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DBI potential, DBI inflation action and general Lagrangian relative to phantom, K-essence and quintessence

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Abstract. We derive a Dirac-Born-Infeld (DBI) potential and DBI inflation action by rescaling the metric. The determinant of the induced metric naturally includes the kinetic energy and the potential energy. In particular, the potential energy and kinetic energy can convert into each other in any order, which is in agreement with the limit of classical physics. This is quite different from the usual DBI action. We show that the Taylor expansion of the DBI action can be reduced into the form in the non-linear classical physics. These investigations are the support for the statement that the results of string theory are consistent with quantum mechanics and classical physics. We deduce the Phantom, K-essence, Quintessence and Generalized Klein-Gordon Equation from the DBI model.

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Contents

1 Introduction

String theory can impose a specific non-trivial kinetic behavior through the Dirac-Born-Infeld (DBI) action that arises naturally in consideration of D_p-brane motion within a warped compactification [1–3]. The field properties are related to the geometric position of a three-dimensional brane within higher dimensions, and the brane tension and potential functions are (in principle) given by string theory, through the AdS/CFT correspondence [4–10].

Field theories of the DBI type have attracted much attention in recent years, which is due to their critical role in inflationary models based on string theory [11–15]. These scenarios indicate that the inflaton relates with the D_p-brane moving on a 6-dimensional compact submanifold of spacetime, which means that the inflaton is interpreted as an open string mode. This interpretation of the inflaton implies that the effective field theory is well motivated by string computations.

Since the dynamics of a D_p-brane is described by a DBI action in string theory and characterized by a nonstandard kinetic term, the inflation could turn out with steep potentials in contrast with usual slow-roll inflation. Many models of the inflation are based on the motion of D-branes in a higher-dimensional spacetime with DBI action [7, 16–18]. The DBI inflation concludes a more general class, *i.e.*, k-inflation models [19–21].

Furthermore, DBI inflation has the additional nice feature of a natural ending when the branes collide, where the collision itself is useful for reheating and the possible production of cosmic strings [22–25]. String theory dictates both the dynamics of the inflation and its potential, so that one can make precise cosmological predictions from a given set of background parameters [26]. The calculability and the limited numbers of the parameters make brane inflation be an interesting arena to explore the possibilities for cosmology in string theory. Therefore, it is possible to set up a cosmic evolution model, which satisfies the data and is coincident with the fundamental theory. There has been a great deal of work done in a variety of inflationary scenarios and in understanding the inflationary fields [27–34].

In the review of [35], branes and antibranes or branes without the same supersymmetry are both present in different parts of the compact space M. The candidate inflaton is the distance between the branes and antibranes on M [36], while the inflationary potential is generated by interbrane Ramond-Ramond (RR) and gravitational forces. On the other hand, the exit from inflation can occur when the brane and antibrane reach a distance as the length of string from one another, where the lightest stretched string becomes tachyonic. Such brane inflation models were generalized and explored by many studies [37–44].

In this paper, we deduce the DBI potential and DBI Lagrangian, which are different from refs. [45–49]. We show that there always exists a general function potential in the DBI inflation action, and deduce the DBI inflation Lagrangian, where the determinant of the induced metric naturally includes the kinetic energy and the potential energy. In particular, the result is consistent with the form in the non-linear classical physics, since the kinetic energy and potential energy can convert into each other.

The arrangements of this paper are: section 2 is the Lagrangian of DBI inflation; section 3 is the general Lagrangian relative to Phantom, K-essence and Quintessence and corresponding generalized Klein-Gordon equation; the last section is summary and conclusion.

2 The Lagrangian of DBI inflation

In the usual DBI inflation scenario, the Lagrangian of the system is in the case of single field DBI inflation, where the determinant of the induced metric only contains kinetic energy. The Lagrangian of the usual DBI infaltion arises in type IIB string theory, in terms of the volume swept out by a D3-brane in a warped geometry coupled to gravity [25]. The DBI Lagrangian presenting in previous works ignores the possibility that the potential energy can convert into kinetic energy in the determinant. The potential energy and kinetic energy can not convert into each other at any order, which is not in agreement with the limit of classical physics.

