Novel Phase Transitions in XY Antiferromagnets on Plane Triangulations

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"Novel phase transitions in XY Antiferromagnets on Plane Triangulations"

The XY model

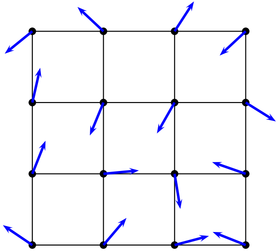
XY model on graph $G = (V, E)$:

- Configuration space is $(S^1)^V$
- Gibbs measure (at temperature T)

$$\mathrm{d}\mu(\mathbf{s}) \propto \exp(-H(\mathbf{s})/T) \prod_{v \in V} \mathrm{d}\mathbf{s}_v$$

  - $\mathrm{d}\mathbf{s}_v$ is uniform measure on S^1
  - Hamiltonian is $\mathrm{O}(2)$ symmetric

$$H(\mathbf{s}) = -J \sum_{uv \in E} \mathbf{s}_u \cdot \mathbf{s}_v$$



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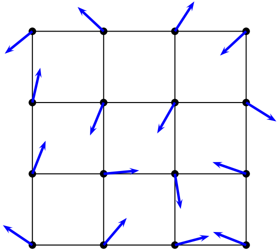

On two-dimensional lattices:

- Mermin-Wagner Theorem forbids magnetic ordering at $T > 0$
- Ferromagnetic model has Kosterlitz-Thouless transition:
  - For all $T \leq T_c$

$$\mathbb{E}(\mathbf{s}_0 \cdot \mathbf{s}_x) \sim |x|^{-\eta(T)} \qquad \mathrm{as} |x| \to \infty$$

  - Quasi-long range order
  - $\eta(T)$ is an increasing function of T on $[0, T_c]$

Josephson Junction Arrays

Josephson Junction Array in a uniform transverse magnetic field

- Planar graph $G = (V, E)$
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  - Edges are Josephson junctions
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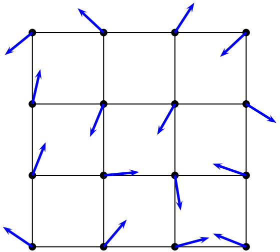
  - $A_{uv} = 2\pi \int_u^v \mathbf{A} \cdot \mathrm{d}\mathbf{l}$



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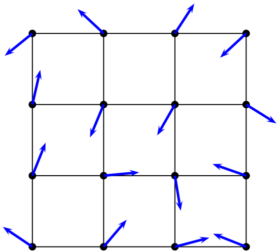

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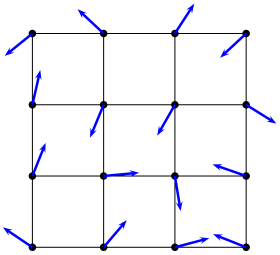

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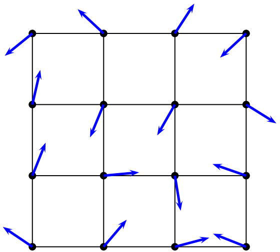

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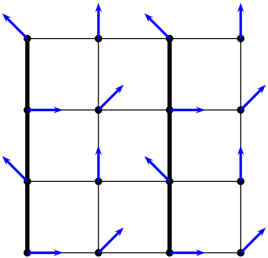

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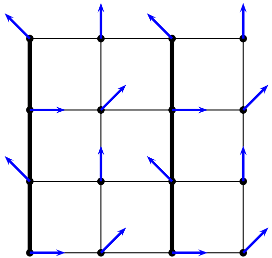
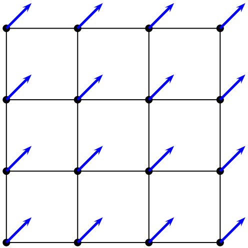


Ground states of XY model

- Ferromagnetic couplings ($J = +1$)
  - Ground states totally ordered
  - Generated by $\mathrm{SO}(2)$



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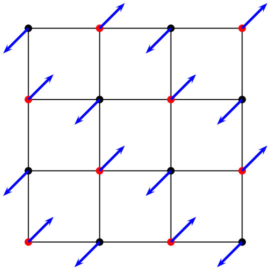

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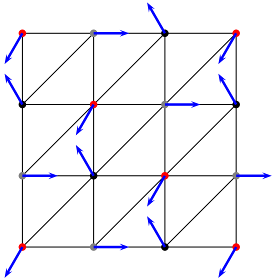

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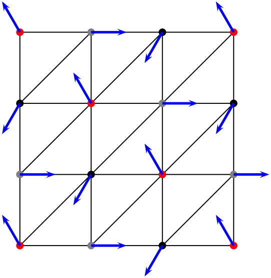

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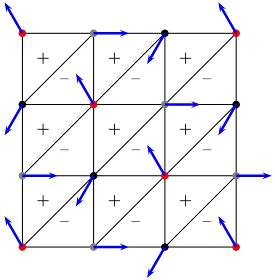

