FROM CONFORMAL INVARIANCE TO QUASISTATIONARY STATES

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Quasistationary states

Stochastic models with long-range interactions

Long relaxation times (τ) which increase with the size of the system (L)

$\tau \sim L^m$ ($m \approx 1.7$ for many models)

Reviews: Mukamel, Campa,Dauxois, Ruffo 2008,2009, Politi,Torcini, 2010

Dictionary: Absorbing state

If in the stationary state of a system onde finds with probability 1 only one configuration, the configuration is the absorbing state

A System with states

$$|a\rangle \quad (a = 1, 2, \dots) \quad \rightarrow \quad \text{probability} \quad P_a(t)$$

$$|b\rangle \quad \rightarrow \quad |a\rangle \qquad \text{rate} \quad -H_{a,b}$$

The master equation

$$\frac{d}{dt} P_a(t) = -\sum_b H_{a,b} P_b(t),$$

H is an $N \times N$ intensity matrix (eigenvalues $\mathrm{Re}(E(k)) \geq 0$)

$$H_{a,b} \leq 0, \quad \sum_{a=1}^{N} H_{a,b} = 0$$

The stationary state $\rightarrow$ ground state $E_0 = 0$

$$\langle 0| = \sum_a 1 \langle a|, \quad |0\rangle = \sum_a P_a |a\rangle, \quad P_a = \lim_{t \rightarrow \infty} P_a(t)$$

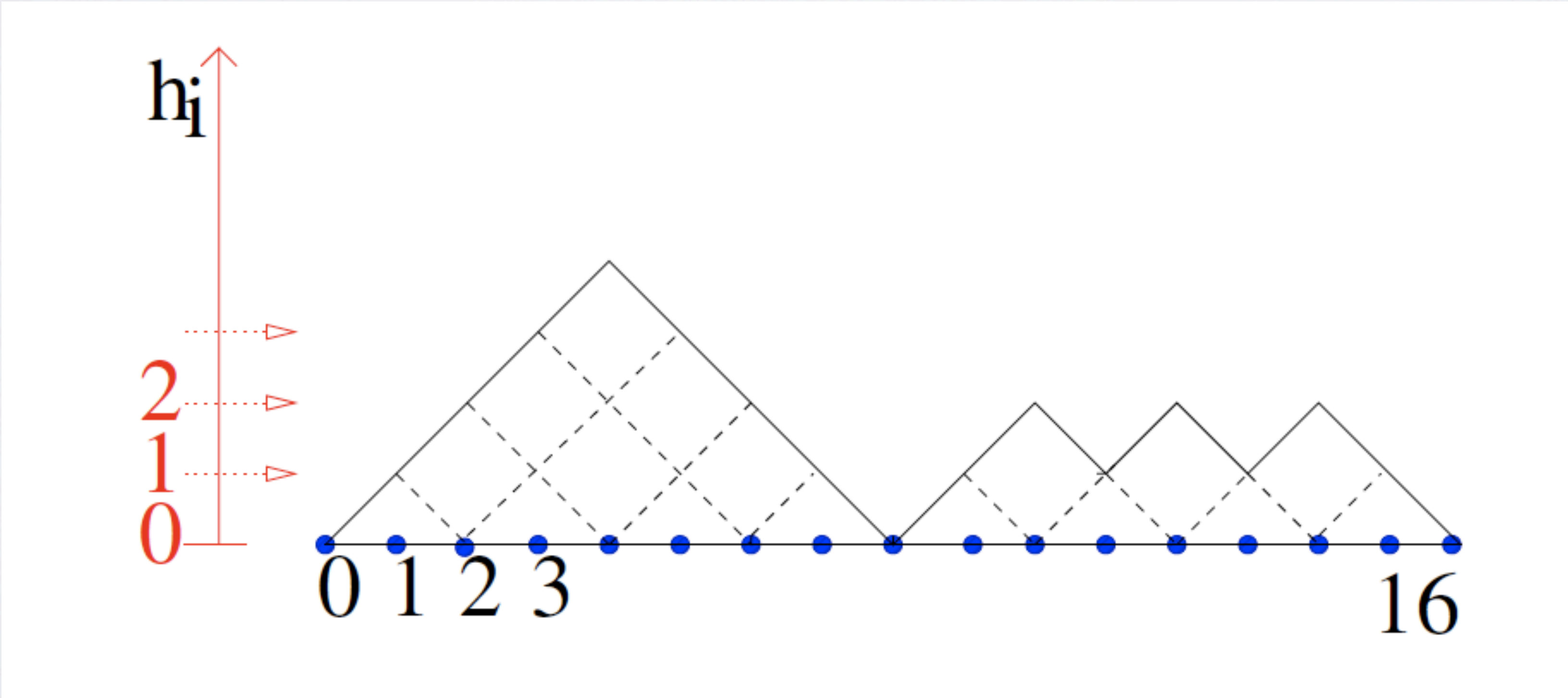
Configuration space: Dyck paths

One dimensional lattice with $L+1$ sites ($L = 2n$) where we attach non negative heigths h_i ($i = 0, 1, \ldots, L$)

