ADHM: Quivers and monopoles

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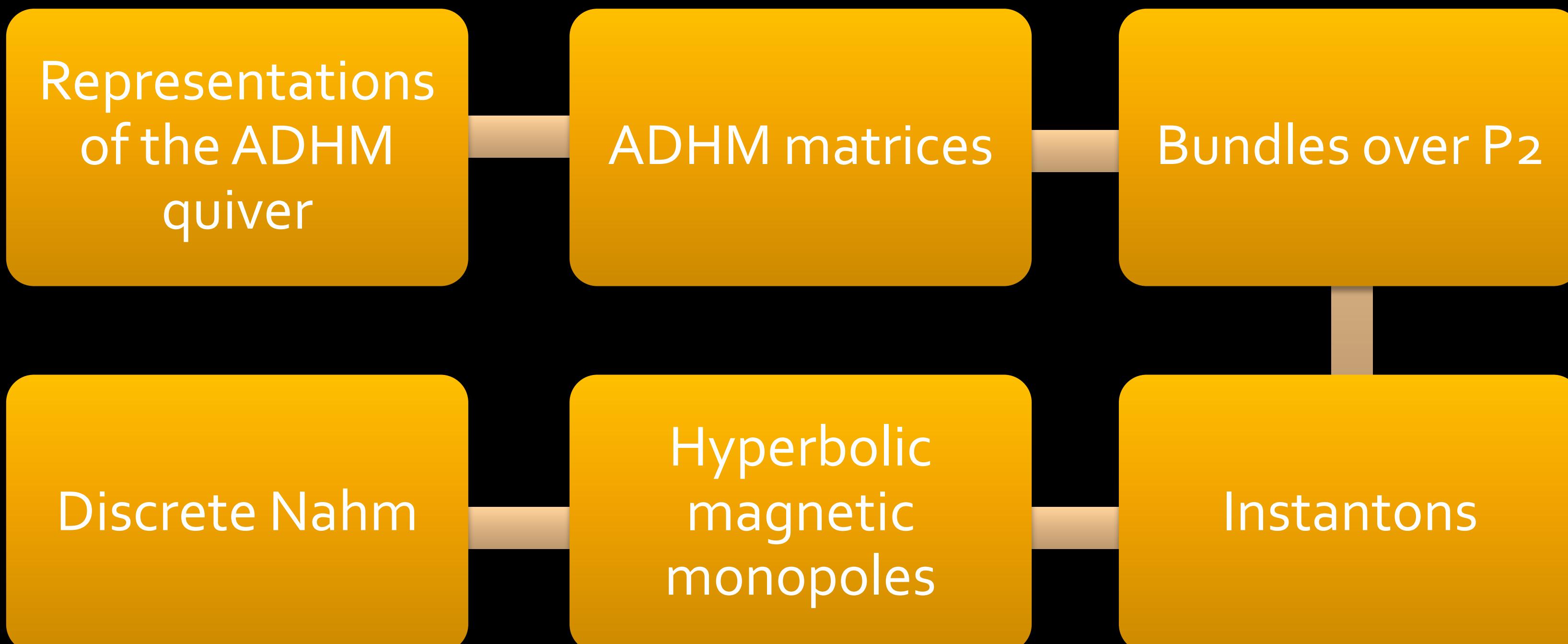
Outline


Representations of the ADHM quiver
ADHM matrices
Bundles over P2
Instantons
Hyperbolic magnetic monopoles
Discrete Nahm

