Dynamics of Compact Relativistic Bodies

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Dynamics of fully relativistic, compact elastic bodies

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◊ Bodies $\mathcal{B}_1$ and $\mathcal{B}_2$ are composed of elastic matter

- asteroids, moons, comets, planets, stars, neutron stars

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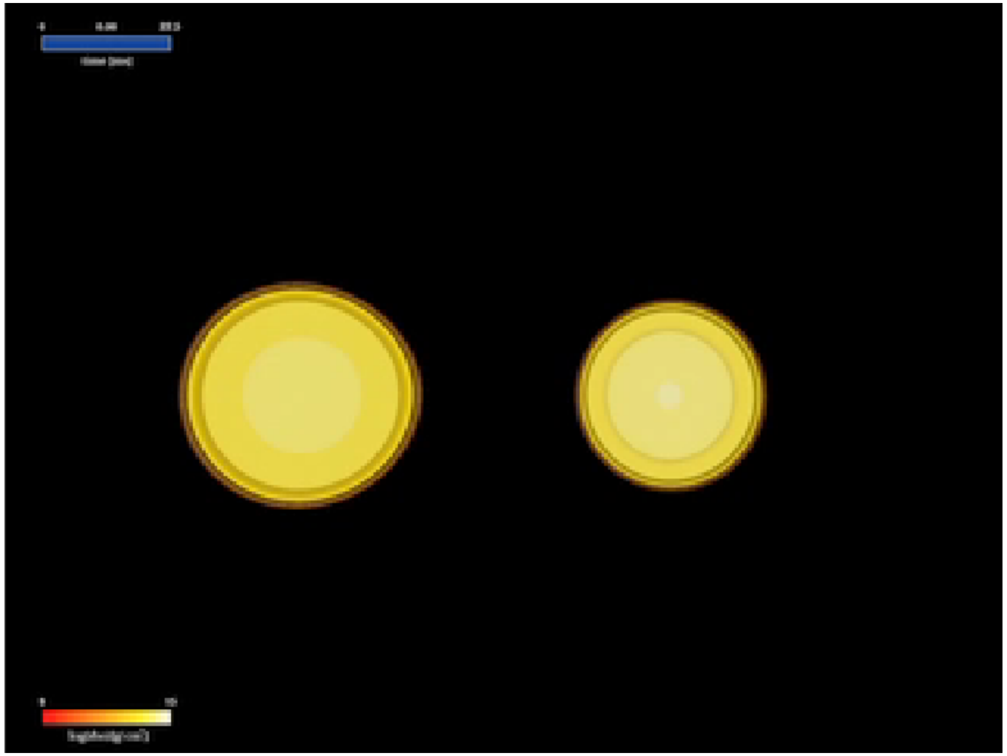
◇ Bodies are fully relativistic.

Basic Problem: Prove the existence of appropriate solutions that represent the evolution of fully relativistic, compact elastic bodies.

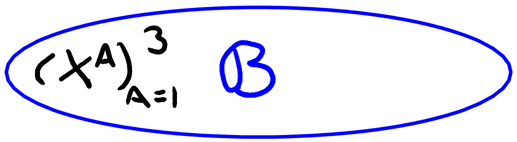


<u>Ref:</u> Rezzolla et. al., Class. Quantum Grav. <u>27</u> (2010) 114105

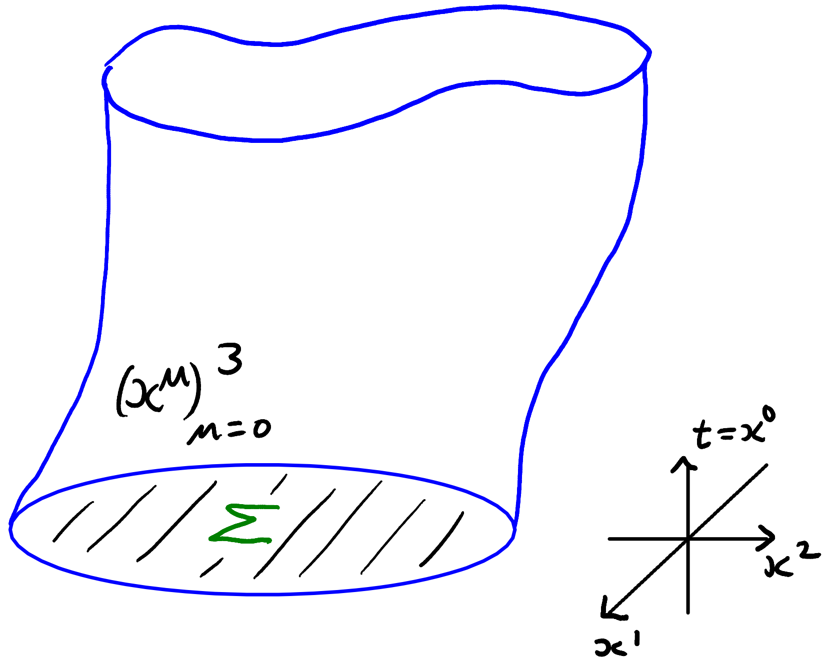
Relativistic Elasticity

Relativistic Elasticity

3-d Material Manifold



Spacetime evolution



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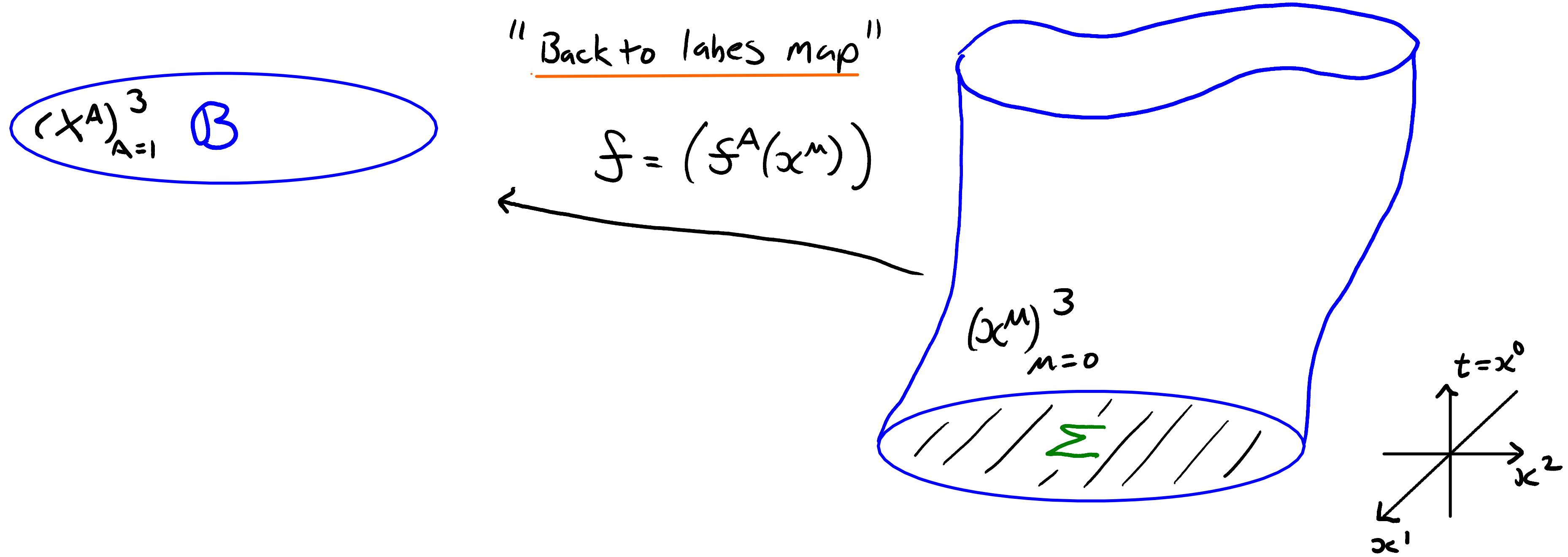