On the other hand, the non-canonical kinetic term in DBI models leads to the loop corrections, which are enhanced by slow-varying parameter and small sound speed. Thus, in general the loop-corrections in multi-DBI models can be large [9]. String theory is a developing theory in particle physics that attempts to reconcile quantum mechanics and general relativity. String theory is a theory of everything, a manner of describing the known fundamental forces and matter in a mathematically complete system. And the results of string theory are consistent with quantum mechanics and classical physics.

In this section, we derive the generic DBI action different from the previous works [1–3], by rescaling the metric. In order to make potential energy and kinetic energy in this set-up consistently appear, we show that the determinant of the induced metric could naturally include the kinetic energy and potential energy. In this case, the potential energy and the kinetic energy can convert into each other, and match the limitation of classical mechanics.

In a general field DBI inflation, the field ϕ responsible for inflation is relative to the degree of freedom associated with a (3+1)-dimensional world volume with metric $\mathrm{d}s_4^2$ moving in a six-dimensional "throat", where the (3+1)-dimensional volume looks like a particle moving along the radial r and compacted by a 5-dimensional orbifold, the corresponding metric is [48]

$$\mathrm{d}s_{10}^2{=}h^2(r)\mathrm{d}s_4^2{+}h^{-2}(r)(\mathrm{d}r^2{+}r^2\mathrm{d}s_{x_5}^2). \tag{2.1}$$

By rescaling the metric, we generalize eq. (2.1) to a more general symmetry situation

$$\mathrm{d}s_{10}^2{=}h^2(r)U(r)\mathrm{d}s_4^2{+}h^{-2}(r)U(r)(\mathrm{d}r^2{+}r^2\mathrm{d}s_{x_5}^2). \tag{2.2}$$

For $U(r) = 1$, eq. (2.2) is reduced into eq. (2.1). In principle, our universe may exist in various parts of compactification, including other warped throats. The construction involves wrapped D-branes and orientifold planes [11, 33, 38].

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Then the induced metric on D_3-brane is

$$G_{\alpha\beta}=h^2(r)U(r)g_{\mu\nu}\frac{\partial x^\mu}{\partial\sigma^\alpha}\frac{\partial x^\nu}{\partial\sigma^\beta}+h^{-2}(r)U(r)\frac{\partial r}{\partial\sigma^\alpha}\frac{\partial r}{\partial\sigma^\beta}+h^{-2}(r)U(r)r^2g_{ab}\frac{\partial x^a}{\partial\sigma^\alpha}\frac{\partial x^b}{\partial\sigma^\beta}, \quad (2.3)$$

where $\mu,\nu,\alpha,\beta(=0,1,2,3)$ describe the geometry of our universe; $x^4=r$; $a,b=5,6,7,8,9$ are relative to the geometry of s_{x_5}. Because x^a,x^b are not dependent on $\sigma^\alpha,\sigma^\beta$, we have

$$h^{-2}(r)U(r)r^2g_{ab}\frac{\partial x^a}{\partial\sigma^\alpha}\frac{\partial x^b}{\partial\sigma^\beta}=0. \quad (2.4)$$

From the first term of eq. (2.3), the potential is equal to the product of the general scalar function and the prime Newton potential in the limit of classic physics, i.e. $g'_{00}=\Lambda g_{00}$. This potential comes from the geometry of the DBI background.