RSOS rules

$$h_{i+1} - h_i = \pm 1, \qquad h_0 = h_L = 0, \qquad h_i \geq 0$$

Catalan number $\longrightarrow$ $C_n = \frac{1}{n+1}\binom{2n}{n}$



Observables

Average local heigh $h(i, L, t)$

Average local density of contact points $g(i, L, t)$

Average density of peaks and valleys

#(peaks+valleys)/L $\tau(L, t)$

Stationary Quantities no t dependence

$$(h(i, L, t) \rightarrow h(i, L))$$

Peak adjusted raise and peel model

Monte Carlo simulations

a) Sequential updating

With a probability $R_p = \frac{p}{L-1}$ it hits a peak ($p \geq 0$ parameter)

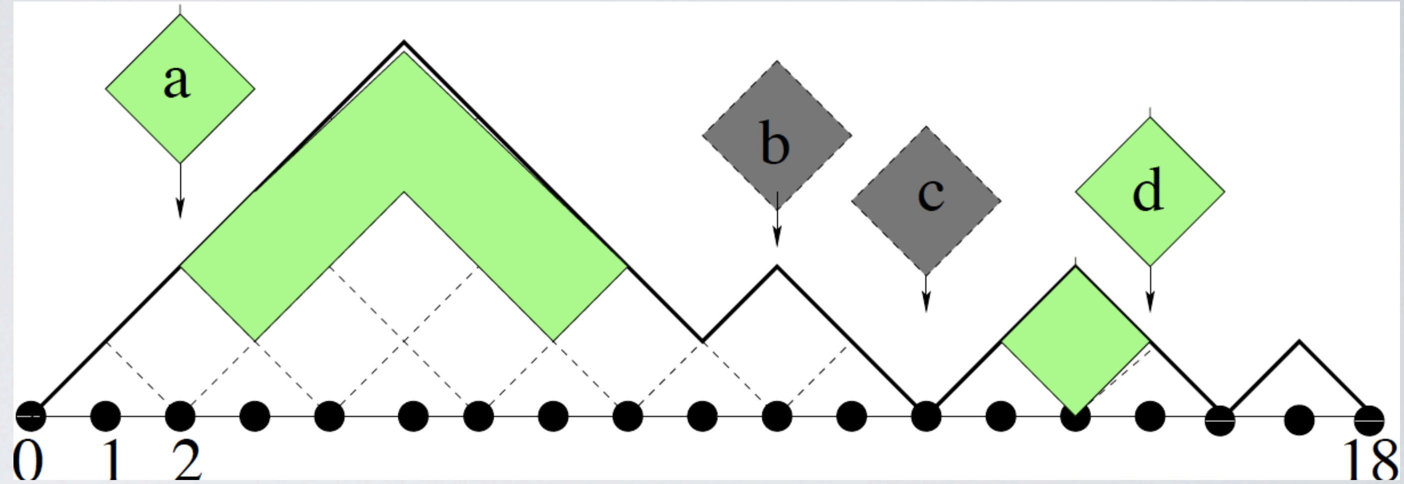
With a probability Q_c it hits the remaining $L - 1 - n_p^c$ sites

(n_p^c = #of peaks) $$Q_c = \frac{q_c}{L-1}$$

$$n_p^c R_p + (L - 1 - n_p^c) Q_c = 1 \qquad q_c = 1 - \frac{n_p^c (p-1)}{L - 1 - n_p^c}$$

$p = 1 \rightarrow$ no n_p^c and L dependence (RPM)

b) Effects of a hit by a tile



$$H_{ac} = -r_{ac} q_c \qquad (c \neq 0)$$

$$r_{ac} \rightarrow \text{rates} \qquad p = 1 \qquad \text{(RPM)}$$

$$q_c = 1 - \frac{n_c(p-1)}{L-1-n_c}$$

configurations with large
are more stable

$$p \leq 2\frac{L-1}{L}$$

Statement: For $0 \leq p < 2$ the finite-size scaling properties are the same as for $p = 1$ (RPM) which is integrable ($U_q(sl(2))$ invariant), XXZ model, $q = e^{i\pi/3}$,T-L algebra (semi-group $e_i^2 = e_i$) known properties

Conformal Invariance

Exact results RPM

Exact stationary state (exact results)

Density of contact points

$$g(x,L) = c(\frac{L}{\pi}\sin(\frac{\pi x}{L}))^{-1/3}$$

$$c = \frac{\sqrt{3}}{6\pi^{5/6}}\Gamma(1/6) = 0.753149\ldots$$

(one-point function)

Density of peaks and valeys

$$\lim_{L\to\infty} \tau(L) = \frac{3}{4}$$

Finite-size scaling spectrum

$$\lim_{L\to\infty} E_i(L) = \frac{\pi v_s(p)\Delta_i}{L} \qquad i = 0,1,2,\ldots$$