Relativistic Elasticity

3-d Material Manifold

$(X^A)_{A=1}^{3}$ $\mathcal{B}$

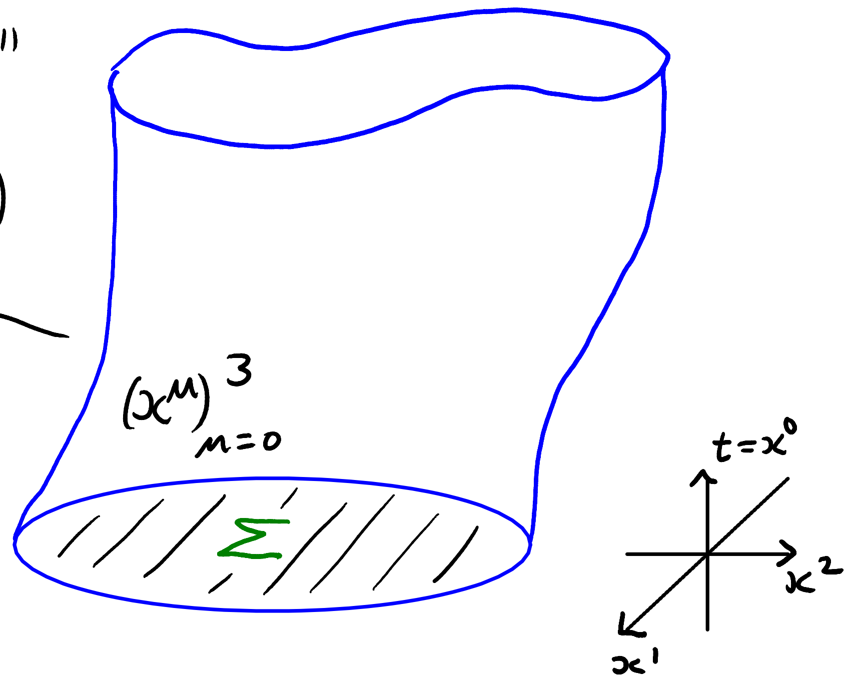
Volume form

Ω_{ABC}

Spacetime evolution

"Back to labels map"

$$f = (f^A(x^\mu))$$

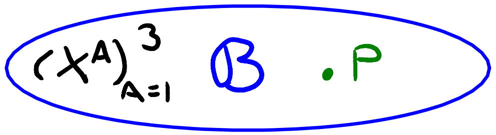


Spacetime metric

$$g = g_{\mu\nu}\, dx^\mu dx^\nu$$

Relativistic Elasticity

3-d Material Manifold



Volume form

$$\Omega_{ABC}$$

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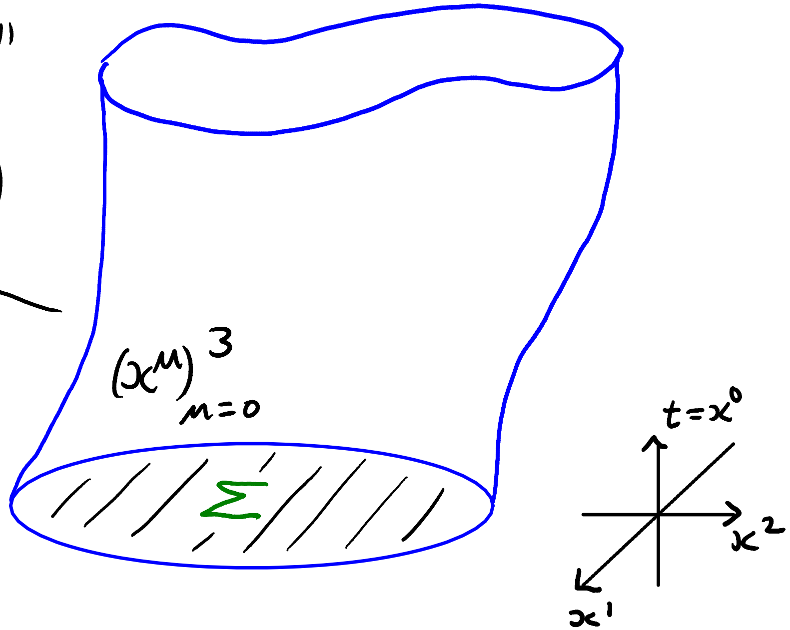
Spacetime evolution



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Relativistic Elasticity

3-d Material Manifold

$(X^A)^3_{A=1}$ $\mathcal{B}$ $\cdot P$

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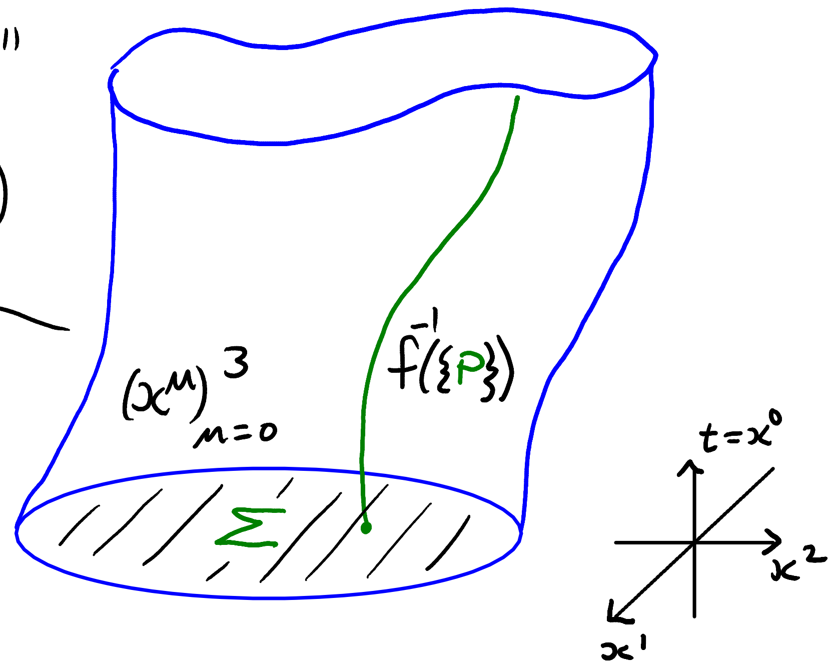

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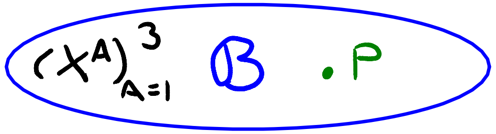

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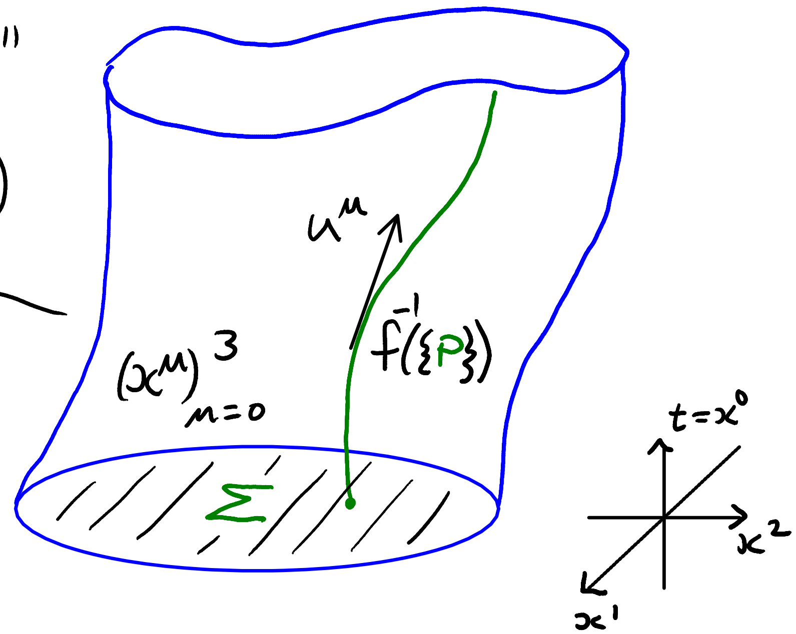

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$$g = g_{\mu\nu}\, dx^\mu dx^\nu$$

Four-velocity u^μ

$$u^\mu \partial_\mu f^A = 0$$

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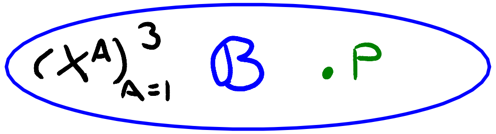
Relativistic Elasticity

3-d Material Manifold



Volume form

$$\Omega_{ABC}$$

Number density n

$$u^\mu = \frac{1}{3!\,n}\,\epsilon^{\mu\nu\lambda\rho}\,\omega_{\nu\lambda\rho}$$

where $\omega = f^*\Omega$ & $\epsilon = \mathrm{vol}(g)$

Spacetime evolution

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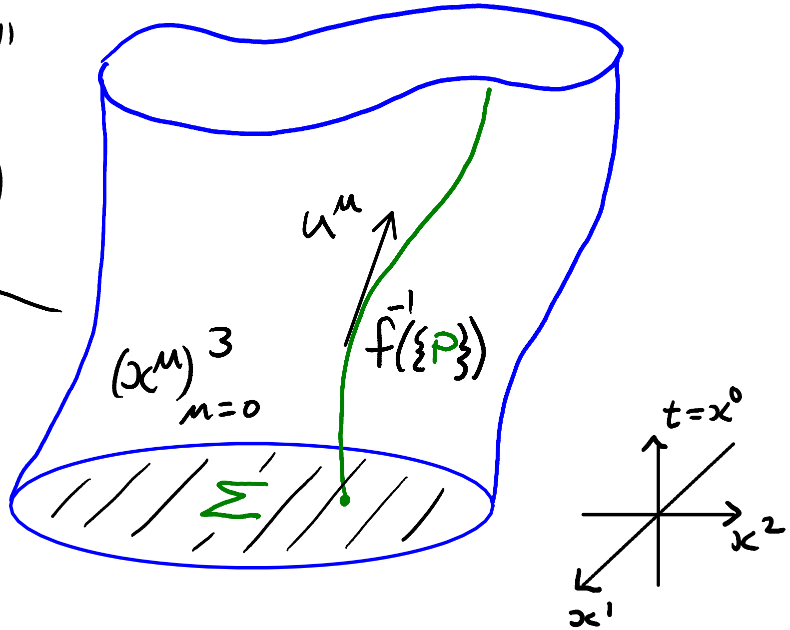

