Crossover from isotropic to directed percolation

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Outline

1. Percolation
2. Directed Percolation
3. Biased Directed Percolation

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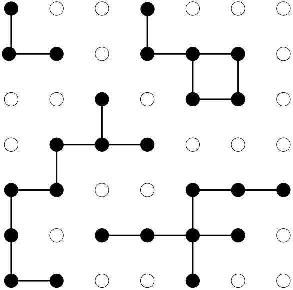

Figure 1: A typical percolation configuration, with four clusters.

- Site percolation on square lattice
- Occupation probability p
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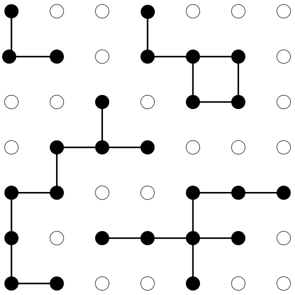

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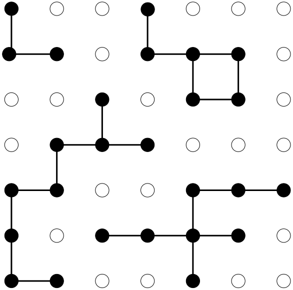

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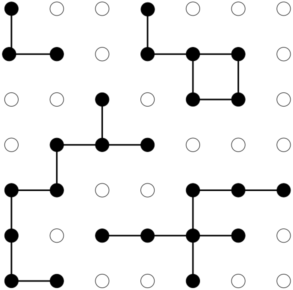

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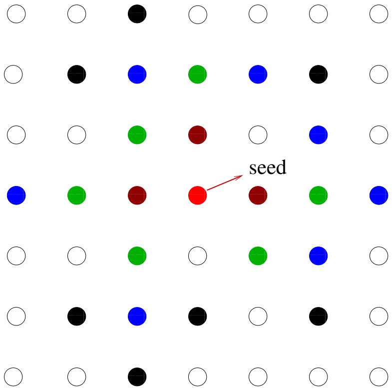

Figure 2: Percolation illustrated as a stochastic process.

- Use an algorithm to generate the cluster from the origin.
- Let an active seed affect its nearest neighbors with probability p.
- Distinguish different shells (time) by colors.
- Reformulate percolation as a stochastic process.

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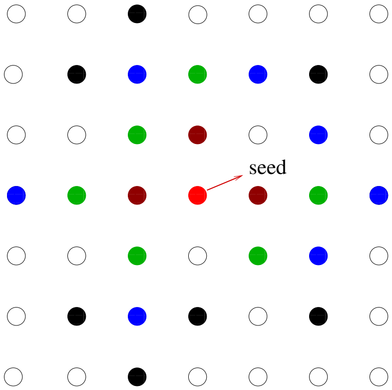

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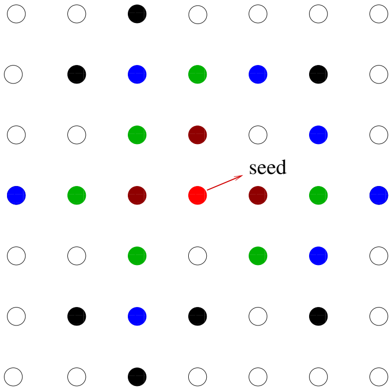

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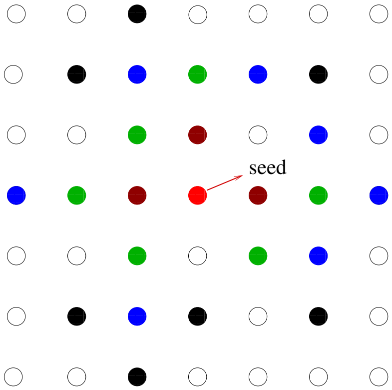

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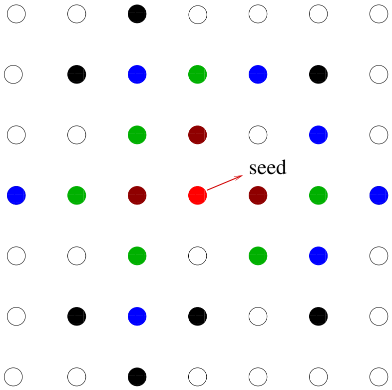

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Percolation

For infinite system ($L \to \infty$), there is a critical p value (p_c),

- when $p \leq p_c$, no infinite cluster exists
- when $p > p_c$, an infinite cluster exists with non-zero probability.

