Electromagnetic spikes

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Overview

- Heuristic picture of initial singularity
- What is a Bianchi spacetime?
- What is a Gowdy spacetime?
- The electromagnetic spike solution

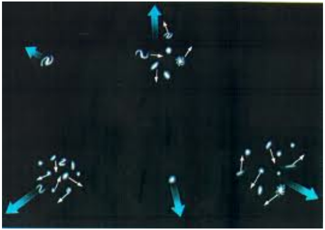
The basic scenario and the Einstein equations

- A spacetime is a manifold together with a Lorentzian metric $(M, g_{\alpha\beta})$; signature $-+++$
- In mathematical cosmology one usually assumes $M = I \times S$ where S is **spatially compact**
- The units are chosen such that the velocity of light and Newtons gravitational constant equal to one ($c = G = 1$):

$$G_{\alpha\beta} + g_{\alpha\beta}\Lambda = 8\pi T_{\alpha\beta}$$

  where $T_{\alpha\beta}$ is the energy-momentum tensor which describes the **matter content**. Usually it is divergence free (Conservation of energy-momentum).

The Universe as a fluid (standard model)



The equation of state $P = f(\rho)$ relates the pressure P with the energy density ρ. The velocity of the fluid/observer is u^α

$$T_{\alpha\beta} = (\rho + P)u_\alpha u_\beta + P g_{\alpha\beta}$$

The Euler equations of motion coincide with the requirement that $T_{\alpha\beta}$ has to be divergence-free. Usually one assumes a linear relation:

$$P = (\gamma - 1)\rho$$

Standard picture about behavior at the initial singularity

- Hawking and Penrose theorems tell us geodesical incompleteness under general physically reasonable assumptions
- But what happens? **BKL**-picture [Belinskii, Khalatnikov, Lifshitz]
  1. homogeneous ("Mixmaster"):
  2. Vacuum dominated ("matter does not matter")
  3. oscillatory ("chaotic")
- ...has probably to be refined

A **generic** singularity is characterized by asymptotic locality. Each spatial point evolves **independently** from its neighbors in an **oscillatory** manner represented by a sequence of **Bianchi type I and II vacuum** models.

Why study homogeneous spacetimes?

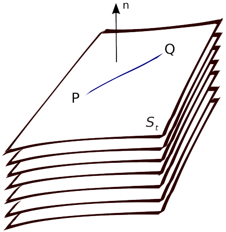
- Simplifying assumption
- Generalizes Friedman-Lemaître-Robertson-Walker spacetimes
- Are believed to play an important role towards the singularity acting as building blocks

Homogeneous and isotropic spacetimes:

- FLRW closed ($k = 1$) $\subset$ Bianchi IX
- FLRW flat ($k = 0$) $\subset$ Bianchi I and VII_0
- FLRW open ($k = -1$) $\subset$ Bianchi V and VII_h with $h \neq 0$

What is a Bianchi spacetime?

- Definition of a **homogeneous spacetime**: A spacetime is said to be (spatially) **homogeneous** if there exist a one-parameter family of spacelike hypersurfaces S_t foliating the spacetime such that for each t and for any points $P, Q \in S_t$ there exists an **isometry** of the spacetime metric ${}^4g$ which takes P into Q
- Definition of **Bianchi spacetime**: it is defined to be a **spatially homogeneous** spacetime whose isometry group possesses a 3-dim subgroup G that acts **simply transitively** on the spacelike orbits.
- *"Existence and uniqueness of an isometry group which possesses a 3-dim subgroup"*

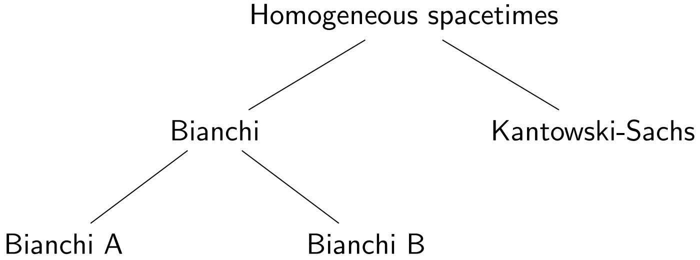


Lie group structure of Bianchi spacetimes

- Manifold structure is $M = I \times G$, G being a **Lie group**
- Bianchi spacetimes have 3 Killing vectors ξ and they can be classified by the **structure constants** C^i_{jk} of the associated **Lie algebra**: $[\xi_j, \xi_k] = C^i_{jk}\xi_i$
- They fall into 2 catagories: A and B. Bianchi class A is equivalent to $C^i_{ji} = 0$ (unimodular). In this case $\exists$ unique symmetric matrix with components ν^{ij} such that $C^i_{jk} = \epsilon_{jkl}\nu^{li}$
- Relation to Geometrization of 3-manifolds and Thurstons classification [cf. Ringström (2013)]

