Numerical space-times near space-like and null infinity

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Introduction

The defining problem of Numerical Relativity

The binary black-hole problem

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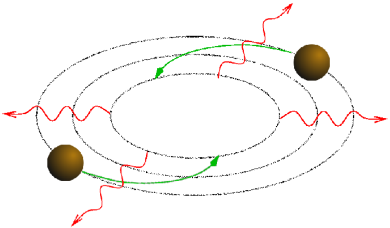

- Two compact objects (BH/NS) in gravitational interaction
- Inspiraling orbits due to gravitational radition

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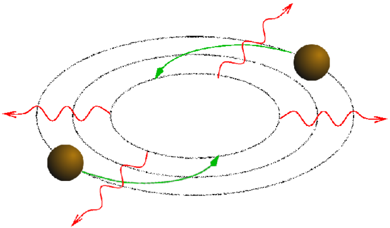

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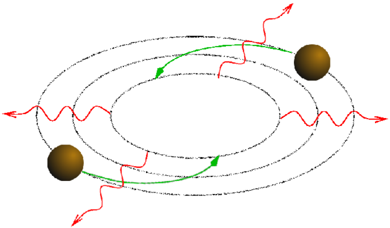

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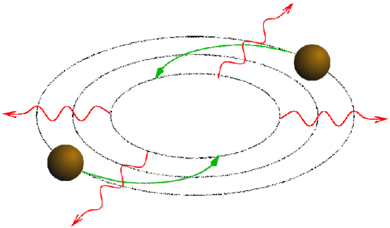

- Two compact objects (BH/NS) in gravitational interaction
- Inspiraling orbits due to gravitational radition
- Compute the emitted waves
- Can be partly handled by approximations
- But computer simulation is necessary for details

Introduction

Successes

Numerical Relativity has had several breakthroughs in the last decade

- Binary black-hole problem can be considered as solved
- Long-term stable evolution of full 3D scenarios
- Gravitational wave-form computations are possible for many different parameter sets
- Gravitational wave recoil ('kicks') can be considered as theoretical prediction to be verified by observations (see Camossa et al)

Introduction

Remaining problems

Yet, some largely unexplored issues remain

- Insights into the formation of singularities and horizons
- Cosmic censorship
- The outer boundary

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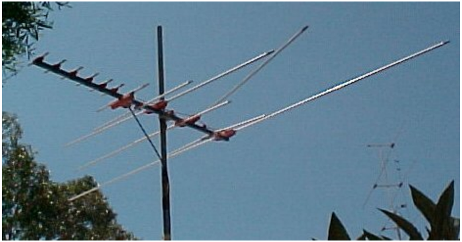
Conformal Infinity

The physical problem



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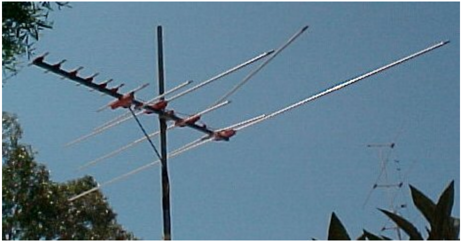

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Conformal Infinity

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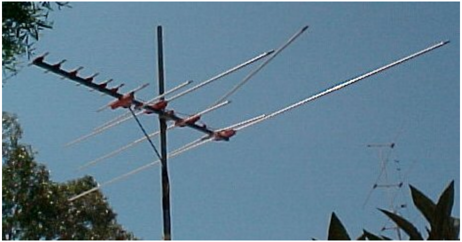

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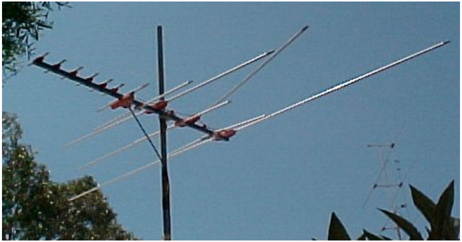

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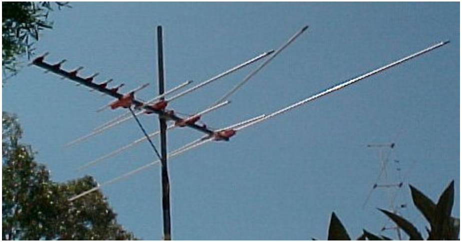

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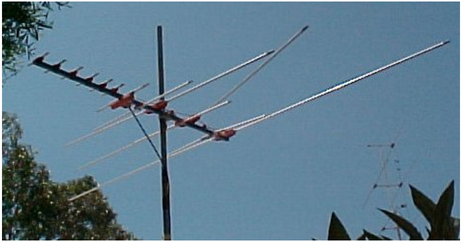