Therefore, we get the DBI action on D_3-brane as follows

$$\begin{aligned}S_{DBI}&=-T\int\mathrm{d}^4\sigma\sqrt{-\det\left[h^2(r)g_{\mu\nu}U(r)+h^{-2}(r)U(r)\frac{\partial r}{\partial\sigma^\mu}\frac{\partial r}{\partial\sigma^\nu}\right]}\\&=-T\int d^4\sigma\sqrt{-\det[h^2g_{\mu\alpha}]}\sqrt{\det\left[\delta^\alpha_\nu U(r)+h^{-4}(r)U(r)g^{\alpha\beta}\frac{\partial r}{\partial\sigma^\beta}\frac{\partial r}{\partial\sigma^\nu}\right]}\end{aligned} \quad (2.5)$$

in which U is an arbitrary function of r.

We define the scalar field $\phi=\sqrt{T_{P_3}}r$ and the brane tension T_{P_3} is a function of the string scale m_s and the string coupling g_s, then we have [31]

$$\begin{aligned}T_{D_p}&=\frac{1}{g_s(2\pi)^p(\alpha')^{(p+1)/2}}\\\frac{m_s}{g_s}&=T_{D_0}=\frac{1}{g_s\sqrt{\alpha'}}\end{aligned} \quad (2.6)$$

and then we obtain

$$T_{P_3}=\frac{m_s^4}{(2\pi)^3g_s}. \quad (2.7)$$

Therefore, we have the DBI action

$$S_{DBI}=-T_{P_3}\int\mathrm{d}^4\sigma h^4(\phi)\sqrt{-\det g_{\mu\alpha}}\sqrt{\det[\delta^\alpha_\nu U(\phi)+U(\phi)h^{-4}(\phi)T^{-1}_{P_3}g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi]}, \quad (2.8)$$

and the DBI Lagrangian is

$$\begin{aligned}L_{DBI}&=-T_{P_3}h^4(\phi)\sqrt{-\det g_{\mu\alpha}}\sqrt{\det[\delta^\alpha_\nu U(\phi)+U(\phi)h^{-4}(\phi)T^{-1}_{P_3}g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi]}\\&=-f^{-1}(\phi)\sqrt{-\det g_{\mu\alpha}}\sqrt{\det[\delta^\alpha_\nu U(\phi)+U(\phi)f(\phi)g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi]},\end{aligned} \quad (2.9)$$

where the inverse brane tension $f(\phi)$ is relative to eq. (2.7) and the warp factor h by

$$f(\phi)=\frac{1}{T_{P_3}h^4(\phi)} \quad (2.10)$$

When we consider $\sqrt{-\det g_{\mu\nu}}\mathrm{d}^4\sigma$ as the invariant volume element of the integral eq. (2.8) and add an integral constant term $T_{p_3}\int \mathrm{d}^4\sigma h^4(\phi)\sqrt{-\det g_{\mu\nu}}=\int \mathrm{d}^4\sigma\sqrt{-\det g_{\mu\nu}}f^{-1}(\phi)$ into eq. (2.8), we finally achieve a DBI new Lagrangian

$$L_{DBI}{=}-f^{-1}(\phi)\sqrt{\det[U(\phi)(\delta^\alpha_\nu + f(\phi)g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi)]}{+}f^{-1}(\phi). \tag{2.11}$$

Defining $U(\phi) = 1 + V'(\phi)$, we have

$$L_{DBI} = -f^{-1}(\phi)\sqrt{\det\left[\delta^\alpha_\nu + U(\phi)\left(f(\phi)g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi + \frac{V'(\phi)}{U(\phi)}\delta^\alpha_\nu\right)\right]}+f^{-1}(\phi), \tag{2.12}$$

taking linear approximation, we obtain

$$L_{DBI}{=}-f^{-1}(\phi)\sqrt{1{+}U(\phi)\left[f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi{+}\frac{V'(\phi)}{U(\phi)}\right]}{+}f^{-1}(\phi), \tag{2.13}$$

where deducing details of eq. (2.13) see appendix A.