Type	ν_1	ν_2	ν_3
I	0	0	0
II	1	0	0
VI_0	0	1	-1
VII_0	0	1	1
VIII	-1	1	1
IX	1	1	1

Subclasses of homogeneous spacetimes



"New" variables

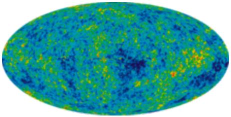
- Decompose the second fundamental form

$$\boxed{k_{ab} = \sigma_{ab} - H g_{ab}}$$

where $H = -\frac{1}{3}k$ is the Hubble parameter ('Expansion velocity').

- Shear variables ('Anisotropy')

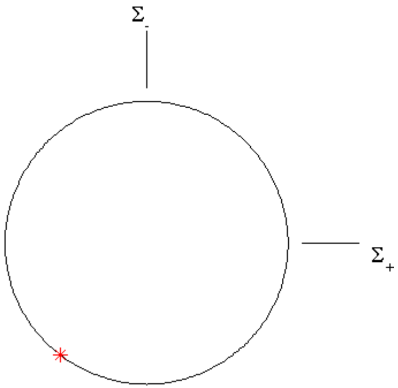
$$\boxed{\Sigma_+ = -\frac{\sigma_2^2 + \sigma_3^3}{2H}; \quad \Sigma_- = -\frac{\sigma_2^2 - \sigma_3^3}{2\sqrt{3}H}; \quad F = \frac{1}{4H^2}\sigma_{ab}\sigma^{ab}}$$



Microwave background (WMAP)

The Kasner solution (Bianchi I vacuum)

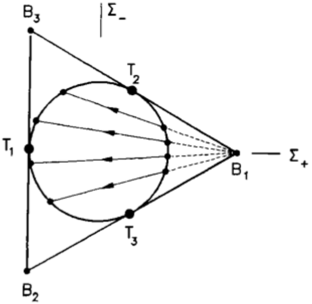
$$
\begin{aligned}
\Sigma_+^2 + \Sigma_-^2 &= 1 \\
p_1 + p_2 + p_3 &= 1 \\
p_1^2 + p_2^2 + p_3^2 &= 1 \\
g_{ij} &= \mathrm{diag}(t^{2p_1}, t^{2p_2}, t^{2p_3})
\end{aligned}
$$



Taub solution (Bianchi II vacuum)

$$\begin{aligned}
\Sigma_+' &= -(2-q)\Sigma_+ + \frac{1}{3}N_1^2 \\
\Sigma_-' &= -(2-q)\Sigma_- \\
N_1' &= (q - 4\Sigma_+)N_1 \\
q &= \frac{1}{2}(3\gamma - 2)(1 - \frac{1}{12}N_1^2) + \frac{3}{2}(2-\gamma)(\Sigma_+^2 + \Sigma_-^2)
\end{aligned}$$

For the vacuum boundary one obtains that $\Sigma_+' = -(2-q)(\Sigma_+ - 2)$ and as a consequence: $\Sigma_- = k(\Sigma_+ - 2)$



Gowdy spacetimes

- Robert Gowdy was the first to systematically analyze the consequences of a two-dimensional isometry group with space-like orbits (before usually the stationary axisymmetric case was considered)
- Model closed empty universes filled with gravitational waves of two polarizations
- Factoring by the symmetry leads to an $1+1$ dim effective equations
- One of the best model systems for obtaining a mathematical understanding of the dynamics of inhomogeneous solutions of the Einstein equations, also for quantum approaches

Gowdy symmetric spacetimes on T^3

- one assumes a compact and connected 2-dimensional Lie group acting effectively on the initial hypersurface. Since the initial hypersurface is a 3dim manifold
- $==>$ the Lie group has to be T^2, i.e. $U(1) \times U(1)$
- The symmetry assumptions imply that the topology has to be T^3, $S^2 \times S^1$, S^3 or the Lens space
- two commuting spacelike Killing vectors, say $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ on T^3 $==>$ T^2 symmetry
- it has Gowdy symmetry if the transformation which simultaneously maps x to $-x$ and y to $-y$ is an isometry.
- it is said to be polarized if the individual mappings are isometries $==>$ polarized Gowdy symmetry

Spikes in Gowdy symmetric spacetimes on T^3

- where studied numerically by Berger and Moncrief and they observed the development of large spatial derivatives near the singularity, which they called spiky features
- These structures where found to occur in the neighborhood of isolated spatial surfaces
- 2001 Weaver and Rendall used a solution generating technique and Fuchsian methods to produce asymptotic expansion for spikes, which where classied in true and false spikes, where the latter are only a rotation artifact
- 2008 Lim found an explicit spike solution in terms of elementary functions. The explicit spike solution suggests a new way to simulate spikes numerically

Lim's solution

Applying solution generating transformations. First inversion in the hyperbolic plane and then Gowdy to Ernst transformation. Then again both.