Define the order parameter P_∞, which is the probability to grow an infinite cluster and

$$P_\infty = \begin{cases} 0 & \text{for } p \leq p_c \\ (p - p_c)^{\beta_\text{P}} & \text{for } p \to {p_c}^+ \end{cases} . \tag{1}$$

β_P is a critical exponent, and $\beta_P = 5/36$ for 2D.

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Around p_c, the correlation length ξ diverges as

$$\xi \sim |p - p_c|^{-\nu} \ , \tag{2}$$

- ξ can be intuitively viewed as the averaged cluster radius
- ν is another critical exponent and $\nu = 4/3$ for 2D

Critical exponents β_{P} and ν label percolation universality class.

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When growing a cluster from an active seed using breadth first scheme, the shortest-path between an occupied site and the seed is ℓ, one has

$$\langle \ell \rangle \propto r^{d_{\min}} \ . \tag{3}$$

At p_c, one expects:

- $P(r) \sim r^{-\beta/\nu}$.
- $P(\ell) \sim \ell^{-\beta/(\nu d_{\min})}$

Around p_c ($\epsilon = p - p_c$), according to scaling theory, one has:

- $P(r, p) = r^{-\beta/\nu} f(\epsilon L^{1/\nu})$.
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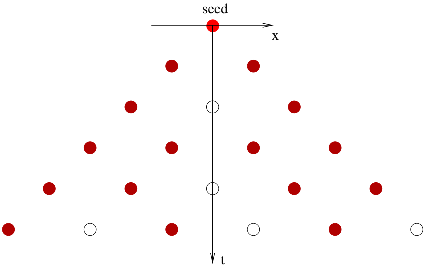

Figure 3: two-dimensional directed percolation

- Rotate the square lattice
- Infection probability p
- Only along the time axis
- Use the same order parameter P_∞
- With a exponent β_{DP}, and $\beta_{DP} \neq \beta_P$.

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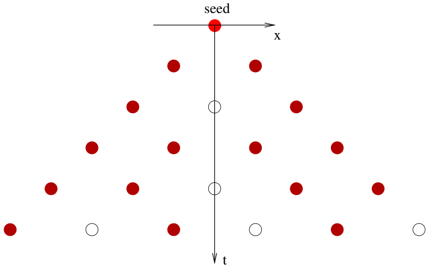

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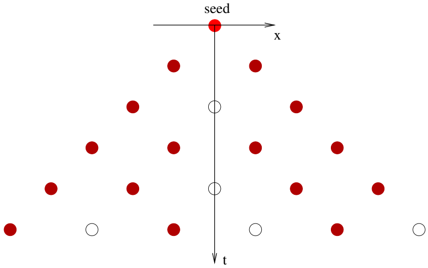

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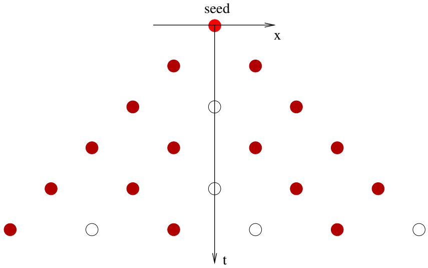

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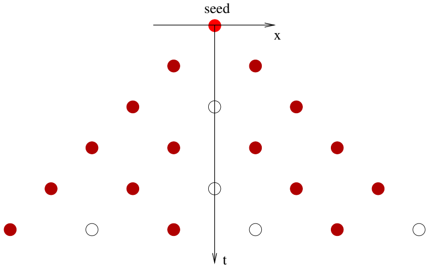