$E_0 = 0$ (for any L stochastic model) $\rightarrow c = 0$

$$v_s(p) = (1 - \frac{3(p-1)}{5})\frac{3\sqrt{3}}{2} \quad (p < 2)$$

Single place where p enters

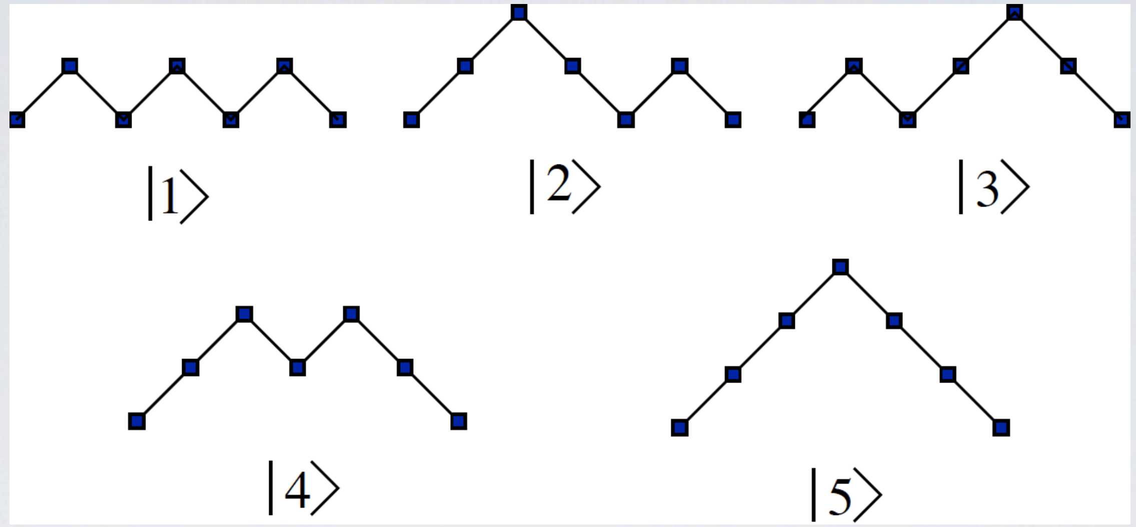
$$Z(q) = \sum_{i=0} q^{\Delta_i} = (1-q)\prod_{n=1}^{\infty}(1-q^n)^{-1}$$

$$\Delta = 0(1), \quad 2(1), \quad 3(1), \quad 4(2), \quad 5(2), \quad 6(4), \quad \cdots$$

What happens at $p = \frac{p(2L-1)}{L}$?

Triller !!

Example L=6

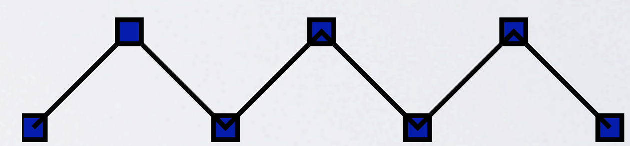


Ground state
(stationary state)

$$\left|\frac{11(5-p)}{2(5-3p)}, \frac{15(5-p)}{4(5-2p)}, \frac{15(5-p)}{4(5-2p)}, \frac{3(5-p)}{5-2p}, 1\right\rangle$$

Nice combinatoric properties only at p=1 ! → $|11, 5, 5, 4, 1>$

If p=5/3 |1> is an absorbing state



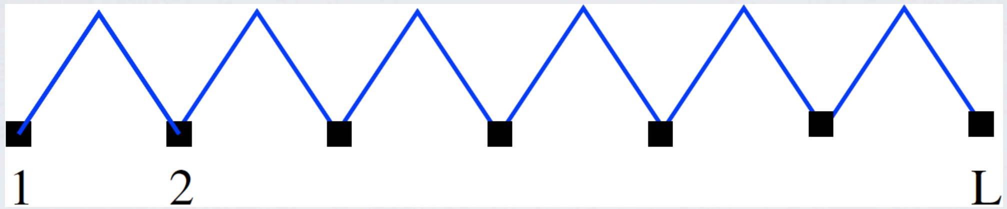
For lattice L → substrate has $n_p = L/2$ peaks

p is restricted → $0 \leq p < 2(L-1)/L$

$$
H = -\left(\begin{array}{c|ccccc}
 & |1\rangle & |2\rangle & |3\rangle & |4\rangle & |5\rangle \\
\hline
\langle 1| & -5+3p & 2[(5-2p)/3] & 2[(5-2p)/3] & 0 & 2[(5-p)/4] \\
\langle 2| & (5-3p)/2 & -5+2p & 0 & (5-2p)/3 & 0 \\
\langle 3| & (5-3p)/2 & 0 & -5+2p & (5-2p)/3 & 0 \\
\langle 4| & 0 & (5-2p)/3 & (5-2p)/3 & -5+2p & 2[(5-p)/4] \\
\langle 5| & 0 & 0 & 0 & (5-2p)/3 & -5+p
\end{array}\right) \quad (2.8)
$$

The limiting case: $p = p_{max} = 2(L-1)/L$

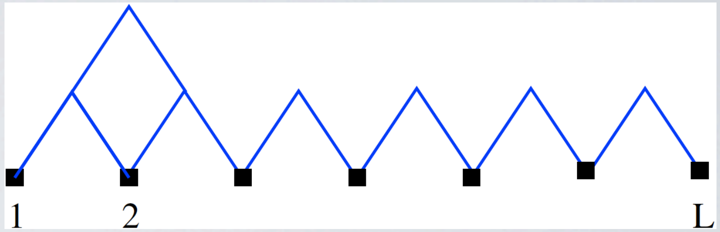
substrate



$$n_p = L/2 \quad R_p = p/(L-1) = 2/L \quad Q_c = 0!$$

substrate is an absorbing state !!!

