Hyperbolic Magnetic Monopoles

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Outline

1. Why study magnetic monopoles?
2. The setting: hyperbolic space, negative curvature.
3. SU(N) Yang-Mills-Higgs, Bogolmonyi and boundary conditions.
4. The spectral curve.
5. Holomorphic spheres for SU(2).
6. The general SU(N) case.

Why study magnetic monopoles?

The Standard Model is a (quantised) Yang-Mills-Higgs (YMH) gauge field theory:

- The strong force – SU(3)
- The weak force – SU(2) × U(1)

Want a Grand Unified Theory (GUT) [Polchinski].

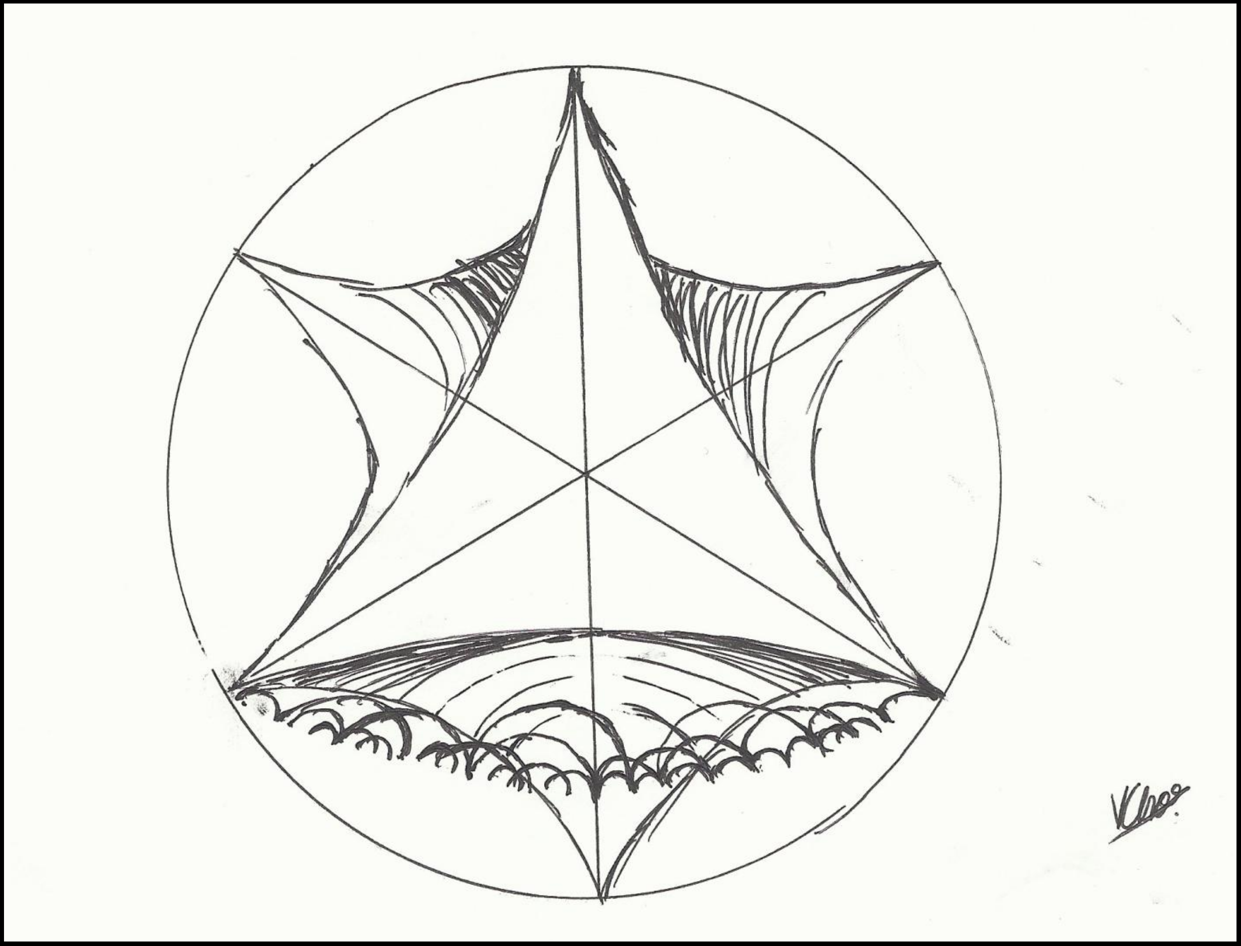
The Dirac Quantisation Condition.