Kasner (Bianchi I vacuum)

Kasner viewed in a rotated frame

Taub (Bianchi II vacuum)

Taub rotating in a spiky manner

spike solution

"Hidden" symmetry

If $(\bar{P}, \bar{Q}, \bar{\lambda})$ is a solution of the vacuum Einstein equations with Gowdy symmetry, i.e. with line element

$$ds^2 = -e^{(\bar{\lambda}-3\tau)/2}(d\tau^2 + e^{2\tau}dx^2) + e^{-\tau}[e^{\bar{P}}(dy + \bar{Q}dz)^2 + e^{\bar{P}}dz^2],$$

then (P, χ, λ)

$$P = 2\bar{P} - \tau \quad \lambda = 4\bar{\lambda} + 4\bar{P} - \tau, \quad \chi = \bar{Q},$$

will be a solution of the Einstein-Maxwell equations with polarized Gowdy symmetry with line element:

$$ds^2 = e^{(\lambda-3\tau)/2}(d\tau^2 + e^{2\tau}dx^2) + e^{-\tau}(e^{P}dy^2 + e^{-P})dz^2,$$

and a Maxwell field described by the vector potential with only one non-zero component, namely $A_3 = \chi(x, \tau)$.

Electromagnetic spike solution

Kasner with constant electromag. potential

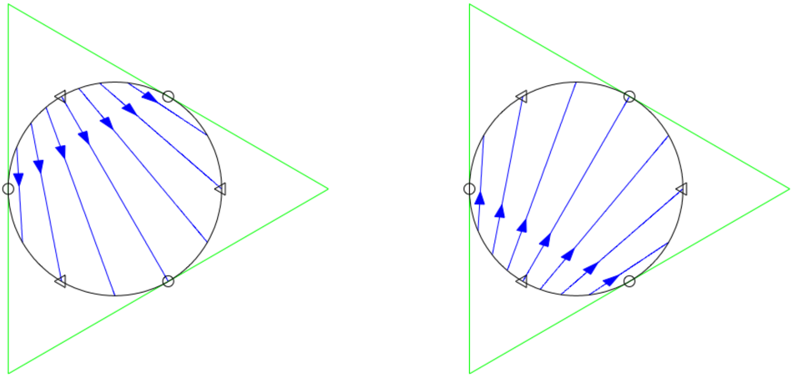
Electric Rosen (Bianchi I with electric field)

Magnetic Rosen

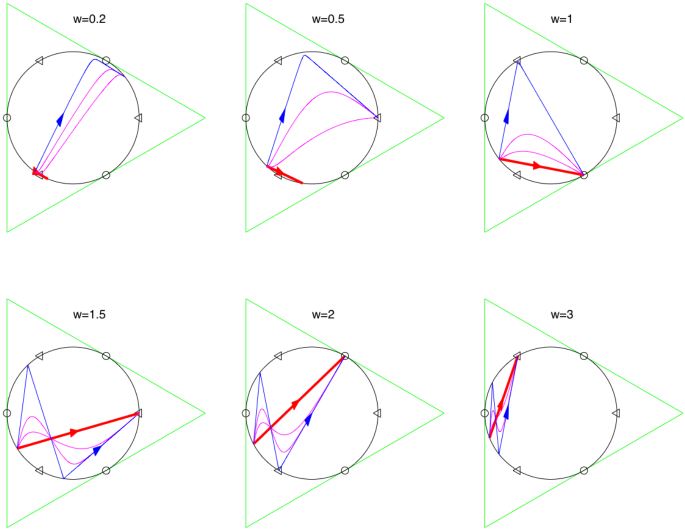
EME electromagnetic spike

MEM electromagnetic spike

Rosen electric and magnetic



MEM Electromagnetic spike



Outlook

- Recurring spikes might be an alternative to the inflationary mechanism for generating matter perturbations and thus act as seeds for the subsequent formation of large scale structure
- Analysis of full Gowdy with Maxwell
- electromagnetic spikes might help to understand structure formation with primordial magnetic fields
- Twisted Gowdy