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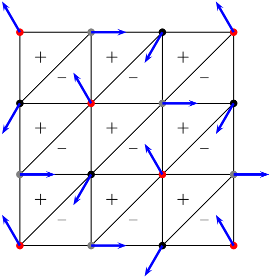

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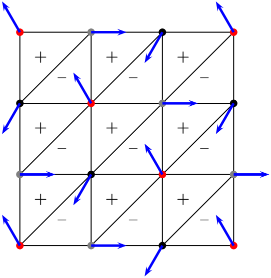

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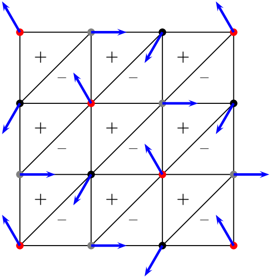

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  - Same ground states arise on any Eulerian plane triangulation
  - Square-lattice FFXY also has same ground state degeneracies

Phase transitions

For both the square-lattice FFXY and triangular-lattice AFXY models:

- At low temperatures:
  - Long-range chiral order
  - Quasi long-range magnetic order on each sublattice
- At high temperatures:
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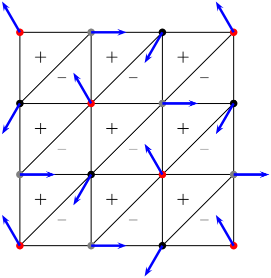
  - $T_{\rm s} < T_{\rm c}$
- What happens on other Eulerian triangulations?

Order parameters

Consider an Eulerian plane triangulation



- Three sublattices $A, B, C \subset V$
- Each sublattice S has its own magnetization

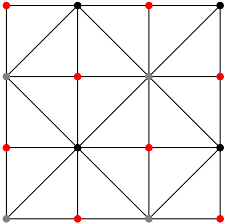
$$M_S = \frac{1}{|S|} \sum_{v \in S} \mathbf{s}_v$$

- The chiral order parameter is

$$M_{\rm c} = \frac{1}{\#\text{faces}} \sum_{ijk \in \text{faces}} \text{sgn}[\sin(\theta_i - \theta_j) + \sin(\theta_j - \theta_k) + \sin(\theta_k - \theta_i)],$$

Union Jack lattice

- Eulerian triangulation which is also a Laves lattice



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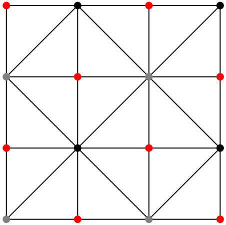

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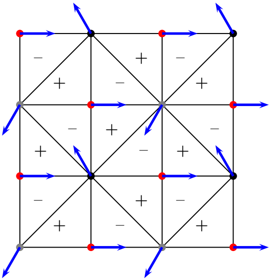

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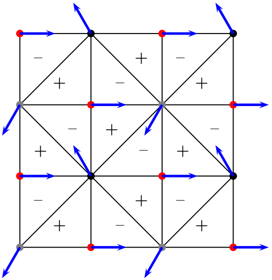

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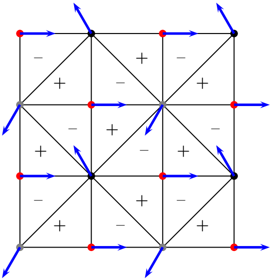

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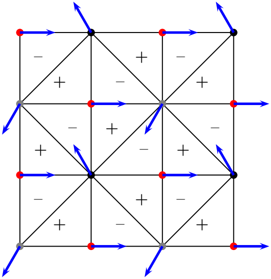

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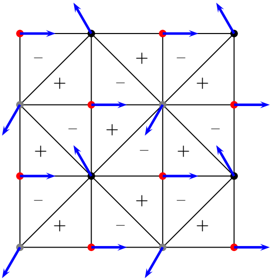
- Fit Q_c to finite-size scaling ansatz

$$Q_\mathrm{c} = Q_\mathrm{c}^* + a_1 t\, L^{1/\nu} + a_2 t^2 L^{2/\nu} + \ldots, \qquad t := (T - T_\mathrm{c})$$

  - Gives $T_\mathrm{c} = 0.4316(1)$, and $\nu = 1.01(1)$



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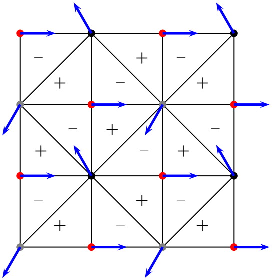
$$\chi_\mathrm{c} = L^{2-\eta} \left(a_0 + a_1 t\, L^{1/\nu} + a_2 t^2 L^{2/\nu} + \dots \right)$$

  - Gives $\eta = 0.252(6)$



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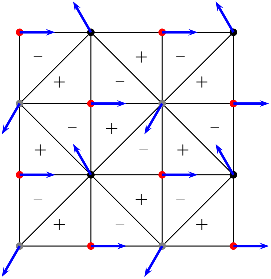

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- Compare exact Ising values $\nu = 1$ and $\eta = 1/4$