Kapustin-Witten paper on S-duality and the Geometric Langlands.

Example of integrable system/soliton behaviour.

The Setting

Hyperbolic space, $\mathbb{H}^3$ with geodesics specified by points in $\mathbb{P}^1 \times \mathbb{P}^1$ (twistor space).



SU(N) YMH and Bogolmonyi

The **YMH equations** are conditions on

- An SU(N) principal bundle connection A .
- A section of the adjoint bundle (scalar field) called the Higgs field Φ .

All of which are $\mathfrak{su}(\mathsf{N})$-valued.

The **Bogolmonyi equations** minimise the energy of the YMH equations. They are nonlinear conditions on the curvature F_A and the derivative of the Higgs field $D_A\Phi$.

Boundary Conditions

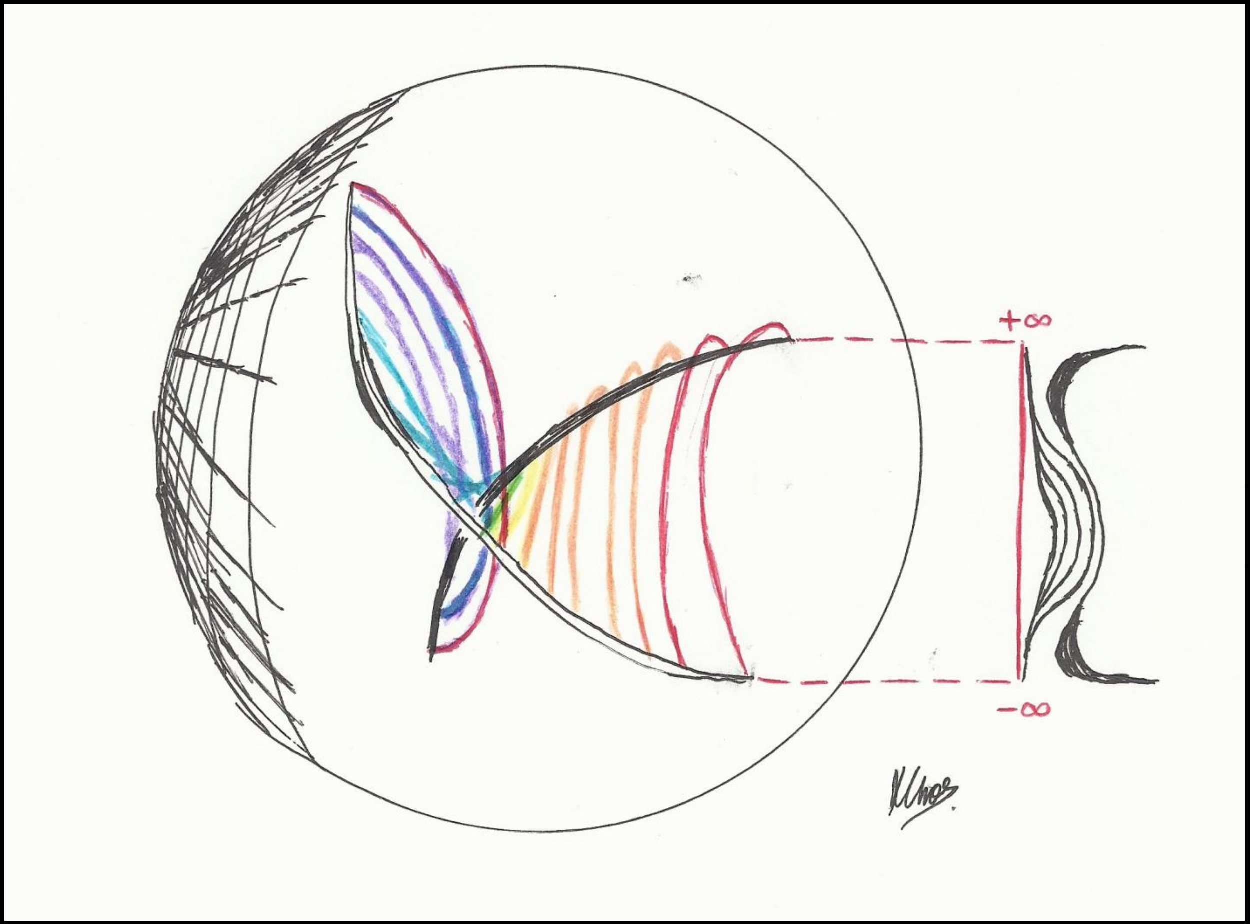
Hyperbolic monopoles are determined by their boundary data [Austin and Braam].