The term $U(\phi)$ can arise in different places within the string theory. Firstly if the brane is actually a non-BPS one [7], then the scalar field mode is actually tachyonic and the potential is therefore of the usual runaway form. If there are N multiple coincident branes, then the world-volume field theory is a U(N) non-Abelian gauge theory and the potential term is simply a reflection of the additional degrees of freedom. Through the dielectric effect, one can also see that this configuration is related to a D_5-brane wrapping a two-cycle within the compact space and carrying a non-zero magnetic flux along this cycle. Both of these configurations lead to an additional potential multiplying the usual DBI kinetic term [1–3].

Further, we generally define $V'(\phi) = f(\phi)\xi V(\phi)$ (ξ is a general parameter). Thus, eq. (2.13) can be rewritten as

$$L_{DBI}{=}-f^{-1}(\phi)\sqrt{1{+}U(\phi)f(\phi)\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi{+}\xi\frac{V(\phi)}{U(\phi)}\right]}{+}f^{-1}(\phi). \tag{2.14}$$

One expects open or closed string interactions to generate a scalar potential V; however, the precise form of such an interaction depends upon many factors such as the number of additional branes and geometric moduli, the number of nontrivial cycles in the compact space, and the choice of embedding for branes on these cycles. Typically, one can only compute this in special cases in the full string theory. There are also additional terms coming from coupling of the brane to any background Ramond-Ramond form fields.

Eq. (2.14) shows the potential energy and kinetic energy can convert into each other in any order when expanding the expression. Eq. (2.14), or eq. (2.11), is quite different from the usual DBI Lagrangian [25, 34]

$$L_{DBI}{=}-f^{-1}(\phi)\sqrt{1{+}f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{+}f^{-1}(\phi) + \mathrm{V}(\phi). \tag{2.15}$$

By defining the new inverse brane tension $F(\phi) = f(\phi)U(\phi)$ and $f^{-1} = F^{-1}(\phi)/U^{-1}(\phi)$, we have

$$L_{DBI}{=}-\frac{F^{-1}(\phi)}{U^{-1}(\phi)}\sqrt{1{+}F(\phi)\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi{+}\xi\frac{V(\phi)}{U(\phi)}\right]}{+}\frac{F^{-1}(\phi)}{U^{-1}(\phi)}. \tag{2.16}$$

Taking Taylor expansion of eq. (2.16), we have

$$\begin{aligned}L_{DBI} = & -\frac{F^{-1}(\phi)}{U^{-1}(\phi)}\left[1+\frac{1}{2}F\left(\phi)(g^{\mu\nu}\partial_\nu\phi\partial_\nu\phi+\xi\frac{V(\phi)}{U(\phi)}\right)\right]+\frac{F^{-1}(\phi)}{U^{-1}(\phi)} \\ & +\frac{F^{-1}(\phi)}{U^{-1}(\phi)}\frac{1}{8}\left[F(\phi)\left(g^{\mu\nu}\partial_\nu\phi\partial_\nu\phi+\xi\frac{V(\phi)}{U(\phi)}\right)\right]^2 \\ & -\frac{F^{-1}(\phi)}{U^{-1}(\phi)}\frac{1}{16}\left[F(\phi)\left(g^{\mu\nu}\partial_\nu\phi\partial_\nu\phi+\xi\frac{V(\phi)}{U(\phi)}\right)\right]^3+\ldots\ldots\end{aligned} \tag{2.17}$$

Eq. (2.17) reveals that the potential energy can convert into kinetic energy in any order, and the potential energy emerges from the Lagrangian eq. (2.16) naturally.

Taking linear approximation of eq. (2.17), we have

$$L_{DBI}=-U(\phi)\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\frac{1}{2}\xi\frac{V(\phi)}{U(\phi)}\right). \tag{2.18}$$

The Taylor expansion eq. (2.17) of eq. (2.16) is consistent with the form in the non-linear classical physics, since the kinetic energy and potential energy can convert into each other in any order in the non-linear classical physics. This is an important example, which shows us that the results in string theory can return to the classical physics seen detailed in section 4.