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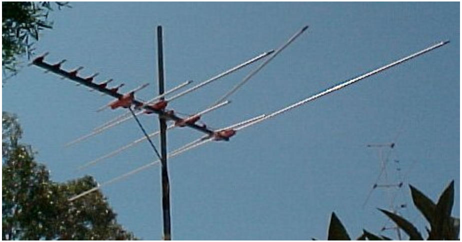

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- BUT: non-linear theory, so modes do not separate
- Gravitational waves are well defined only at infinity
- Accurate simulations need to take the limit $r \to \infty$

Conformal Infinity

R. Penrose

Ignore the scale of space-time, focus on its shape (locally)

Conformal Infinity

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Ignore the scale of space-time, focus on its shape (locally)

Mathematically, regard physical metric g_{ab} defined "up to scalings"

$$g_{ab} \mapsto \Omega^2 g_{ab}$$

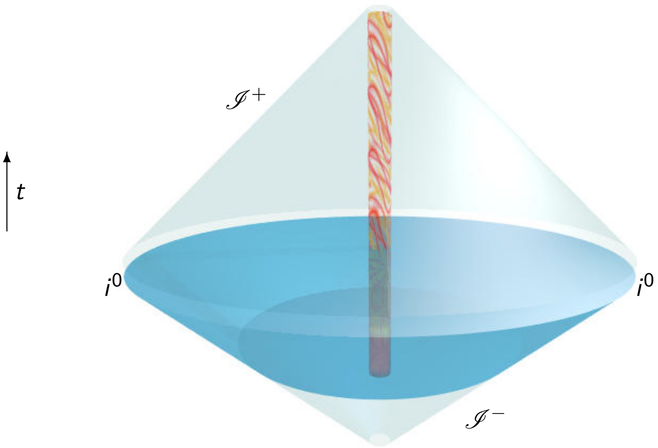
- Causal relations and wave propagation remain unchanged
- Can attach boundary points, which correspond to 'points at infinity'
- 'Infinity' becomes a submanifold, has a causal structure; local geometry

Consequence

Conformal structure is fundamental for space-times

Conformal Infinity

Schematic picture of an asymptotically flat space-time



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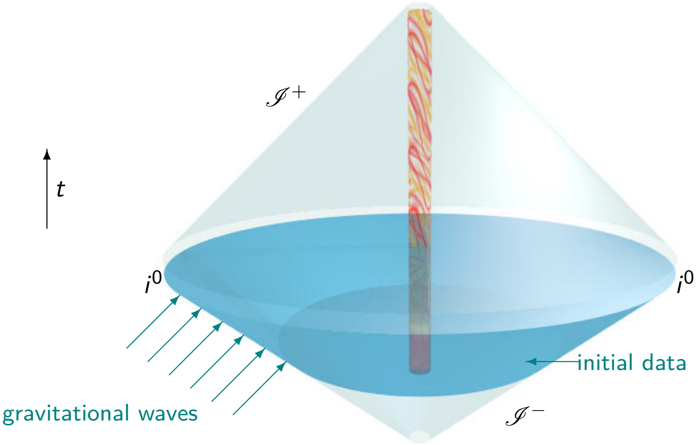

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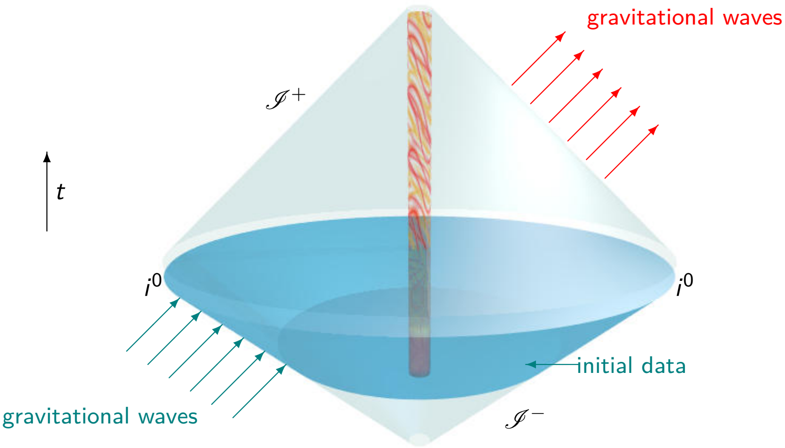