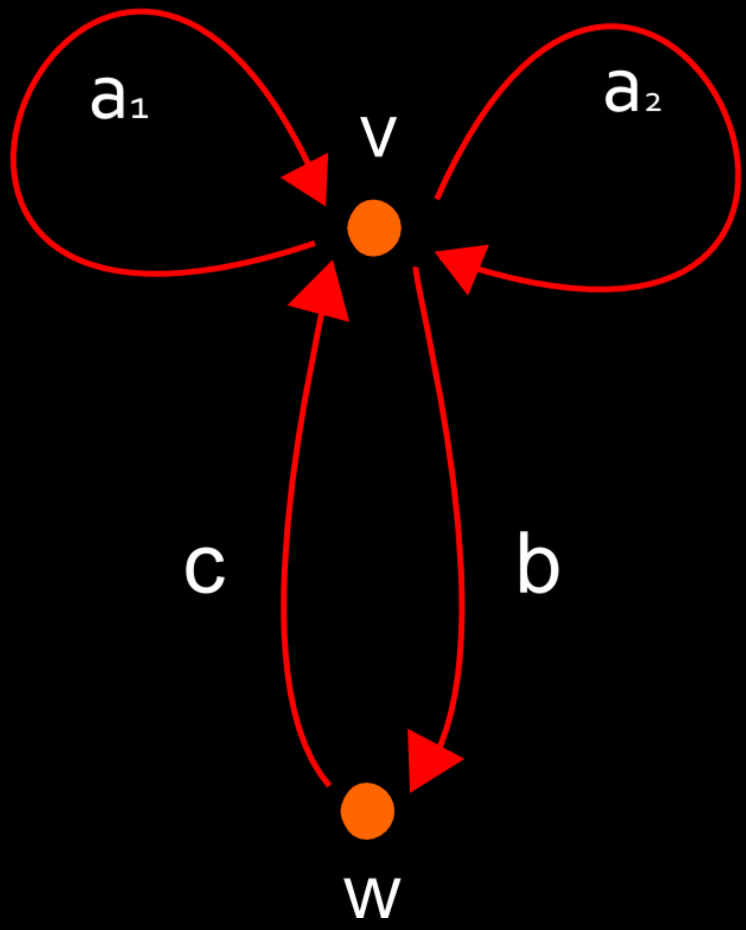
ADHM construction

Atiyah-Drinfeld-Manin-Hitchin Construction (1978).

ADHM quiver

Nakajima (1994).

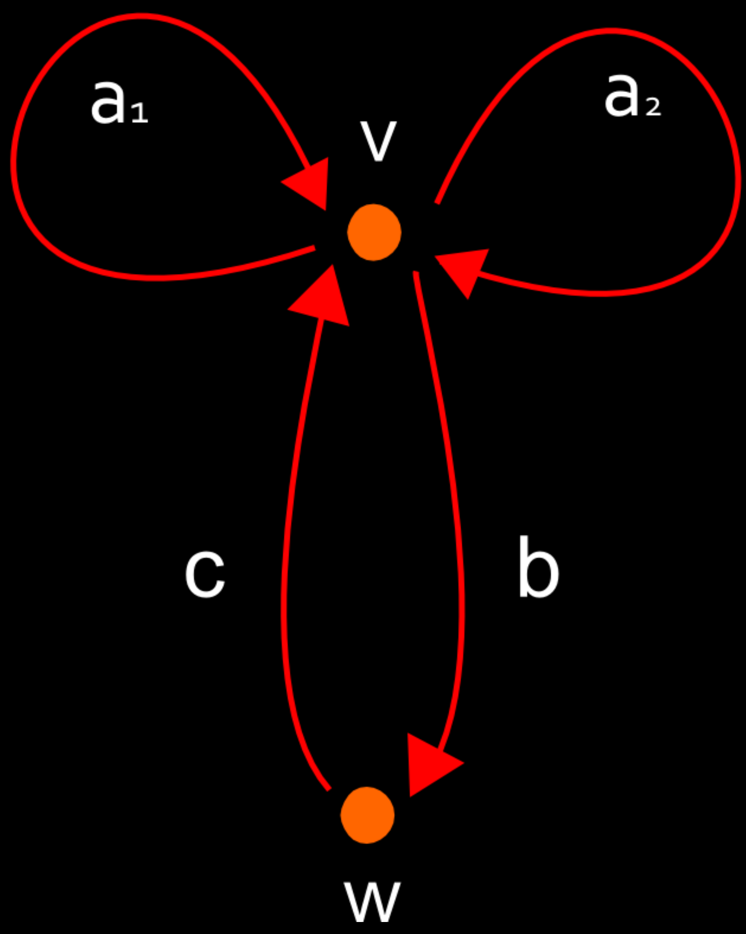
Active area of research.