In this section, we derive the action eq. (2.12), or eq. (2.13), in a different way from that given in the literatures [1–3], by rescaling the metric. This dependence is seen from the DBI action, which involves the pullback of the metric onto the worldvolume of a Dp-brane. Therefore, we can see that there is a general potential function in the DBI action with a more general symmetry, by rescaling the metric. We deduce the Lagrangian eq. (2.12), or eq. (2.13), with potential $U(\phi)$ in a new way, where the determinant of induced metric naturally includes the kinetic energy and potential energy.

In addition, we obtain the linear approximation eq. (2.18) of Lagrangian eq. (2.16) for the DBI Lagrangian corresponding to eq. (2.13). It shows that the DBI Lagrangian in eq. (2.16) includes the usual one [25], which is a special example of eq. (2.18). Furthermore, eq. (2.16) guarantees that the kinetic energy and the potential energy can transform into each other in any order, and never suffers from the problem of not corresponding in high order. The forms of the Lagrangian eqs. (2.13) and (2.16) will play an important role in the following sections of this paper.

3 The general Lagrangian relative to phantom, K-essence and quintessence and corresponding generalized Klein-Gordon equation

Using $U(\phi) = 1 + f(\phi)\xi V(\phi)$, from eq. (2.18) we have

$$L_{DBI\ L} = L_p + L_k \tag{3.1}$$

where

$$L_P = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{\xi}{2}V(\phi), \tag{3.2}$$

$$L_K = W(\phi)X(\phi), \tag{3.3}$$

in which $W(\phi) = -f(\phi)\xi V(\phi), X(\phi) = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.

When $\xi = 2$, the eq. (3.2) is the Lagrangian of Phantom [25]. Eq. (3.3) is the Lagrangian of k-essence with one order $X(\phi)$ [21].

For $f(\phi)V(\phi) = -1$ and $\xi = 2$, using eq. (3.1), eq. (3.2) and eq. (3.3), we have

$$L_{DBI\ L}=\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi), \tag{3.4}$$

eq. (3.4) is the same as Quintessence [30]. Eqs. (3.1)–(3.4) reveal that the model of metric eq. (2.2) contains Phantom, Quintessence and k-essence with one order $X(\phi)$ naturally. Specially, when considering the non-linear Lagrangian eq. (2.17) and simliar to the discussion above in this section, we can deduce k-essence with high order $X(\phi)$ naturally.

On the other hand, eq. (2.18) can be rewritten as

$$L_{DBI\ L} = -(1 + f(\phi)\xi V(\phi))\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\xi V(\phi), \tag{3.5}$$

From eq. (3.5) we achieve the generalized Klein-Gordon Equation

$$\partial_\mu[(1 + f(\phi)\xi V(\phi))\partial^\mu\phi] - \frac{1}{2}\xi\frac{\partial V(\phi)}{\partial\phi} - \xi\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\frac{\partial(f(\phi)V(\phi))}{\partial\phi} = 0. \tag{3.6}$$

When $f(\phi)V(\phi) = 1$ and $\xi = 1$, we have

$$\partial^\mu\partial_\mu\phi - \frac{1}{4}\frac{\partial V(\phi)}{\partial\phi} = 0. \tag{3.7}$$

When $V(\phi) = -\frac{4}{3}m^2\phi^3$, we have

$$\partial^\mu\partial_\mu\phi + m^2\phi^2 = 0. \tag{3.8}$$

This is the usual Klein-Gordon Equation.

Using eq. (2.14) and similar to the studies in this section,people can achieve more general results and richer equations from the DBI general Lagrangian.