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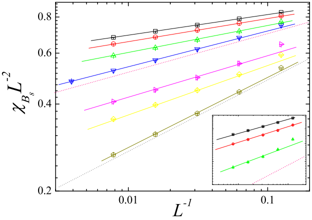
- We observe $\chi \sim L^{2-\eta(T)}$



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$T = 0.36, 0.39, 0.42$ for A_4 (inset)

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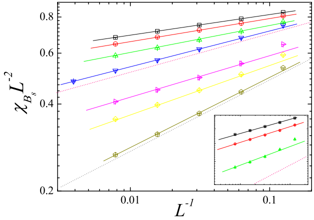

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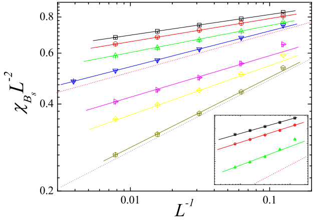

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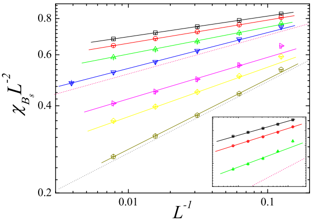

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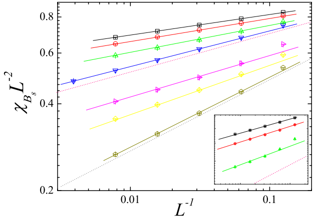

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Sublattice magnetizations disorder at different temperatures

Transition on A_4

- Magnetic transition of A_4 spins at T_c appears to be Ising
  - Very good fits of Binder parameter $Q_{A_4} = \langle M_{A_4}^2 \rangle^2 / \langle M_{A_4}^4 \rangle$ to

$$Q_{A_4} = Q_{A_4}^* + a_1 t\, L^{1/\nu} + a_2 t^2 L^{2/\nu} + \dots ,$$

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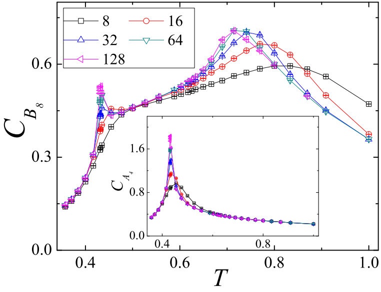
B_8 and C_8 undergo Ising transition at T_c
Separates two QLR-ordered phases
(C_{B_8} specific heat on B_8)



Bisected hexagonal lattice

- There are three Laves lattices which are Eulerian triangulations:
  - Triangular lattice $[6^3]$
  - Union Jack lattice $[4, 8^2]$
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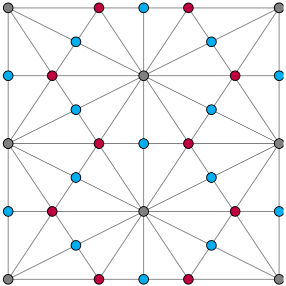

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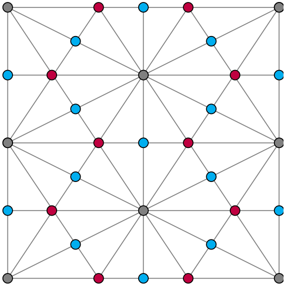

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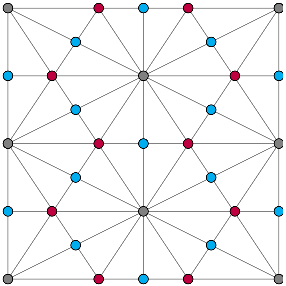

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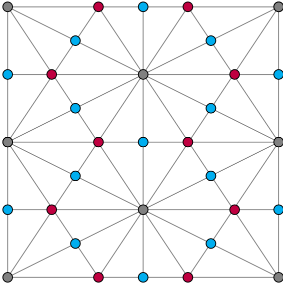

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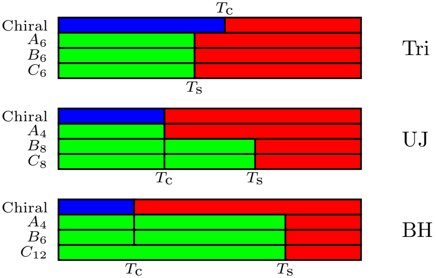
- A_4 and B_6 have Ising transition at T_{c}

Discussion



- A_4 disorders at T_c on UJ
- $T_s > T_c$ on UJ and BJ
- $T_s < T_c$ on $\triangle$ and $\square$
- $T_c(Tri) > T_c(UJ) > T_c(BH)$
- $T_s(Tri) < T_s(UJ) < T_s(BH)$

- For all three Eulerian Laves triangulations the AFXY model has a distinct Ising chiral transition
  - Should be generic on all Eulerian plane triangulations
- Magnetic transitions strongly dependent on specific lattice topology
- We observe Ising magnetic transition in the standard XY antiferromagnet
  - Both QLR-ordered to disordered and QLR-ordered to QLR-ordered