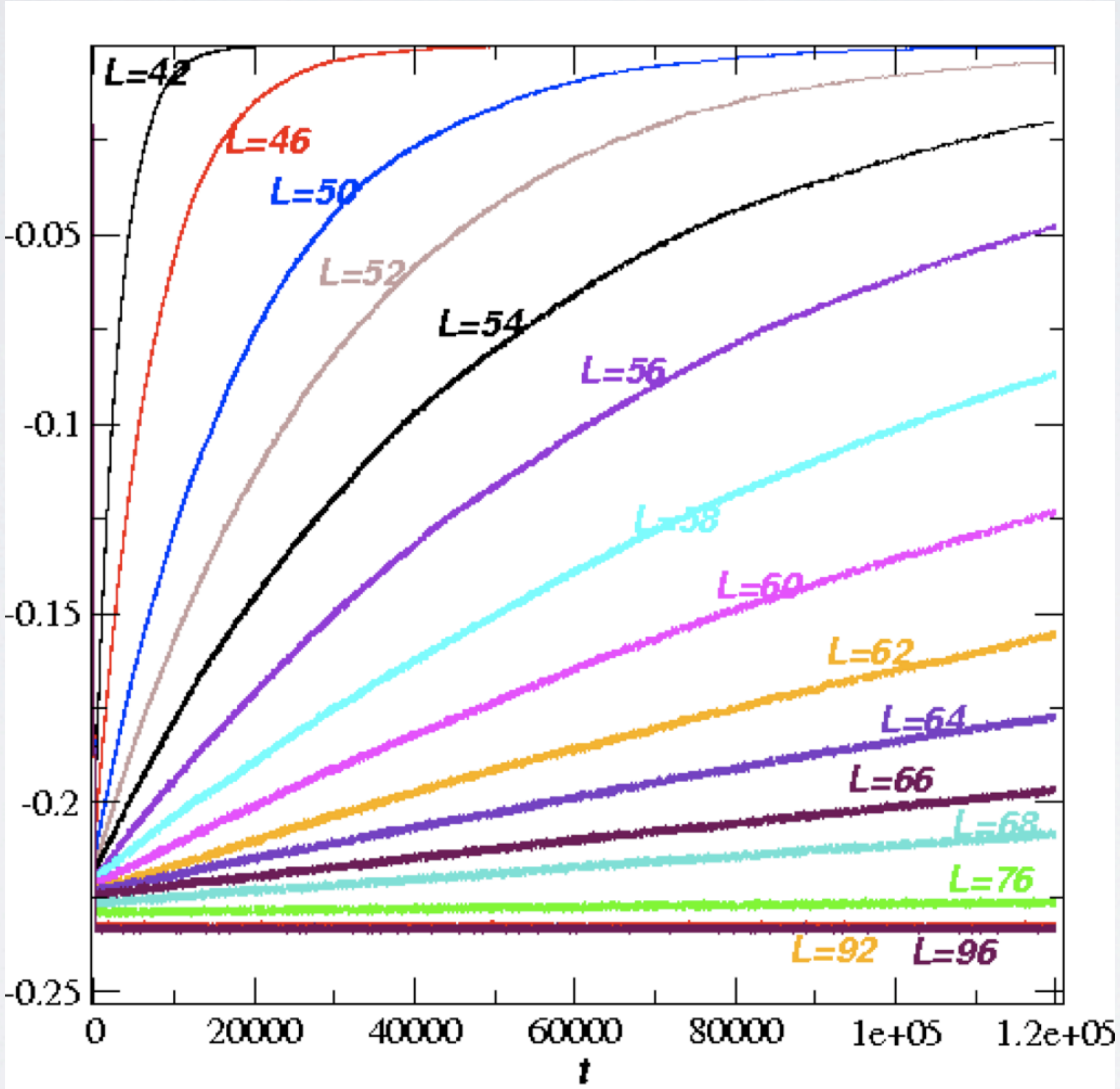
Small lattices starting with:



$\tau(L)$

$$2\frac{n_p}{L-1} - 1$$

substrate gives zero



$L < 46$

Exponential fall-off

$L \sim 62 - 76$

linear fall-off

$L > 92$

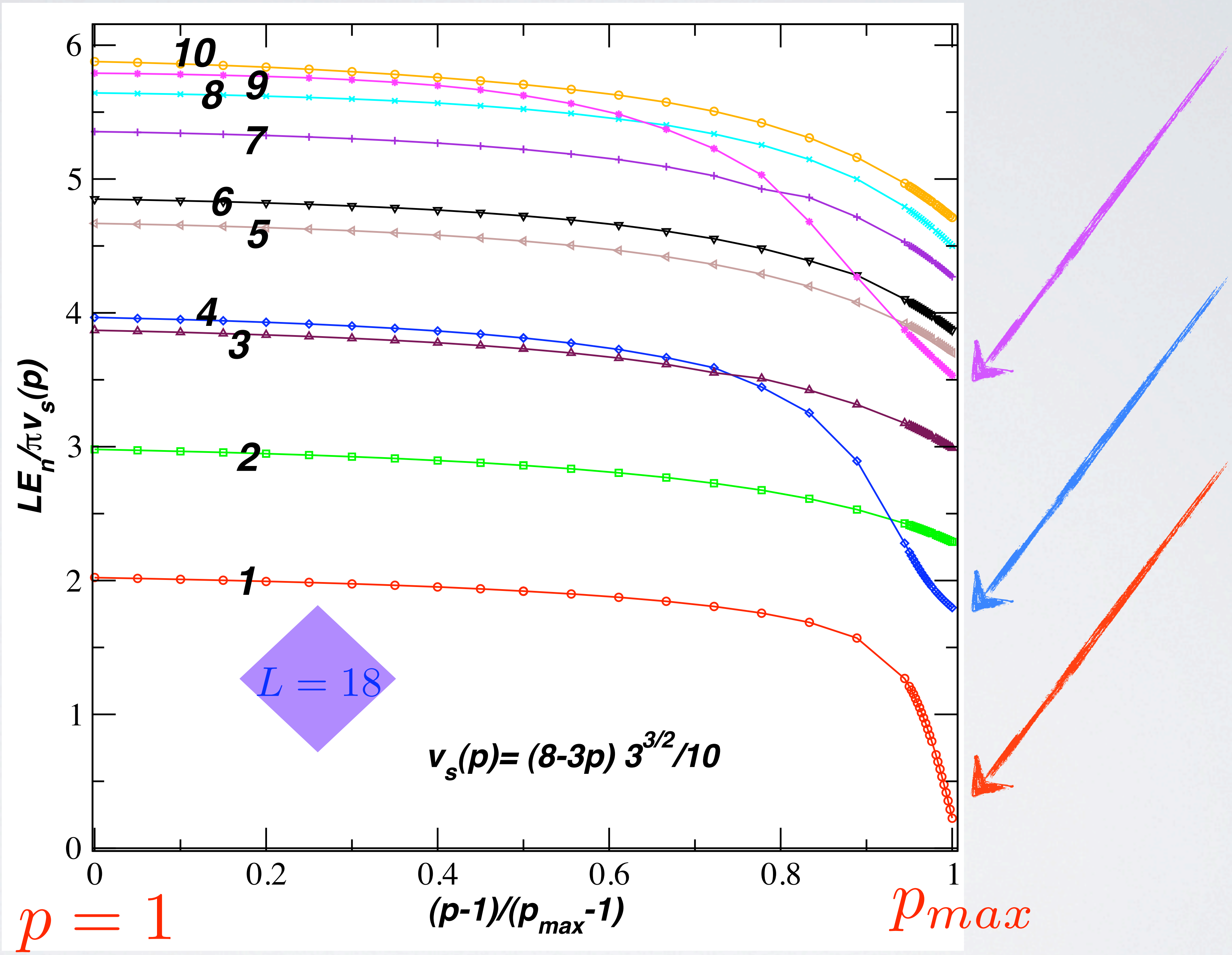
almost no variation