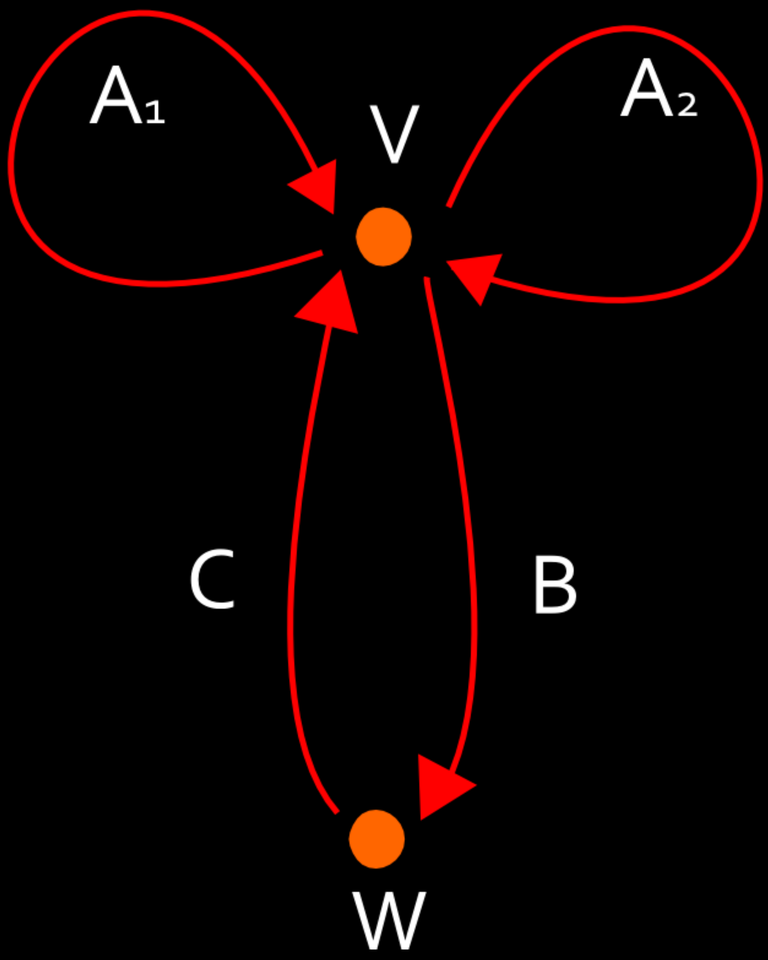
ADHM quiver

- Vertices v,w
- Directed edges a1, a2, b, c
- The relation:

$$[a_1, a_2] + cb = 0$$

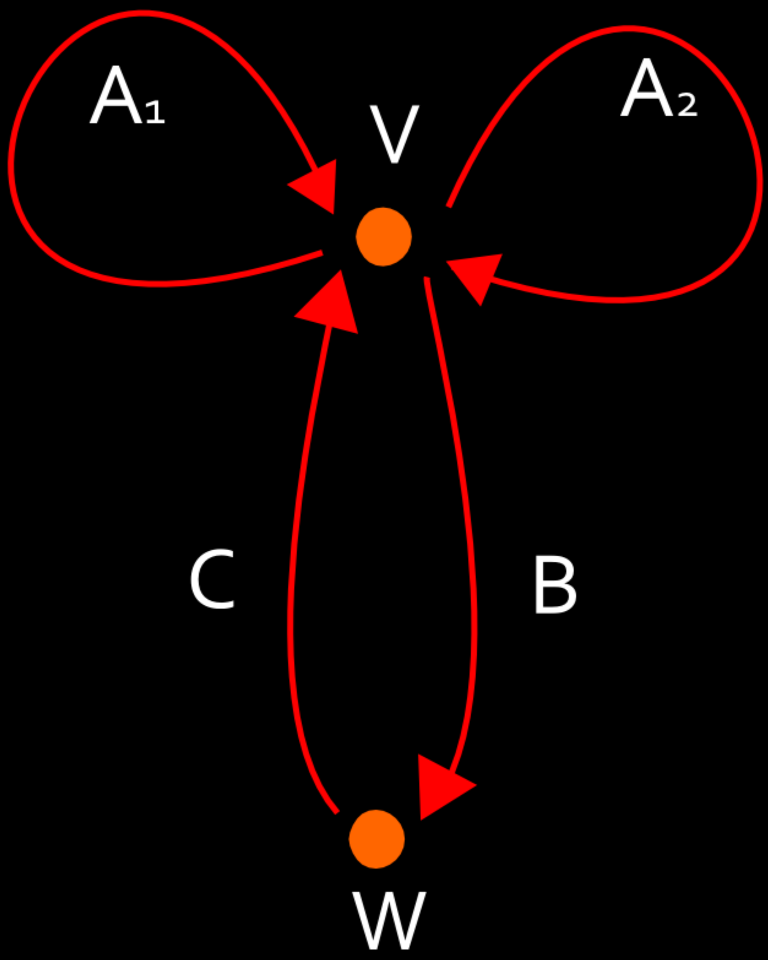


ADHM quiver representations



n=(dim V, dim W)=(k,l)