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where $\omega = f^* \Omega$ & $\epsilon = \mathrm{vol}(g)$

$$\boxed{\nabla_\mu (n u^\mu) = 0}$$

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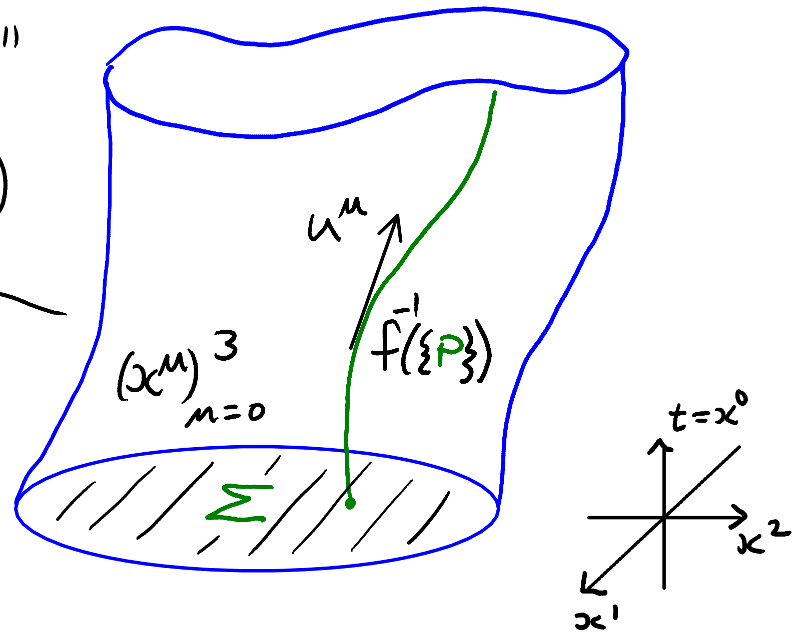

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Field Equations

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Lagrangian

$$S[g,f] = \int_M \rho(f, \partial f, g)\sqrt{-|g|}\, d^4x$$

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Euler-Lagrange equations

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$$\frac{\delta S}{\delta f} = 0 \Rightarrow \boxed{-\mathcal{E}_A := \frac{1}{\sqrt{-|g|}}\partial_\mu\left(\sqrt{-|g|}\,\frac{\partial \rho}{\partial(\partial_\mu f^A)}\right) - \frac{\partial \rho}{\partial f^A} = 0}$$

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Diffeomorphism Covariance

$$\rho = \rho(f, H)$$

where

$$H^{AB} = g^{\mu\nu}\partial_\mu f^A \partial_\nu f^B$$

Stress Energy Tensor

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Alternate form

$$\boxed{T_{\mu\nu} = \rho u_\mu u_\nu + t_{\mu\nu}}$$

where

$$t_{\mu\nu} = n \tau_{AB} \partial_\mu f^A \partial_\nu f^B$$

with

$$\epsilon := \frac{\rho}{n}$$

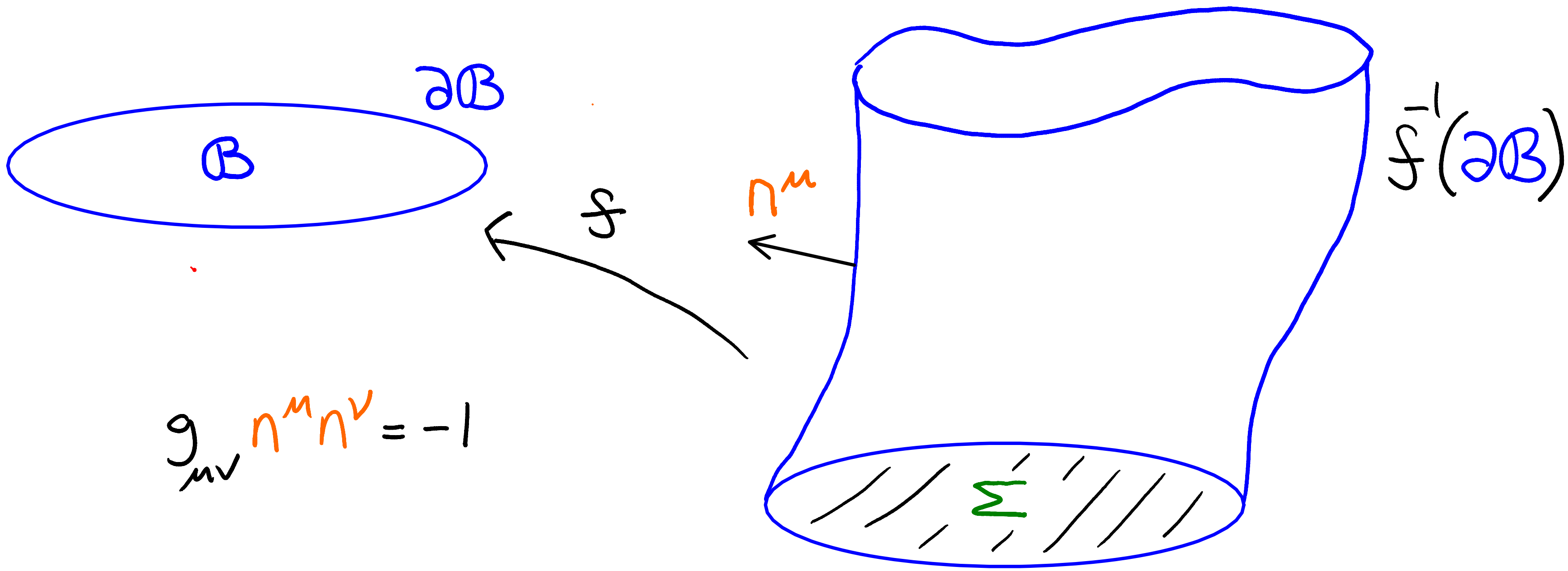
(Stored energy function)

and.

$$\tau_{AB} := 2\frac{\partial\epsilon}{\partial H^{AB}}$$

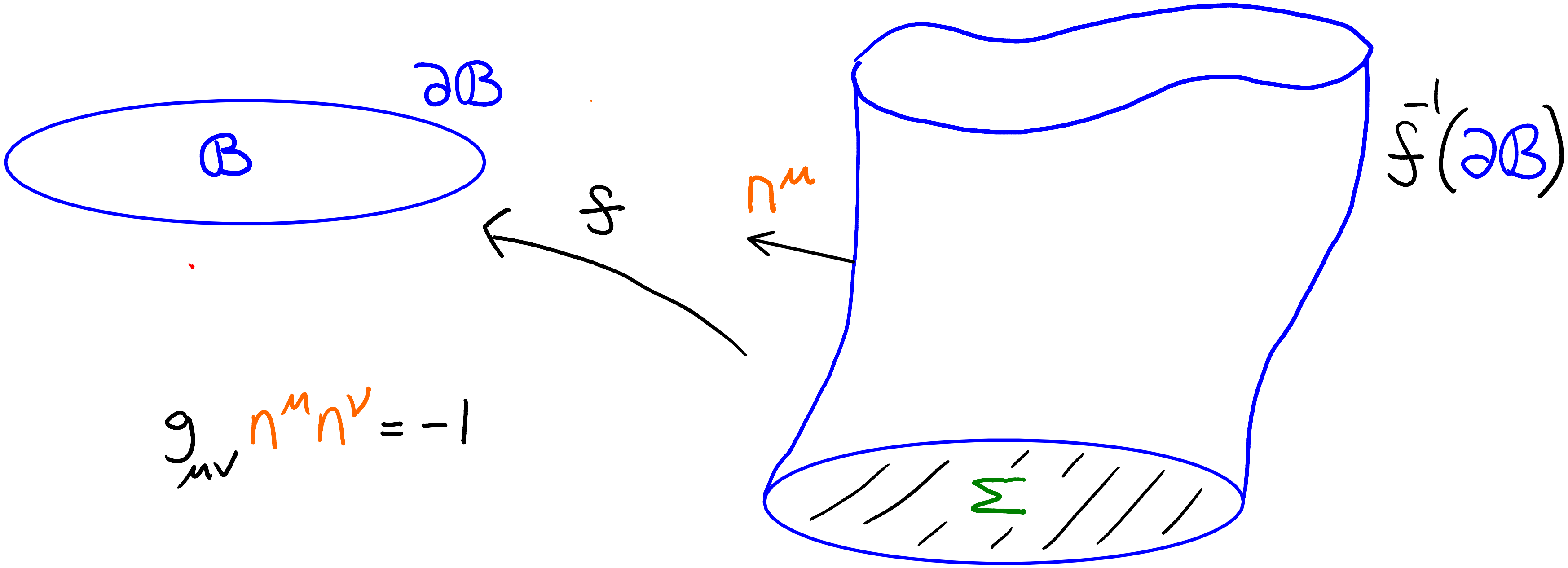
(2nd Piola-Kirchhoff stress tensor)