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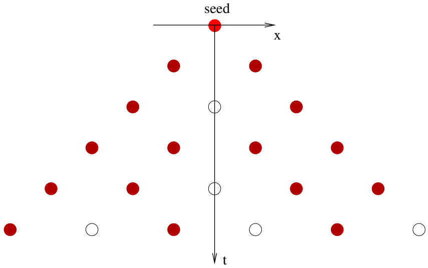

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Directed Percolation

Compared with Percolation,

- DP is a non-equilibrium statistical mechanics model
- have anisotropic correlation

$$\xi_{\parallel} \sim |p - p_c|^{-\nu_{\parallel}} \quad , \xi_{\perp} \sim |p - p_c|^{-\nu_{\perp}} \ . \tag{4}$$

- Independent exponents ($\beta_{\rm DP}$, $\nu_{\parallel}$,$\nu_{\perp}$) label the DP universality class.
- DP class is a fundamental class in non-equilibrium statistical mechanics, like the Ising class for equilibrium statistical mechanics.

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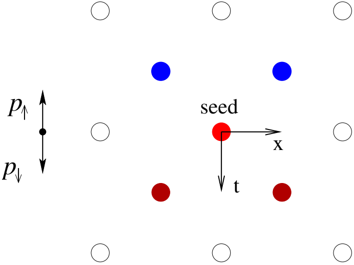
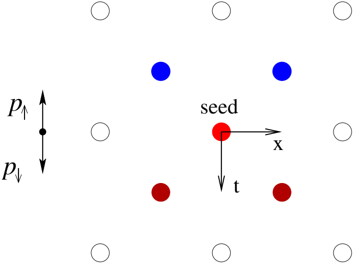


Figure 4: Rules for BDP

Compared with DP,

- no direction limit
- but with anisotropic infection probabilities
  Along time axis:
  $p_\downarrow = pp_d$
  Against time axis:
  $p_\uparrow = p(1 - p_d)$
- When $p_d = 1/2$, BDP = Percolation
- When $p_d = 0, 1$, BDP = DP

Biased Directed Percolation



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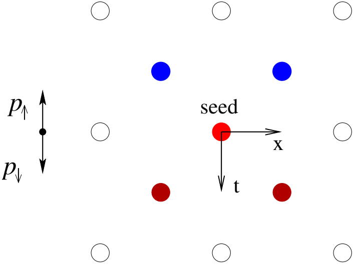

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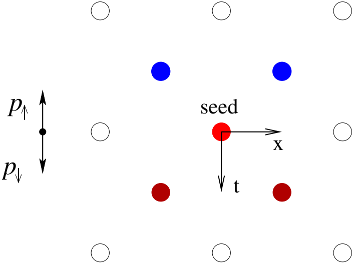

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Biased Directed Percolation

Consider a new region $1/2 < p_d < 1$,

- we still use order parameter P_∞, what is β' characterizing P_∞?
- Correlation length

$$\xi'_{\parallel} \sim |p - p_c|^{-\nu'_{\parallel}} \quad , \xi'_{\perp} \sim |p - p_c|^{-\nu'_{\perp}} \,. \tag{5}$$

what are the values of $\nu'_{\parallel}$ and $\nu'_{\perp}$?

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Biased Directed Percolation

With some fixed $1/2 < p_d < 1$ values, we sample quantity $P(t,p)$ and assume

$$P(t,p) = t^{-\beta'/\nu'_{\parallel}} f(\epsilon t^{1/\nu'_{\parallel}}) \tag{6}$$

where $\epsilon = p - p_c$.

- Do the Taylor expansion to the equation
- Perform the least-squares fits of the Monte Carlo data to the expansion

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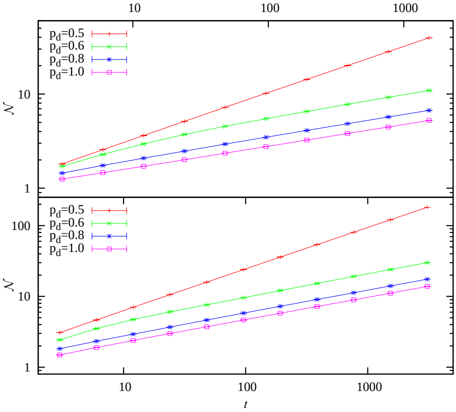

Figure 5: Exponent η with various p_d values.