ADHM quiver representations



n=(dim V, dim W)=(k,l)

R=(A_1,A_2,B,C)

A_1:V→V

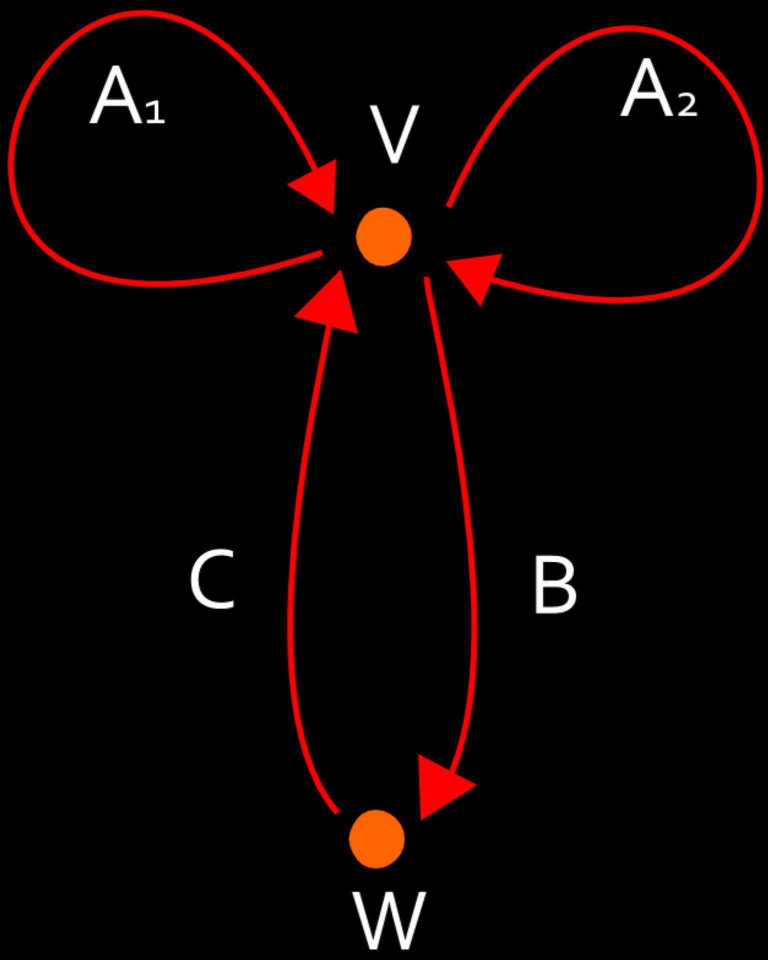
A_2:V→V

B:V→W

C:W→V

$$[A_1, A_2] + CB = 0$$

ADHM quiver representations



n=(dim V, dim W)=(k,l)

$R=(A_1,A_2,B,C)$

$A_1: V \to V$

$A_2: V \to V$

$B: V \to W$

$C: W \to V$

$$[A_1, A_2] + CB = 0$$

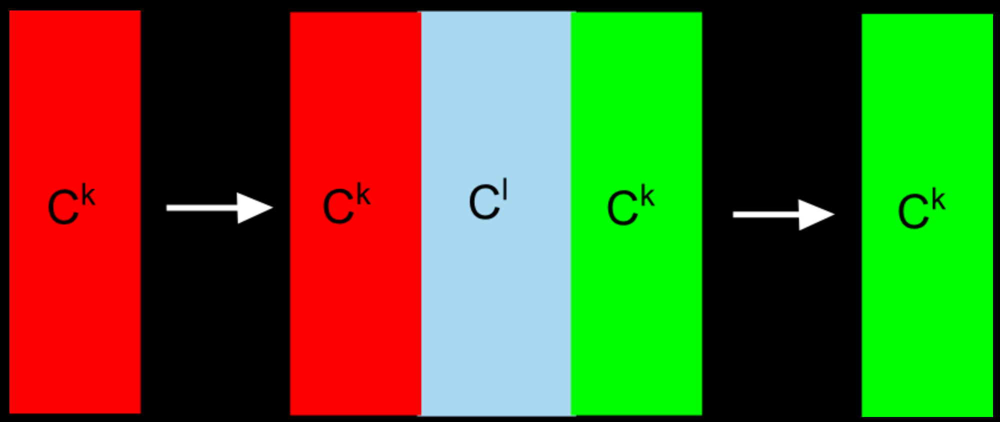
Moduli R//GL(k)