By choice of gauge, the boundary data for the Higgs field Φ takes of form of diagonal matrices with

- Mass data: $\lambda_1, \lambda_2, \ldots, \lambda_{N-1}$
- Charges (always integral): $n_1, n_2, \ldots, n_{N-1}$

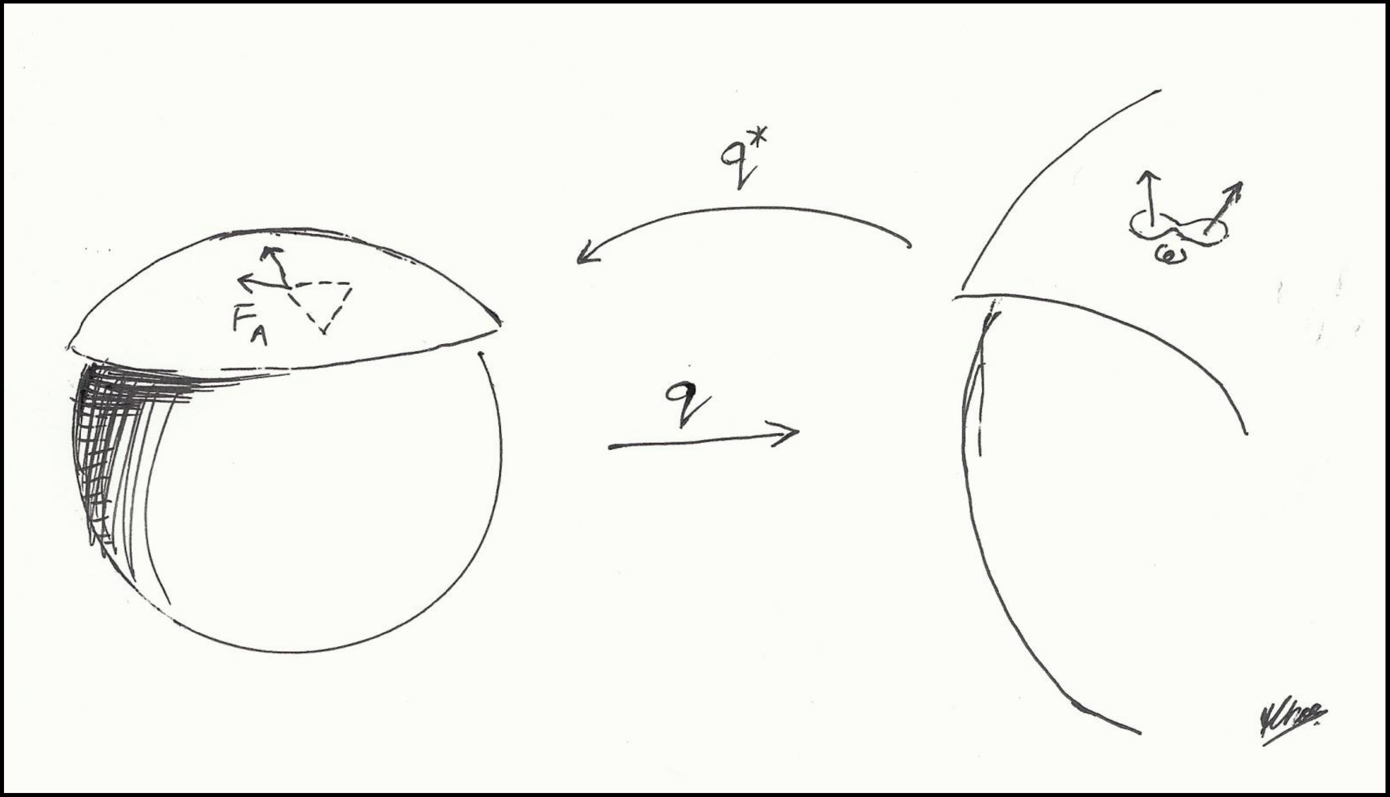
SU(2) Spectral Curve, $\Sigma \subset \mathbb{P}^1 \times \mathbb{P}^1$