According to Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu}, \tag{3.9}$$

one can see that different matter tensor $T_{\mu\nu}$ will cause different curved spacetime $G_{\mu\nu}$. That is, matter tells spacetime how to curve. Therefore, different matter models will cause different geometric metrics, i.e., different gravitational models, the discussions above in this paper give the different gravitational models by using the DBI Lagrangian. Because this paper derives different matter Lagrangians, e.g., the Phantom, K-essence and Quintessence, by which one can derive the corresponding different matter tensor $T_{\mu\nu}$, then which will cause different geometric metrics, i.e., give the corresponding different gravitational models.

4 The Lagrangian of classical mechanics

Inserting $\phi = \sqrt{T_{P_3}} r$ in the paragraph above eq. (2.6) into eq. (2.18), we have the Lagrangian of classical mechanics

$$\begin{aligned} L_{DBI\ L} &= -[1 + f(\phi)\xi V(\phi)]\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\xi V(\phi) \\ &= -[1 + f(\sqrt{T_{P_3}}r)\xi V(\sqrt{T_{P_3}}r)]T_{P_3}\frac{1}{2}[g^{00}\partial_0 r\partial_0 r + 2g^{0i}\partial_0 r\partial_i r + g^{ij}\partial_i r\partial_j r] \\ &\quad -\frac{1}{2}\xi V(\sqrt{T_{P_3}}r). \end{aligned} \tag{4.1}$$

For simplicity and convenience, we reduce eq. (4.1) to the Lagrangian of classical point particle in(1+1)-dimensional spacetime and take $r = x(t)$ and $g^{\mu\nu} = (1, -1)$, then we have

$$L = -\frac{1}{2}[1 + f(x)\xi V_1(x)]T_{P_0}\partial_0 x\partial_0 x - \frac{1}{2}\xi V_1(x) + \frac{1}{2}[1 + f(x)\xi V_1(x)]T_{P_0} \tag{4.2}$$

where $V_1(x) = V(\phi(x))$ is in one dimension, $f(x) = f(\phi(x))$ in one dimension. For classical point particle, we need to use eq. (2.6), and when $\xi f(x)V_1(x) = -g - 1$, we derive the relationship

$$[1 + f(x)\xi V_1(x)]T_{P_0} = -m \tag{4.3}$$

where for point particle we have denoted $m_s = m$ and $g_s = g$.

For $\xi = 2$ and neglecting the last term in eq. (4.2), which now is a constant term and a correction term from the DBI action, then we achieve the classical Lagrangian

$$L = \frac{1}{2}m\partial_0 x\partial_0 x - V_1(x) \tag{4.4}$$

Making the similar discussions deducing eq. (4.4) for eq. (2.17), then we may obtain the corresponding results relative to eq. (4.4), thus people can deduce the non-linear classical Lagrangian, i.e., which shows the non-linear classical physics from eq. (2.17). This is an important example, which shows us that the results in string theory can return to the classical physics.

5 Summary and conclusion

In this paper, we derive the action eq. (2.12) or eq. (2.13) in a different way from that given in the literatures [1–3], by rescaling the metric. This dependence is seen from the DBI action, which involves the pullback of the metric onto the worldvolume of a Dp-brane. The determinant of the induced metric naturally includes the kinetic energy and potential energy.

One of our motivations is to study the potential in DBI action, and find a natural way to guarantee that the potential energy in DBI action can convert into kinetic energy in any order. And the Lagrangian in eq. (2.16) for the DBI action naturally includes potential energy. The Taylor expansion of eq. (2.16) is consistent with the form in the non-linear classical physics, since the kinetic energy and potential energy can convert into each other in any order.

The technical difference between this work and previous is that we present the induced metric with a generic symmetry by rescaling the metric, where the potential arises in the determinant naturally. Meanwhile, the important feature in our treatment is that kinetic energy and potential energy varies in any order, and never suffers from the problem of not corresponding in any high order. We deduce the Phantom, K-essence, Quintessence and Generalized Klein-Gordon Equation from the DBI model.