Boundary Conditions



$$g_{\mu\nu} n^\mu n^\nu = -1$$

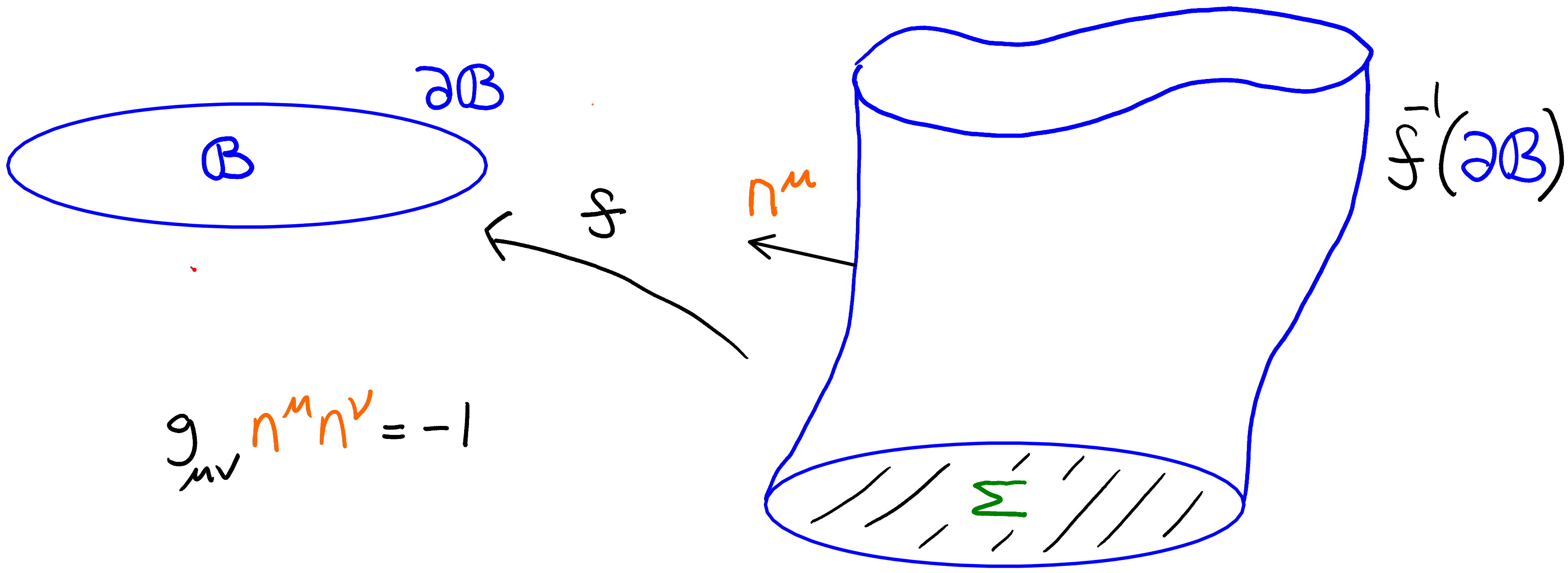
Boundary Conditions



$$g_{\mu\nu} n^\mu n^\nu = -1$$

$$\boxed{T_\mu{}^\nu n_\nu \Big|_{f^{-1}(\partial\mathcal{B})} = 0}$$

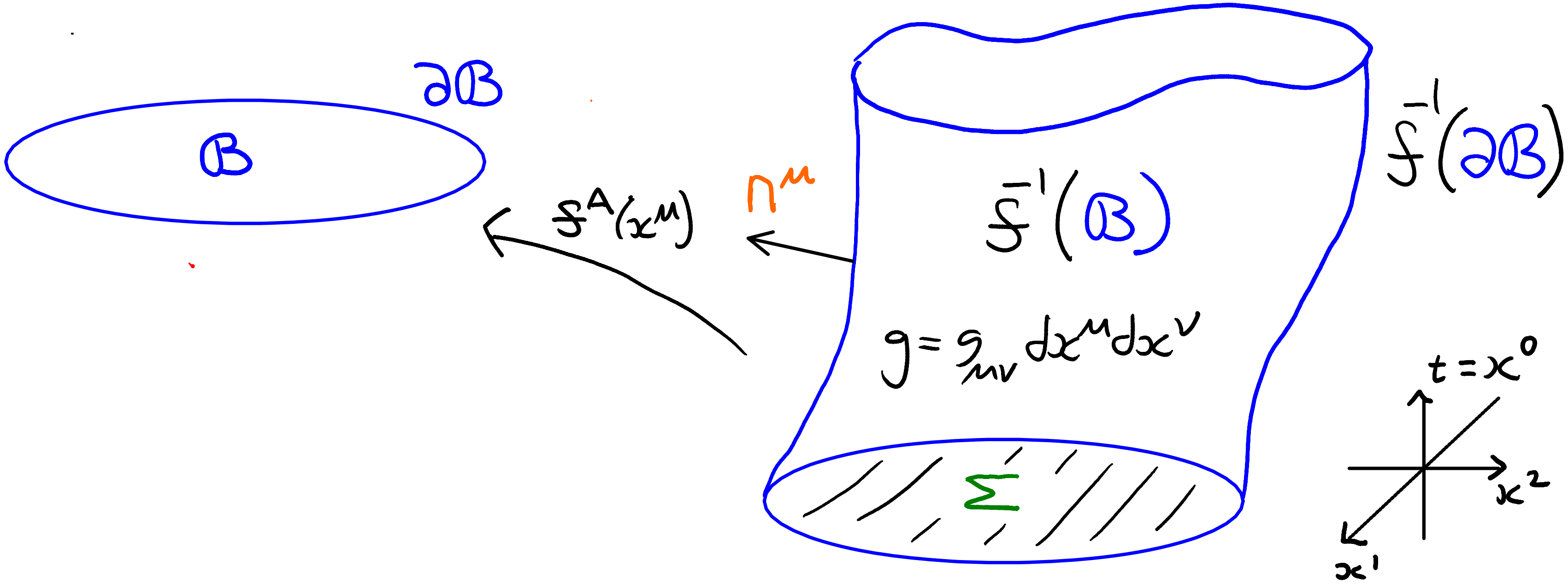
Boundary Conditions



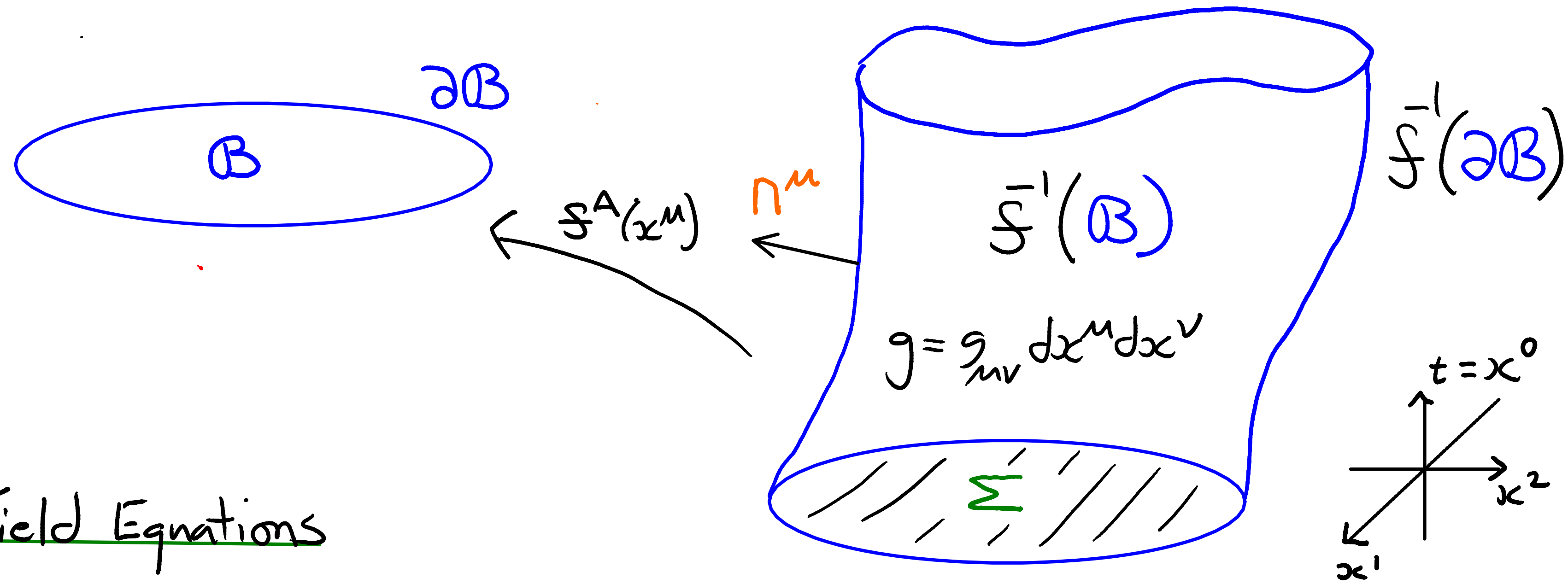
$$g_{\mu\nu} n^\mu n^\nu = -1$$

$$\boxed{T_\mu{}^\nu n_\nu \Big|_{f^{-1}(\partial B)} = 0 \iff \tau_{AB}\, n^\mu \partial_\mu f^B \Big|_{f^{-1}(\partial B)} = 0}$$