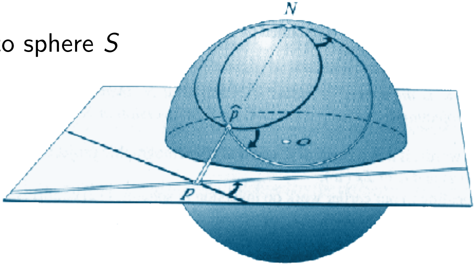
Conformal Infinity

Embedding into larger manifold

Compare with stereographic projection

- Euclidean plane E is embedded into sphere S
- Attach one point N
- 'Endpoint' of all straight lines
- Conformal embedding:

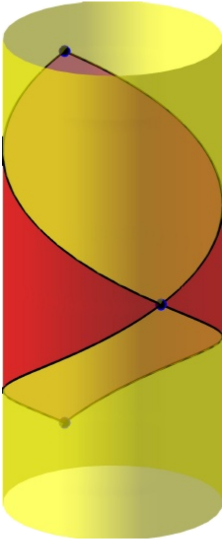
$$g_S = \Omega^2 g_E$$



Conformal Infinity

Embedding into larger manifold

- 'Physical space-time' M contained in a larger conformally equivalent space-time $\hat{M}$
- Submanifold with boundary
- Boundary is smooth except for i^0
- Einstein equations can be generalised to hold on $\hat{M}$
- H. Friedrich: Conformal field equations
- address the conformal class of the 'physical' metric g_{ab}
- Analysis of the structure near i^0 by 'blow-up'



The finite cylinder at spatial infinity

Basic properties

Construction in time-symmetric case

- Choose smooth AFID on $\mathbb{R}^3$,
- Smooth conformal extension to point $i \in S^3$ (cp. stereographic projection)
- Blow up i to a 2-sphere I^0 by including directions of geodesics through i
- evolution eq'ns and conformal Gauss gauge yield the following picture

The finite cylinder at spatial infinity

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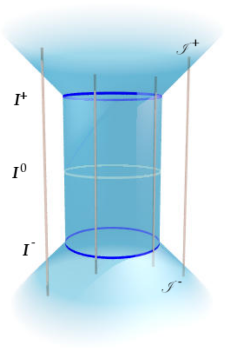

Notable points

- Spatial infinity is represented as a cylinder C
- Any asymp. flat hsf intersects C in a sphere between I^- and I^+
- C is a total characteristic for the time evolution i.e., no outward propagation
- evolution eq'ns yield symmetric hyperbolic system within C

$$A^t \partial_t u + A^{\mathbf{e}} \partial_{\mathbf{e}} u = b$$

- can be used to propagate data from I^0 to I^+

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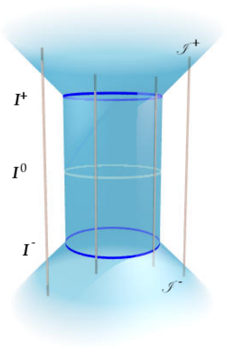

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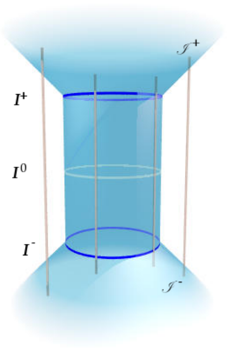

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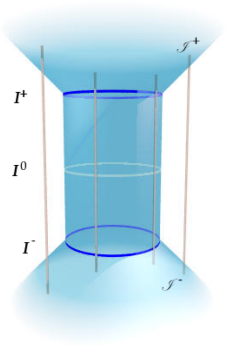

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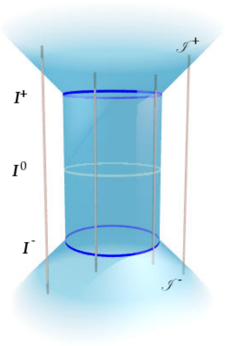

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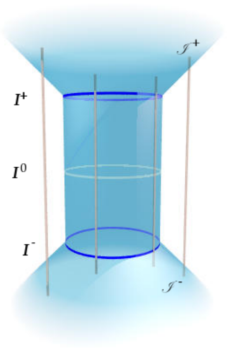
- data need to satisfy conditions at I^0 to exclude logarithms
- restricts the Cotton tensor (and its derivatives) of ID at infinity

$\Longrightarrow$ asymptotic conditions on the conformal class asymptotically static (?)

The finite cylinder at spatial infinity

Numerical study



Numerical study

- Can we extract radiative information ('$1/r$'-term) on $\mathscr{I}^+$ from the solution?
- How does a violation of the conditions influence the smoothness?
- How is it related to the initial data?
- First step: linear perturbation of the gravitational field in Minkowski space-time
- spin-2 zero-rest-mass field equations

→ next talk

Thank you