Monads on P²

Donaldson (1984):

Instantons over $R^4 \leftrightarrow$ Vector bundles over P^2

(with framing, ADHM relation)



$E \longrightarrow \mathbb{P}^2$ with fibres $\ker G_X / \mathrm{im}\ F_X = E|_X$

Monads on P²

$[x : y : z] \in \mathbb{P}^2$

Monads on P²

$[x : y : z] \in \mathbb{P}^2$

$$F_X = F_x x + F_y y + F_z z$$

$$F_x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} F_y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} F_z = \begin{bmatrix} A_1 \\ A_2 \\ B \end{bmatrix}$$

Monads on P²

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$$G_X = G_x x + G_y y + G_z z$$

$$G_x = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} G_y = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix} G_z = \begin{bmatrix} -A_2 & A_1 & C \end{bmatrix}$$

Hyperbolic monopoles

Atiyah (1984).

$$\mathbb{R}^4 - \mathbb{R}^2 = S^1 \times H^3$$

Definition:

Magnetic monopoles on H^3 $\leftrightarrow$ Circle invariant action on R^4

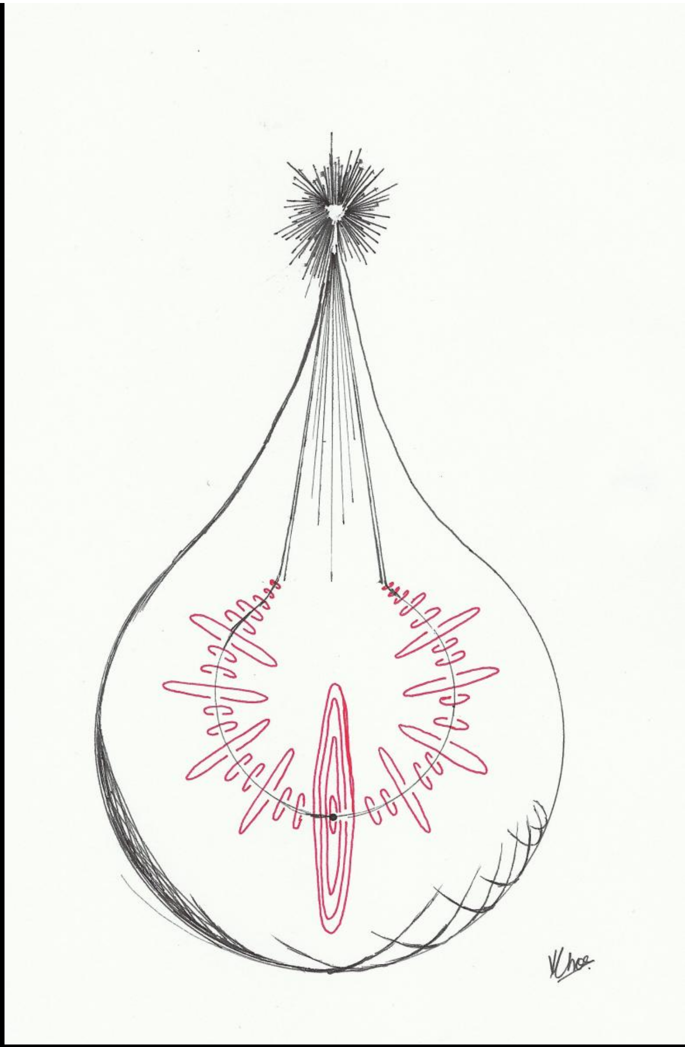
Hyperbolic monopoles

Atiyah (1984).