Coupling Elasticity to the Einstein Field Equations



Coupling Elasticity to the Einstein Field Equations



Field Equations

$$R_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} T^\lambda{}_\lambda\, g_{\mu\nu} \quad \text{in } f^{-1}(\mathcal{B})$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \text{in } f^{-1}(\mathcal{B})$$

$$n^\mu T_{\mu\nu}\Big|_{f^{-1}(\partial\mathcal{B})} = 0$$

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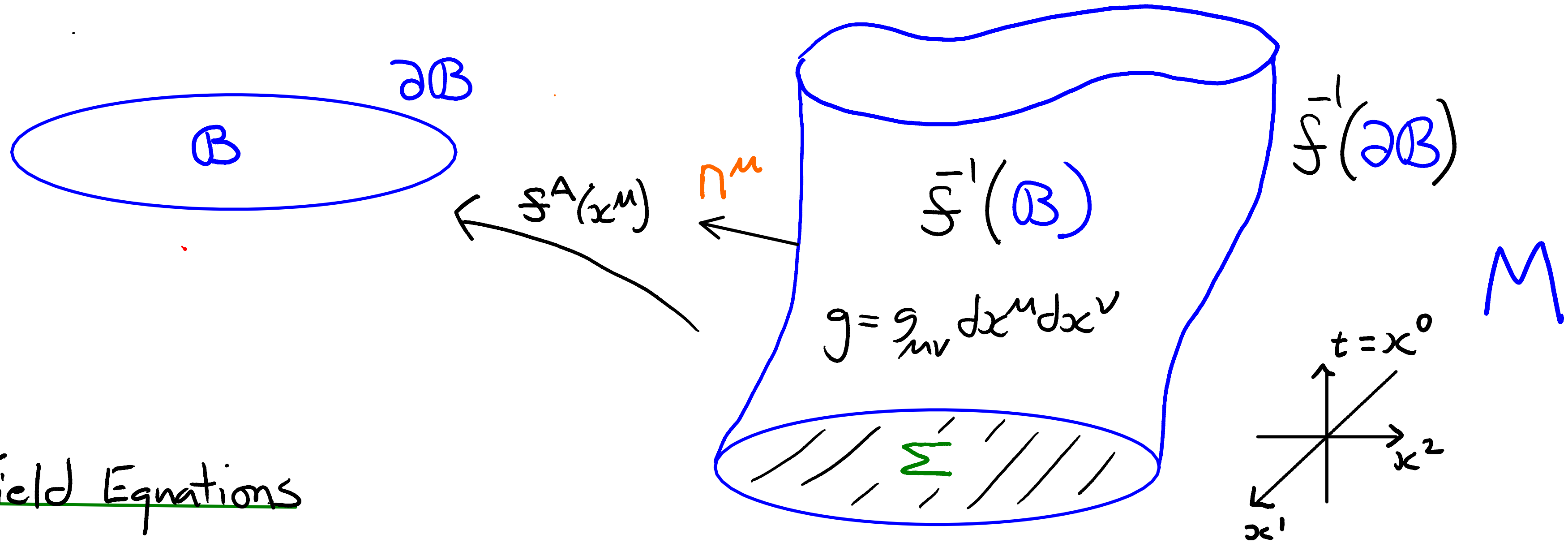


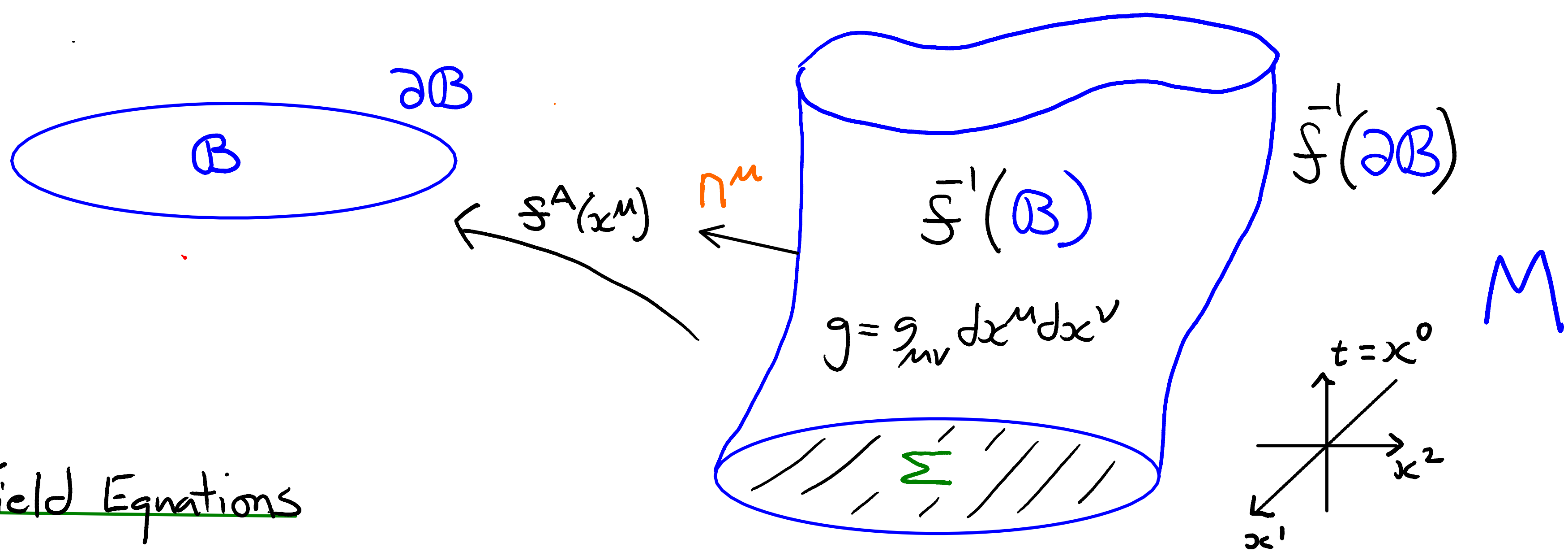
Field Equations

$$R_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} T^\lambda{}_\lambda\, g_{\mu\nu} \quad \text{in } f^{-1}(\mathcal{B}) \text{ or } M \text{ ?}$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \text{in } f^{-1}(\mathcal{B})$$

$$n^\mu T_{\mu\nu}\big|_{f^{-1}(\partial\mathcal{B})} = 0$$

Coupling Elasticity to the Einstein Field Equations



Field Equations

$$\boxed{\begin{aligned} R_{\mu\nu} &= T_{\mu\nu} - \tfrac{1}{2} T^\lambda{}_\lambda\, g_{\mu\nu} \quad &&\text{in } \mathcal{M} \\ \nabla_\mu T^{\mu\nu} &= 0 \quad &&\text{in } f^{-1}(\mathcal{B}) \\ n^\mu T_{\mu\nu}\big|_{f^{-1}(\partial\mathcal{B})} &= 0 && \end{aligned}}$$

Harmonic Coordinates

Harmonic Coordinates

$\diamond$ $$-2R_{\mu\nu} + \left(\nabla_\mu X_\nu + \nabla_\nu X_\mu\right) = -2T_{\mu\nu} + T^\lambda{}_\lambda\, g_{\mu\nu}$$

where

$$X^\mu := g^{\alpha\beta}\,\Gamma_{\alpha\beta}^{\ \mu} = \frac{1}{2} g^{\alpha\beta} g^{\mu\lambda}\left(\partial_\alpha g_{\beta\lambda} + \partial_\beta g_{\alpha\lambda} - \partial_\lambda g_{\alpha\beta}\right)$$