We show that the DBI Lagrangian in eq. (2.16) includes the usual one [25], which is a special example of eq. (2.18). We deduce the linear and non-linear Lagrangians, i.e., which may show the corresponding linear and non-linear classical physics from eq. (2.17). These are important examples, which shows us that the results in string theory can return to the classical physics. These investigations in this paper are the support for the statement that the results of string theory are consistent with quantum mechanics and classical physics. Using the deduced Lagrangians eq. (2.12), eq. (2.16) and eq. (2.18), a lots of works about cosmological investigations may be done, e.g., this paper naturally derives out the Phantom, K-essence, Quintessence (which are largely utilized in the research relative to our universe) and Generalized Klein-Gordon Equation from the DBI model.

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A Linear approximation of general determinant

In general case, we have the relation $A^{\nu}_{\mu} = B^{\nu}_{\mu} + C^{\nu}_{\mu}$. Then we obtain the relation

$$\begin{aligned} detA^{\mu}_{\nu} &= \frac{1}{4!}\delta^{\mu_0\mu_1\mu_2\mu_3}_{\nu_0\nu_1\nu_2\nu_3} A^{\nu_0}_{\mu_0} A^{\nu_1}_{\mu_1} A^{\nu_2}_{\mu_2} A^{\nu_3}_{\mu_3} \\ &= \frac{1}{4!}\delta^{\mu_0\mu_1\mu_2\mu_3}_{\nu_0\nu_1\nu_2\nu_3} [(B^{\nu_0}_{\mu_0} + C^{\nu_0}_{\mu_0})(B^{\nu_1}_{\mu_1} + C^{\nu_1}_{\mu_1})(B^{\nu_2}_{\mu_2} + C^{\nu_2}_{\mu_2})(B^{\nu_3}_{\mu_3} + C^{\nu_3}_{\mu_3})] \\ &= \frac{1}{4!}\delta^{\mu_0\mu_1\mu_2\mu_3}_{\nu_0\nu_1\nu_2\nu_3} B^{\nu_0}_{\mu_0} B^{\nu_1}_{\mu_1} B^{\nu_2}_{\mu_2} B^{\nu_3}_{\mu_3} + \frac{1}{4!}\delta^{\mu_0\mu_1\mu_2\mu_3}_{\nu_0\nu_1\nu_2\nu_3} (C^{\nu_0}_{\mu_0} B^{\nu_1}_{\mu_1} B^{\nu_2}_{\mu_2} B^{\nu_3}_{\mu_3} \\ &\quad + B^{\nu_0}_{\mu_0} C^{\nu_1}_{\mu_1} B^{\nu_2}_{\mu_2} B^{\nu_3}_{\mu_3} + B^{\nu_0}_{\mu_0} B^{\nu_1}_{\mu_1} C^{\nu_2}_{\mu_2} B^{\nu_3}_{\mu_3} + B^{\nu_0}_{\mu_0} B^{\nu_1}_{\mu_1} B^{\nu_2}_{\mu_2} C^{\nu_3}_{\mu_3}) + ... \end{aligned} \tag{A.1}$$

where we omit the high order terms about C^{μ}_{ν}.

For eq. (2.12), $B^{\alpha}_{\nu} = \delta^{\alpha}_{\nu}$, $C^{\alpha}_{\nu} = U(\phi)(f(\phi)g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi + \frac{V'(\phi)}{U(\phi)}\delta^{\alpha}_{\nu})$, we have

$$detA^{\alpha}_{\nu} = 1 + U(\phi)(f(\phi)g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{V'(\phi)}{U(\phi)}) + ... \tag{A.2}$$

in which we have utilized $\delta^{\mu_0\mu_1\mu_2\mu_3}_{\nu_0\nu_1\nu_2\nu_3} = 4!$, $\delta^{\mu_0\mu_1\mu_2\mu_3}_{\nu_0\mu_1\nu_2\nu_3} C^{\nu_0}_{\mu_0} = \delta^{\mu_0}_{\nu_0} 3! C^{\nu_0}_{\mu_0}$ etc.

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