$$\mathbb{R}^4 - \mathbb{R}^2 = S^1 \times H^3$$

Easier:

$$\mathbb{R}^3 - \mathbb{R} = S^1 \times H^2$$



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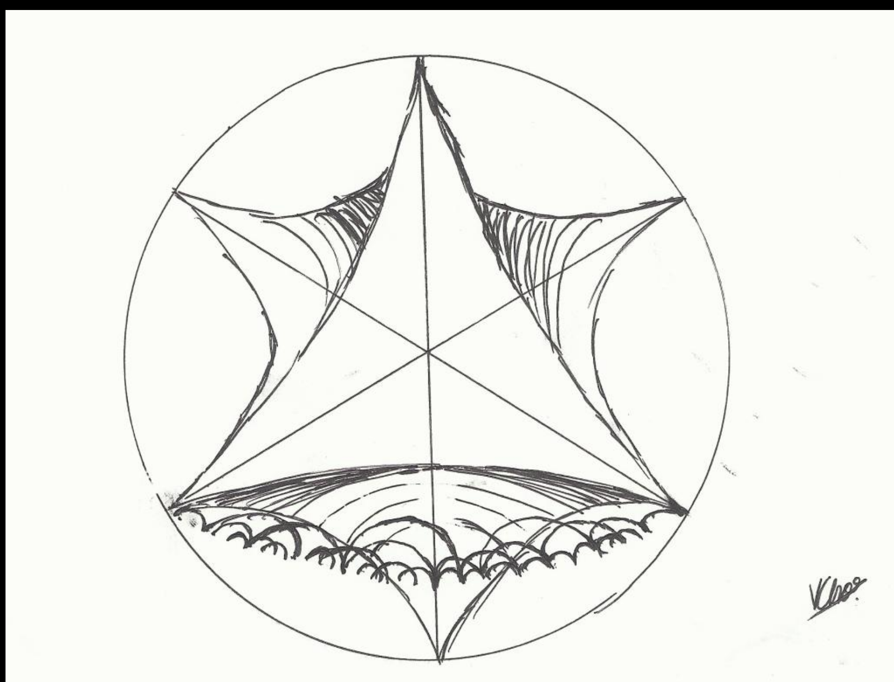
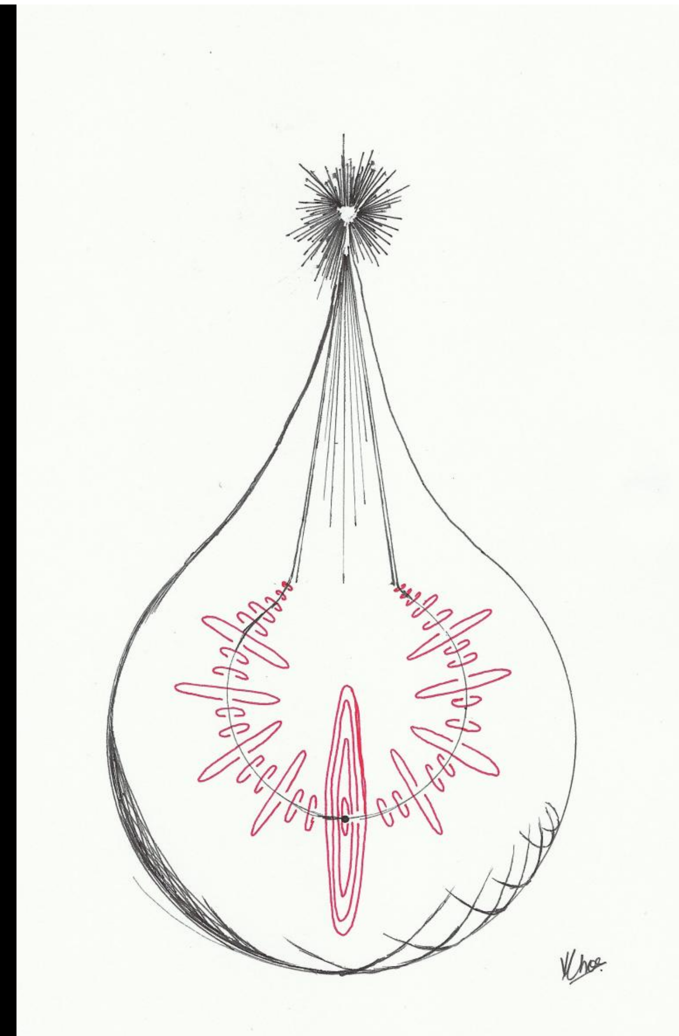