$\tau \sim 0.77$

Conv. Inv. region

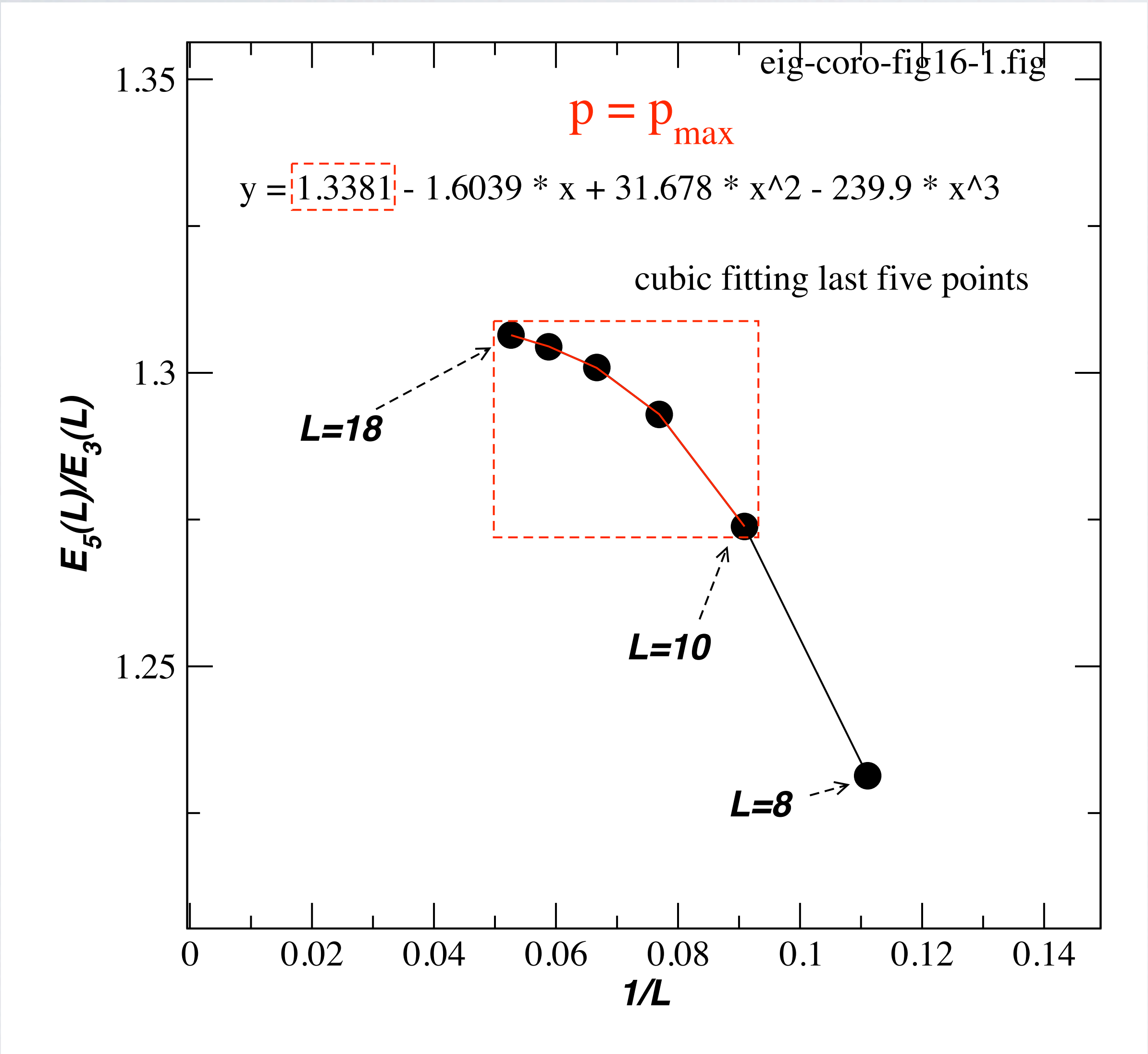
$\tau \sim 0.75$

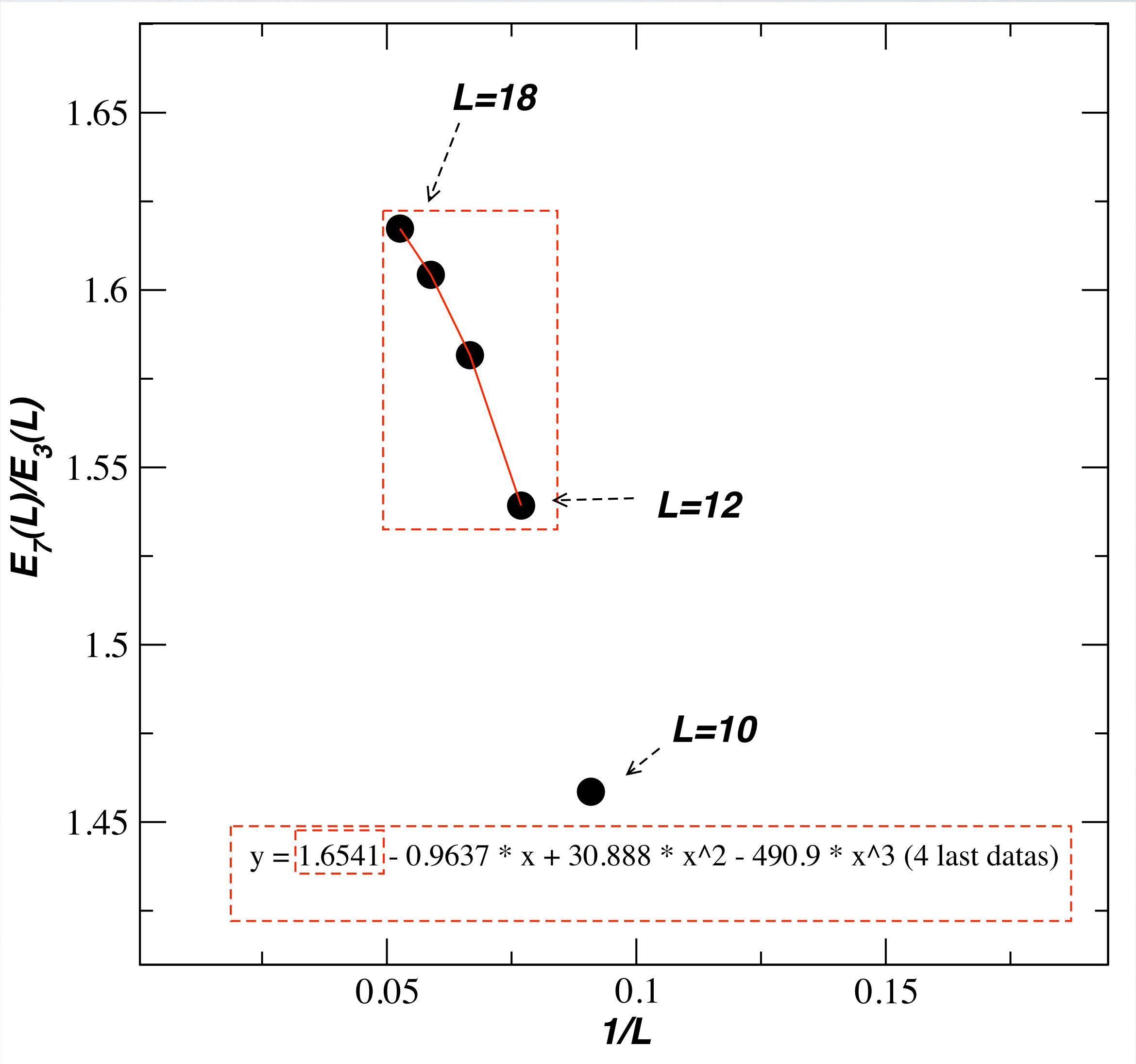
Scaling dimensions: $\Delta = 0(1), 2(1), 3(1), 4(2), 5(2), (4), 7(4)$



