{aligned}
S = \frac{1}{4}\int \mathrm{d}^3x\, e\Big\{\varepsilon^{abc}\Big(&\omega_a{}^{fg}\mathcal{R}_{bc\,fg} - \frac{2}{3}\omega_{af}{}^{g}\omega_{bg}{}^{h}\omega_{ch}{}^{f} - 2\mathcal{R}_{ab}{}^{IJ}V_{c\,IJ} - \frac{4}{3}V_a{}^{IJ}V_{bI}{}^{K}V_{c\,KJ} \\
&- \frac{\mathrm{i}}{2}\Psi_{bc}{}_I^{\alpha}(\gamma_d)_\alpha{}^\beta(\gamma_a)_\beta{}^\gamma\varepsilon^{def}\Psi_{ef}{}^I_\gamma\Big) \\
&+ 8\mathrm{i}X^\alpha X_\alpha - 16\mathrm{i}w^{\alpha IJ}w_{\alpha IJ} - 8w^I y_I \\
&- 8\mathrm{i}\,\psi_a{}_I^\alpha(\gamma^a)_\alpha{}^\beta\big(2w_\beta{}^{IJ}w_J + X_\beta w^I\big) \\
&- 2\mathrm{i}\varepsilon^{abc}(\gamma_a)_{\alpha\beta}\psi_b{}_I^\alpha\psi_c{}_J^\beta(\delta^{IJ}w^K w_K - 2w^I w^J)\Big\}\ .
\end{aligned}
$$

action for $\mathcal{N} = 1, 2, 3, 4$ computed analogously
or by truncation of the $\mathcal{N} = 5$ action
$\mathcal{N} = 6$ action require some generalization but can be done

Conclusion

- We constructed $\mathcal{N} = 1, 2, 3, 4, 5, 6$ conformal SUGRA action in 3D
- construction based on a new formulation of $\mathcal{N}$-extended conformal SUGRA in "conformal superspace" and "Ectoplasms"

Some open problems:

- Construction of non-linear $\mathcal{N} = 3, 4, 5, 6$ TMG. extention to GMG
- What about $\mathcal{N} > 6$?