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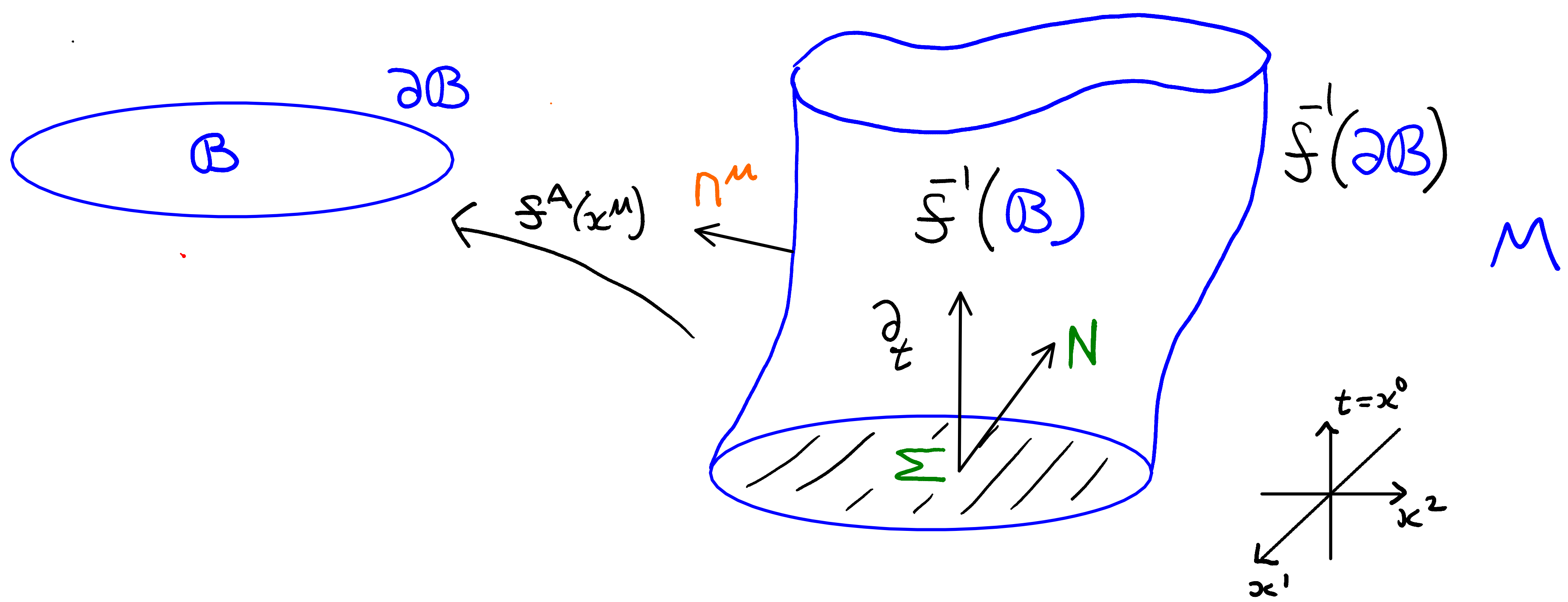
◊ $$g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu} + Q_{\mu\nu}(g,\partial g) = P_{\mu\nu}(g,f,\partial f)$$

Initial Data Complications I: Gravitational Constraint Equations



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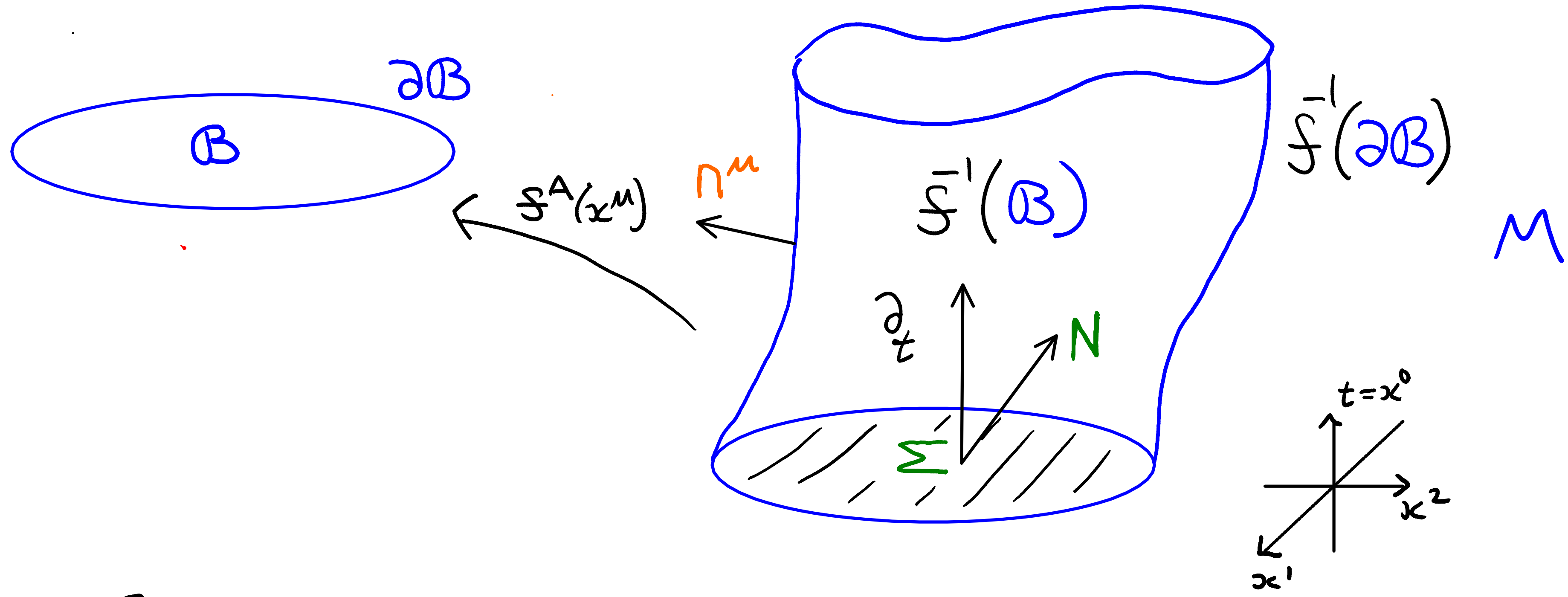


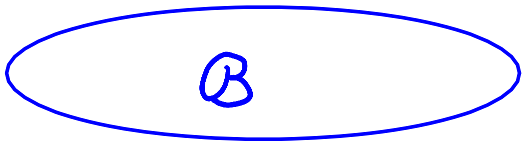
Constraint Equations

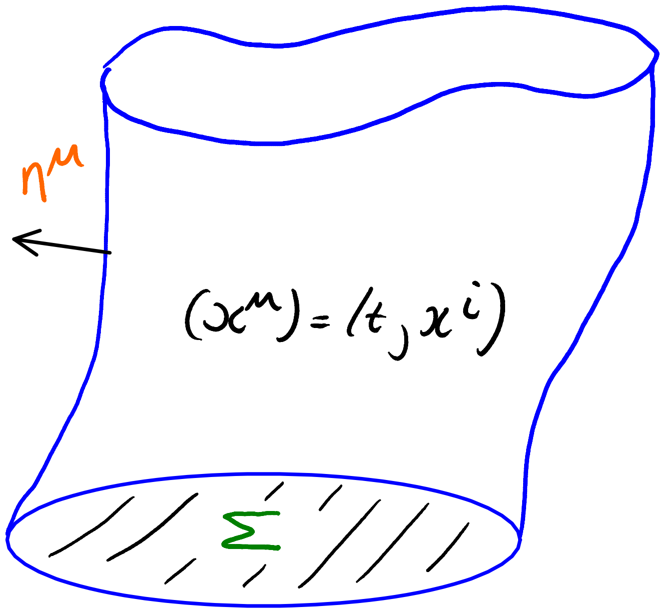
$$\boxed{\left(G^{\mu\nu} - T^{\mu\nu}\right) N_\mu \Big|_\Sigma = 0}$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R^\lambda{}_\lambda \, g_{\mu\nu}$$

Material Representation





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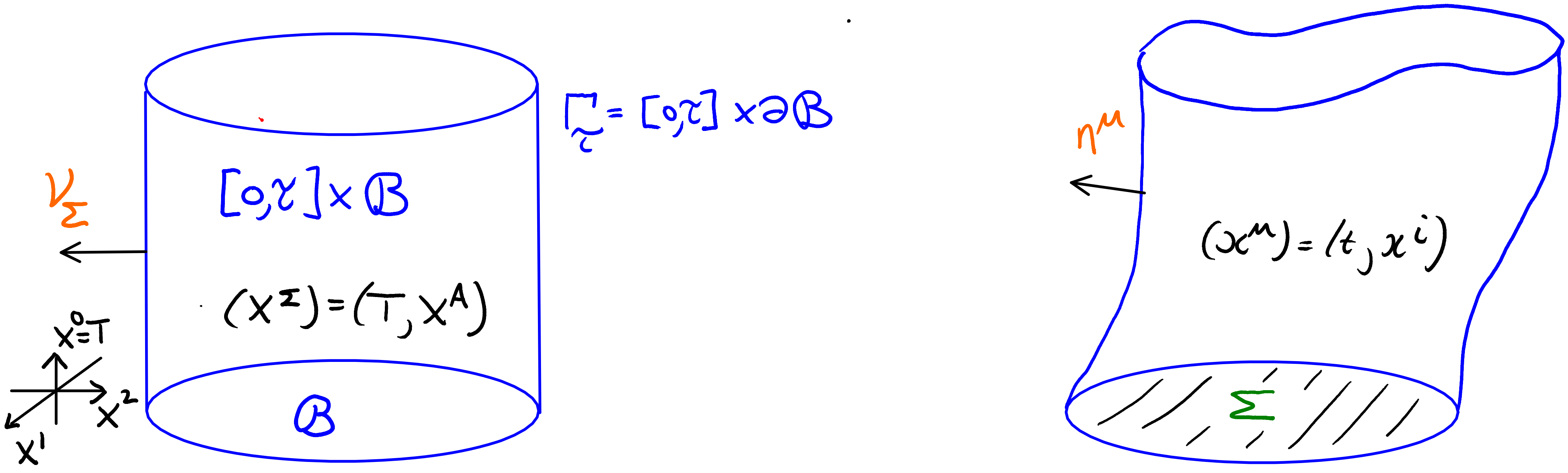