As $p \to p_{max}$ some energies go to zero as the absorbing state

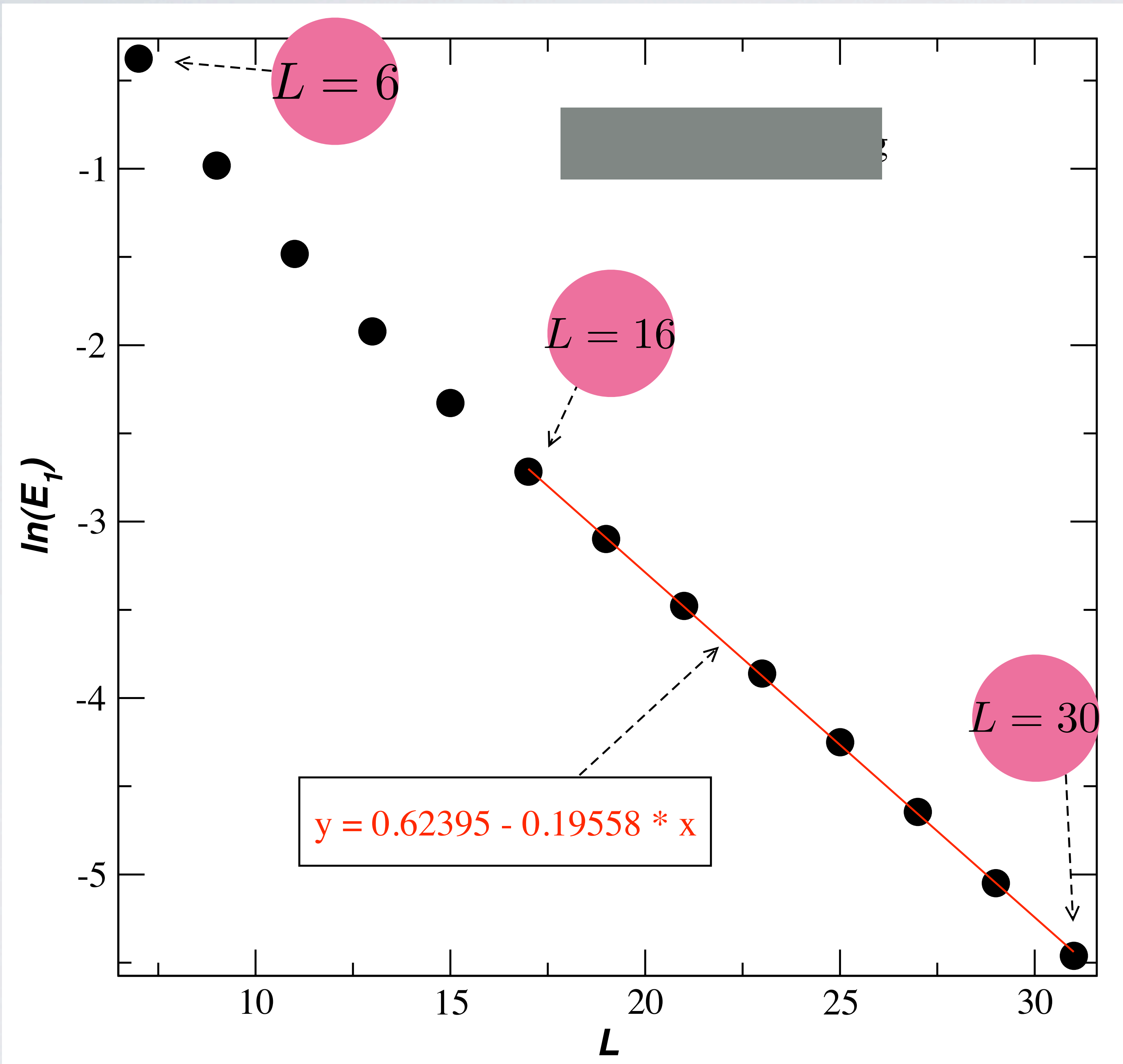
$$BUT \quad E_5(L)/E_3(L) \to 4/3, \qquad E_6(L)/E_3(L) \to 5/3$$





Sound velocity is just $v_s(p_{max})$!!!

First level



$$E_1(L) = 1.86e^{-0.196L}$$

Fourth level

$$E_4(L) = 2.41e^{-0.10L}$$

Origin of at least one quasistationary state

Special property of H in the presence of an absorbing state

$$H_{i,j} \quad (i,j = 0, 1, \ldots, n)$$

absorbing state $\quad H_{i,0} = 0, \quad H_{0,0} = 0; \quad H_{i,j} \leq 0$

$$E_k > 0 \quad K = s, -n; \quad H|k >= E_k|k > \quad H|0 >= 0$$

Identity: $|k >= y_0^{(k)}|0 > + \sum_i y_i^{(k)}|i >$ (sum of components = 0)

$$\tilde{H}_{ij} = h_{ij} \quad (i,j = 1, \ldots, n)$$

$$E_1 \quad \rightarrow \text{ the smallest e.v. of } \tilde{H}_{i,j}$$

$$\text{Perron-Frobenius} \rightarrow y_i^{(k)} \geq 0 \text{ (only for } E_1)$$

$$\rightarrow \quad y_0^{(1)} < 0$$

Solution of differential equations

$$P_0(t,L) = 1 + \sum_k A_k y_0^{(k)} e^{-E_k t}; \quad P_i(t,L) = \sum_k A_k y_i^{(k)} e^{-E_k t}$$