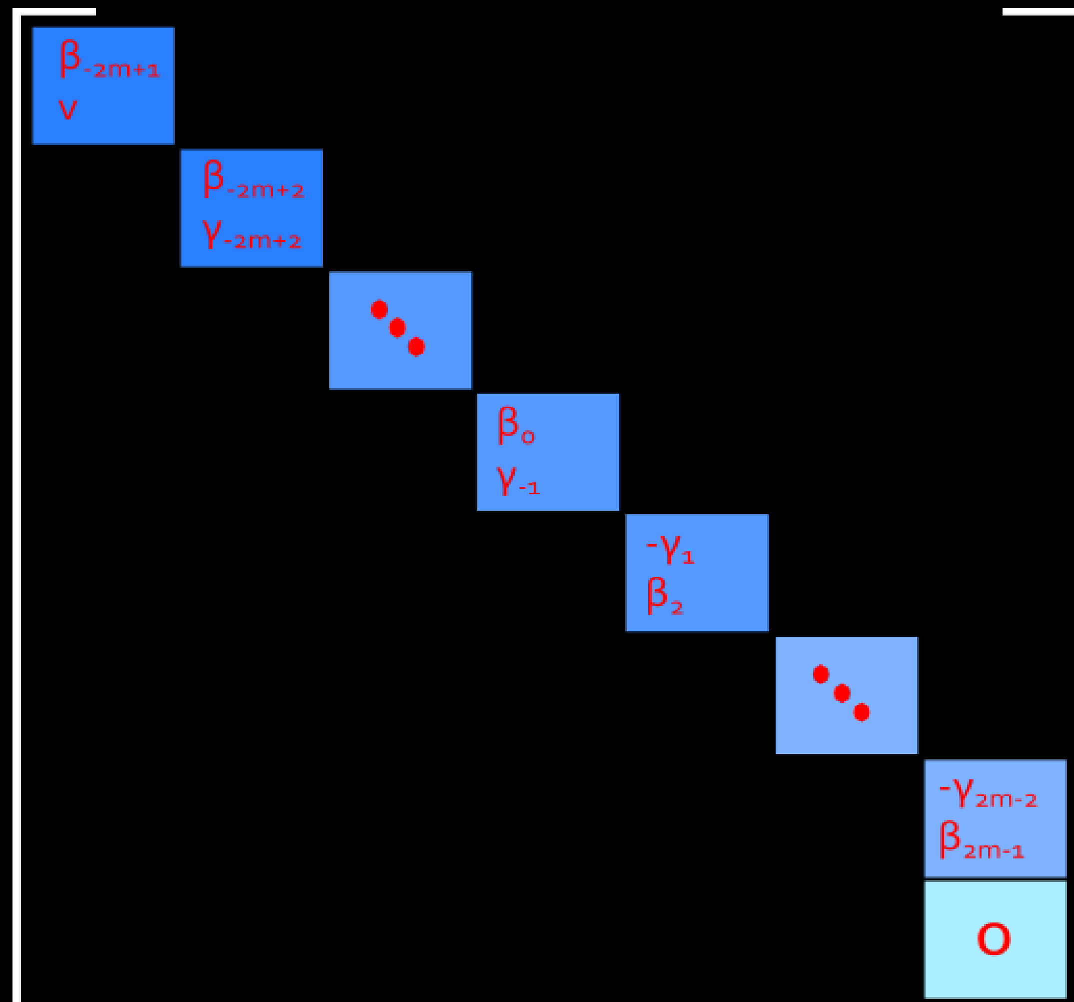
Austin-Braam

Weight space decomposition for the circle action in the SU(2) case:

$$\mathbb{C}^{2k+l} = \mathbb{C}^{k+1}_{2m} \oplus \mathbb{C}^{2k}_{2m-2} \oplus \cdots \oplus \mathbb{C}^{2k}_{-2m+2} \oplus \mathbb{C}^{k+1}_{-2m}$$

Austin-Braam

$$F_z =$$



Nahm and Discrete Nahm

Discrete Nahm

$$\beta_{j-1}\gamma_j - \gamma_j\beta_{j+1} = 0$$

$$[\beta_j^*, \beta_j] + \gamma_{j-1}^*\gamma_{j-1} - \gamma_{j+1}\gamma_{j+1}^* = 0$$

Hyperbolic magnetic monopoles
SU(2) only

Nahm and Discrete Nahm

Discrete Nahm

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Hyperbolic magnetic monopoles
SU(2) only

Nahm's Equations

$$\frac{d\tau}{dz} = [\sigma, \tau]$$

$$\frac{d}{dz}(\sigma + \sigma^*) = [\sigma, \sigma^*] + [\tau, \tau^*]$$

Euclidean magnetic monopoles
SU(n)