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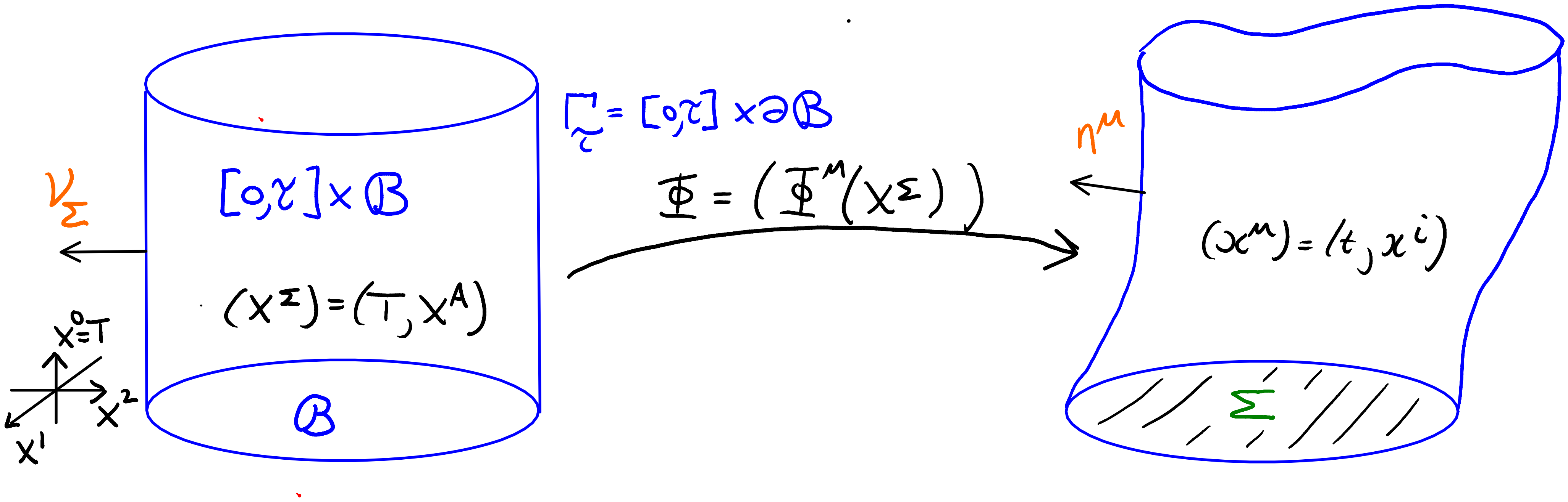

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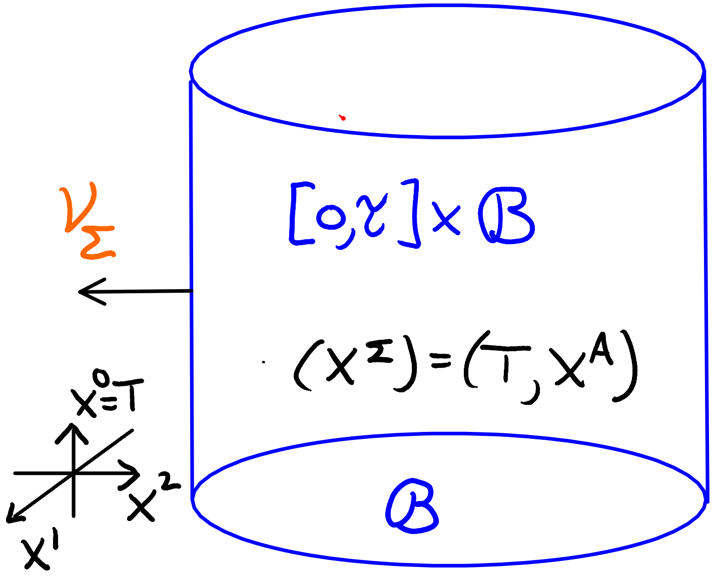
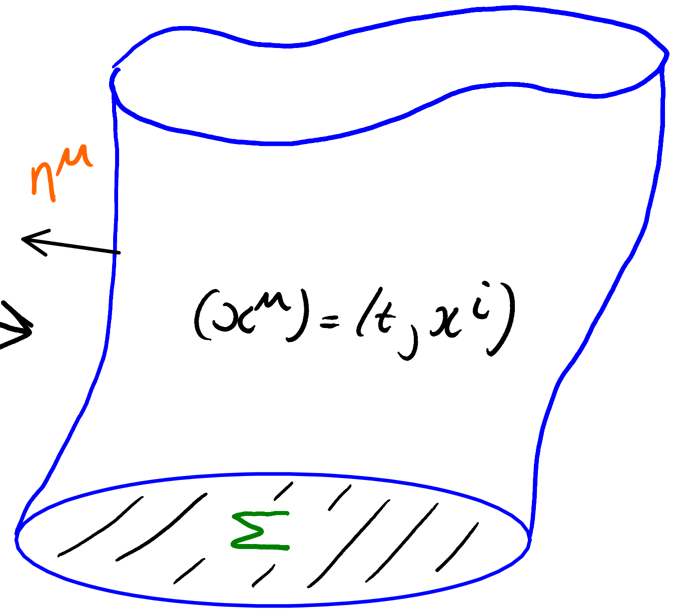


$$\Gamma_\tau=[0,\tau]\times\partial\mathcal{B}$$

$$\Phi=\left(\Phi^\mu(X^\Sigma)\right)$$

$$\boxed{\begin{aligned}&\Phi^0(T,X)=t \Leftrightarrow T=t\\ &f^A\left(t,\Phi^i(T,X)\right)=X^A\end{aligned}}$$



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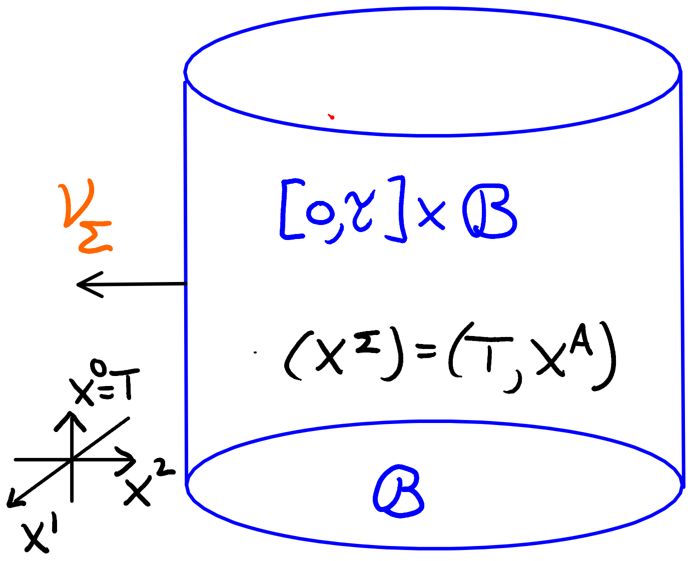

$\Gamma_\tau = [0,\tau]\times\partial\mathcal{B}$

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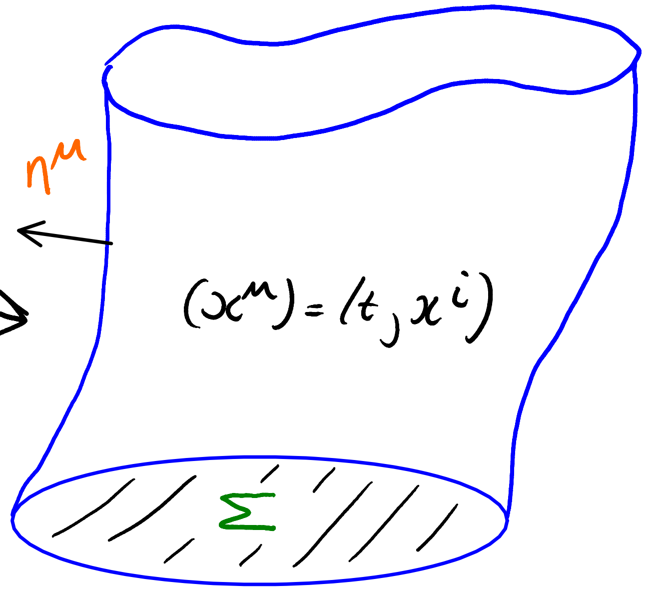


Unknowns:

$$\gamma_{\mu\nu}(T,X) := g_{\mu\nu}\left(T,\Phi(T,X)\right) \qquad (\mu,\nu=0,1,2,3)$$

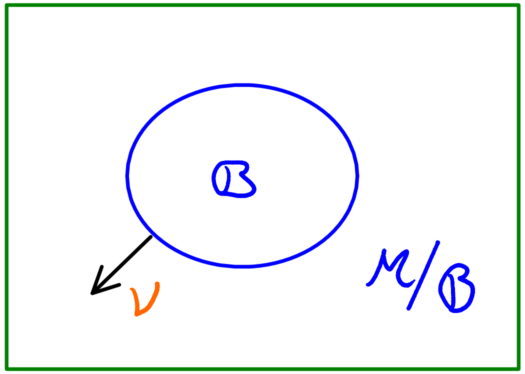
$$\Phi^i(T,X) \qquad (i=1,2,3)$$

Initial Boundary Value Problem

Initial Boundary Value Problem

Extension of Φ^i

$\tilde{\Phi}^i$ defined on $\mathcal{M}$ & $\tilde{\Phi}^i\big|_{\mathcal{B}} = \Phi^i$