A_k given by initial conditions, for t

$$|P(t,L) >= [(1 - a(L)e^{-E_1(L)t})|0 > +a(L)e^{-E_1(L)t} \sum_i p_i(L)|i >$$

$$p_i(L) >= \frac{y_i^{(1)}(L)}{\sum_i y_i^{(1)}(L)}; \quad \sum_i p_i(L) = 1, \quad a(L) = A_1 \sum_i y_i^{(1)}(L)$$

Stationary state

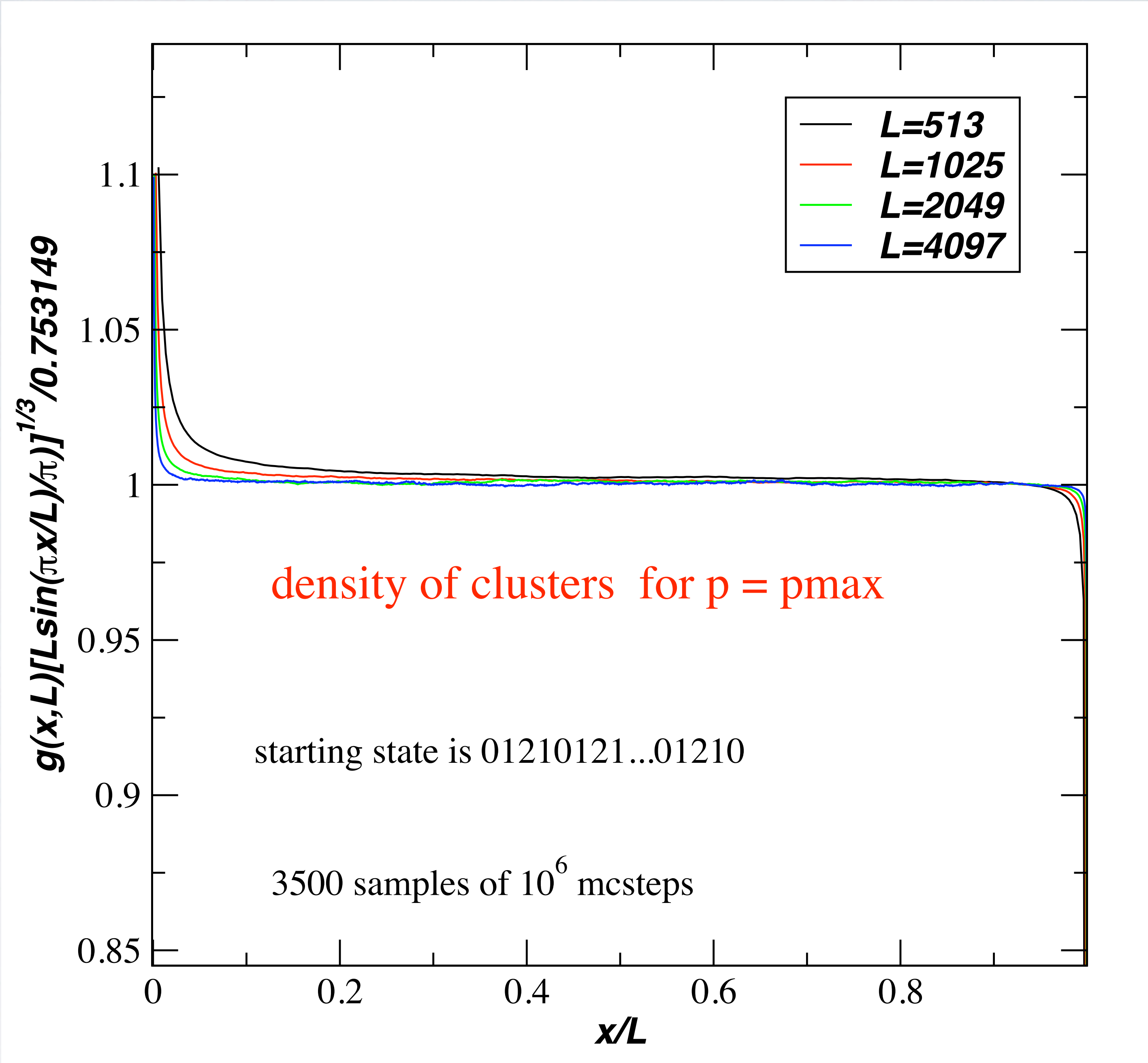
$$|P(\infty, L) >= (1 - a(L))|0 > +a(L)|P_{qs}(L) >$$

In our examples: $a(L) \sim \frac{A}{L}; \quad 1 - \tau(L,t) = 0.25 - \frac{0.8}{L} e^{-E_1(t)t}$

$$E_1 = 1.86 e^{-0.19L}$$

Which properties has the quasistationary state?

Density of contact points $g(x, L)$



Same results as in

the conformal invariant phase !!!

$$0 \leq p < p_{max}$$

Open questions

More QSS?

What kind of left-overs of conformal invariance we have in QSS

Thank you