Sample quantity $\mathcal{N}(t)$, which

- is the number of active sites at time t,
- scales as $\mathcal{N}(t) \sim t^{\eta}$.

Biased Directed Percolation

Phase Diagram

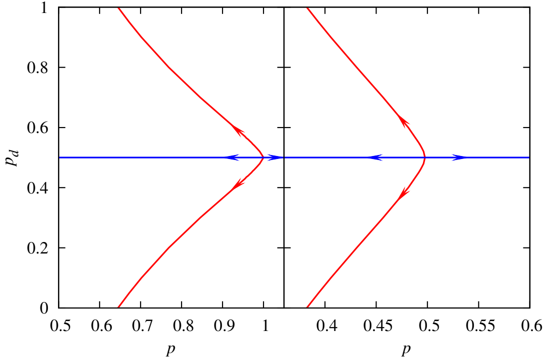

Figure 6: Phase diagram of BDP on two (left) and three (right) dimensions.

- Arrows represent the flow direction of Renormalization Group

Biased Directed Percolation

Consider the critical region around percolation point ($p_d = 1/2$, $p = 1$).

- For $p_d = 1/2$, $p \to 1$, a certain quantity $\mathcal{O}$ scales as

$$\mathcal{O} \sim \ell^{Y_{\mathcal{O}}} f(\epsilon \ell^{Y_\epsilon}) \quad , \epsilon = p - p_c \, , \tag{7}$$

  - $Y_{\mathcal{O}}$ is quantity dependent

- For $p = 1$, $p_d \to 1/2$,

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Consider the critical region around percolation point ($p_d = 1/2$, $p = 1$).

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Crossover exponent ϕ is defined as

$$(1 - p_c) \propto (p_{d,c} - 1/2)^{1/\phi} \tag{9}$$

- points $(p_c, p_{d,c})$ are on the transition line
- ϕ characterizes the crossover behavior from percolation to DP
- $\phi = Y_{\epsilon_d}/Y_\epsilon$ from scaling theory.

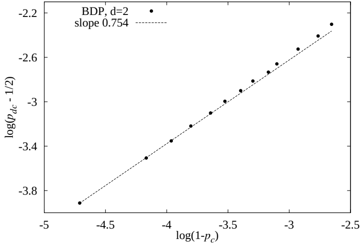

Figure 7: Crossover exponent

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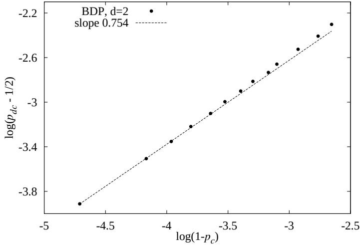

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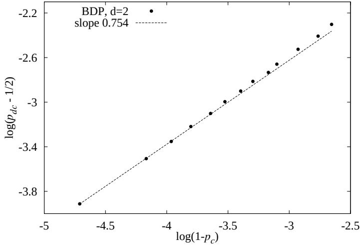

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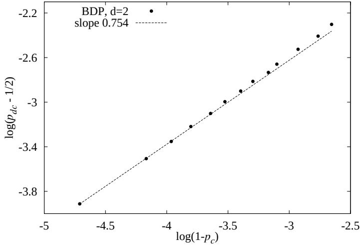

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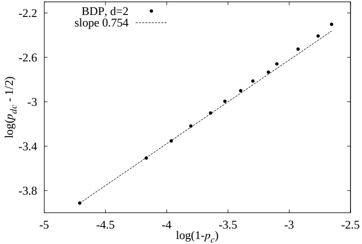

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- Use a simple BDP model to generalize Percolation and DP models
- Study the crossover effect from Percolation to DP
- Is Y_{ϵ_d} (ν') new or related to β, ν, $d_{\min}$?
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