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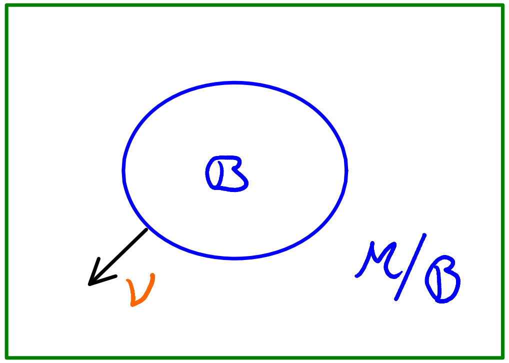

IVBP

$$\partial_\Sigma\left(a^{\Sigma\Lambda}(\gamma,\partial\tilde{\Phi})\partial_\Lambda\gamma_{\mu\nu}\right) = q_{\mu\nu}(\gamma,\partial\gamma,\partial\tilde{\Phi}) + \chi_{\mathcal{B}} P_{\mu\nu}(\gamma,\partial\gamma,\partial\Phi) \text{ in } [0,\tilde{\tau}]\times\mathcal{M}$$

$$\partial_\Sigma\left(F_i^\Sigma(\gamma,\partial\Phi)\right) = w_i(\gamma,\partial\Phi) \quad \text{in} \quad [0,\tau]\times\mathcal{B}$$

$$\nu_\Sigma F_i^\Sigma(\gamma,\partial\Phi)\big|_{\Gamma_T} = 0$$

$$\left(\gamma_{\mu\nu}\big|_{T=0}, \partial_T\gamma_{\mu\nu}\big|_{T=0}\right) = \left(\gamma^0_{\mu\nu}, \gamma^1_{\mu\nu}\right) \text{ in } \mathcal{B}$$

$$\left(\Phi^i\big|_{T=0}, \partial_T\Phi^i\big|_{T=0}\right) = \left(\Phi^i_0, \Phi^i_1\right) \text{ in } \mathcal{B}$$

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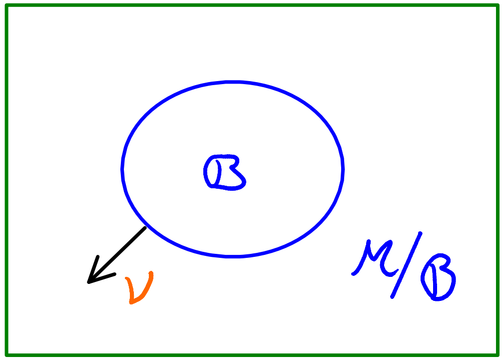

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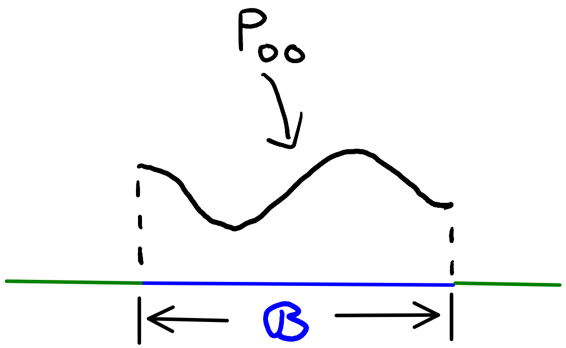

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Elasticity Tensor

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Definition:

$$L^{\mu\nu}_{AB} := \frac{\partial \rho(f, \partial f)}{\partial(\partial_\nu f^B)\,\partial(\partial_\mu f^A)}$$

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Properties:

$$L^{\mu\nu}_{AB} = L^{\nu\mu}_{BA}$$

Elasticity Tensor

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Properties:

$$L^{\mu\nu}_{AB} = L^{\nu\mu}_{BA}$$

Assumptions: $\exists\ \kappa_0, \kappa_1, \mu > 0$ such that

(i) $$\left\langle (\mathbb{P}L)^{ij}_{AB}\, \partial_i \psi^A \,\middle|\, \partial_j \psi^B \right\rangle_{L^2(\Sigma)} \geq \kappa_0 \|\psi\|_{H^1(\Sigma)} - \mu \|\psi\|_{L^2(\Sigma)}$$

where

$$\Sigma = \left(\xi\big|_{t=0}\right)^{-1}(\partial B)$$

(ii) $$L^{00}_{AB}\,\phi^A \phi^B \leq -\kappa_1 |\phi|$$

Function Spaces

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◊ WLOG assume $\mathcal{M} \cong \mathbb{T}^3$

Function Spaces

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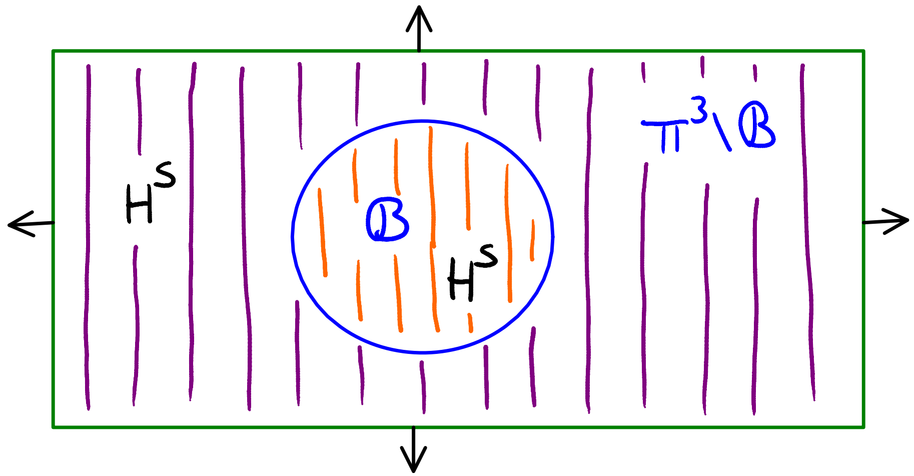

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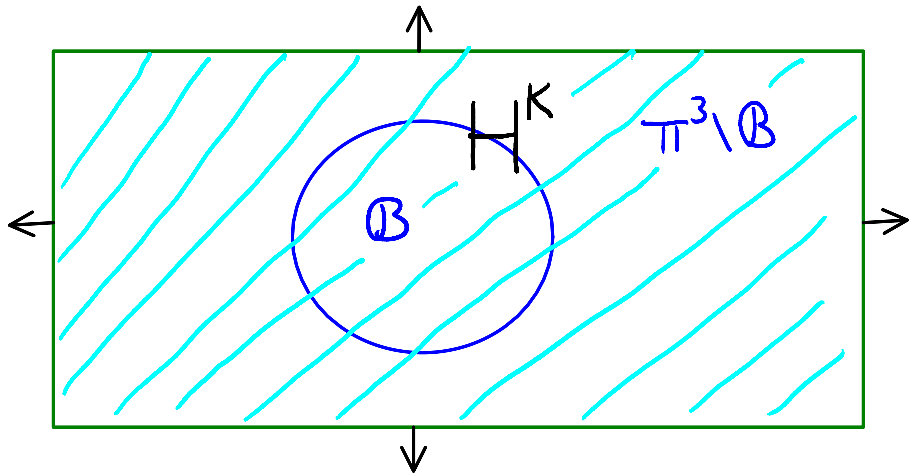

Initial Data Complications II: Compatibility conditions

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$$s \in \mathbb{Z}_{> \frac{5}{2}}$$

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such that $(\gamma_{\mu\nu}, \Phi^i)$ is the unique solution of the IVBP:

$$\partial_\Sigma\left(a^{\Sigma\Lambda}(\gamma, \partial\tilde{\Phi})\partial_\Lambda \gamma_{\mu\nu}\right) = q_{\mu\nu}(\gamma, \partial\gamma, \partial\tilde{\Phi}) + \chi_{\mathcal{B}} P_{\mu\nu}(\gamma, \partial\gamma, \partial\Phi) \text{ in } [0,\tilde{\tau}] \times M$$

$$\partial_\Sigma\left(F_i^\Sigma(\gamma, \partial\Phi)\right) = w_i(\gamma, \partial\Phi) \text{ in } [0,\tilde{\tau}] \times \mathcal{B}$$

$$\nu_\Sigma F_i^\Sigma(\gamma, \partial\Phi)\Big|_{\Gamma_T} = 0$$

$$\left(\gamma_{\mu\nu}\big|_{T=0}, \partial_T \gamma_{\mu\nu}\big|_{T=0}\right) = \left(\gamma^0_{\mu\nu}, \gamma^1_{\mu\nu}\right) \text{ in } \mathcal{B}$$

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Assumption:

$$\boxed{\rho = \rho(n)}$$

where

$$n = \left(\frac{1}{6} H^{AB} H^{CD} H^{EF} \Omega_{ACE} \Omega_{BDF} \right)$$

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Consequences:

◇ $$\boxed{T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}}$$

where

$$p = n \frac{\partial \rho}{\partial n} - \rho$$

◇ $$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

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LEU for the Euler equations
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- incompressible + Newtonian gravity

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