The geodesics for which there are solutions of the scattering given by the Higgs field Φ which decay at $\pm\infty$ lies on a curve ψ [Hitchin].



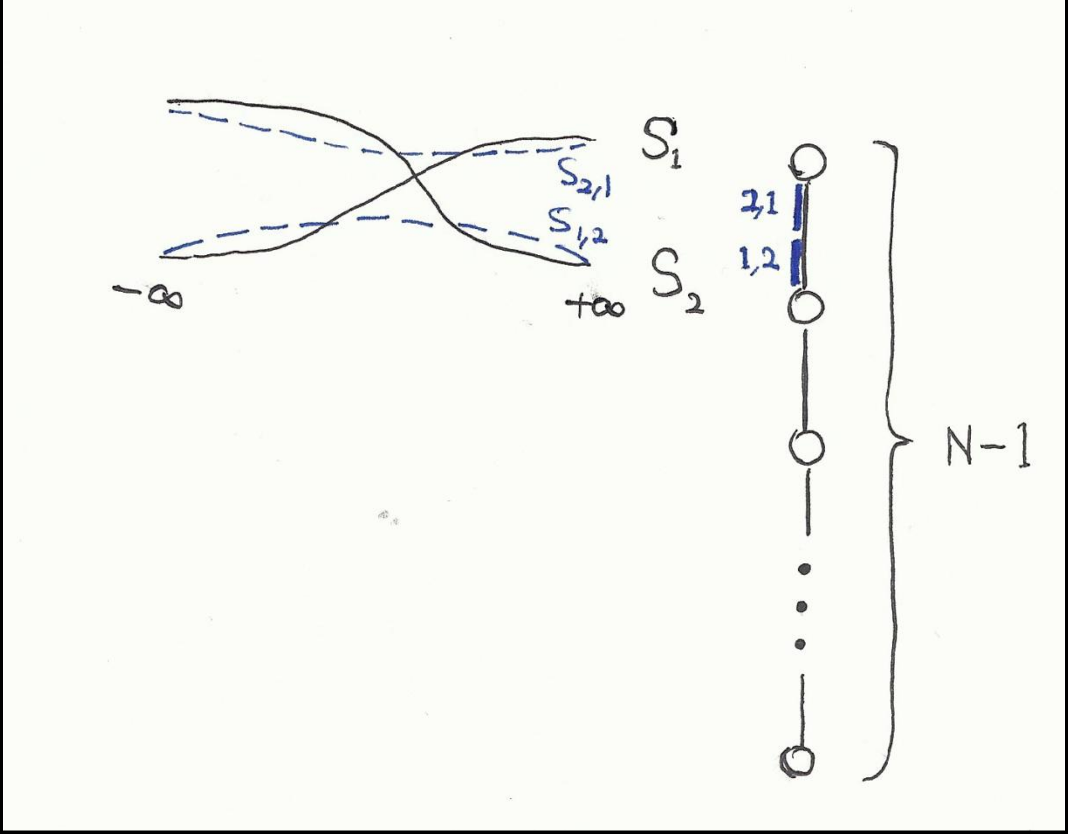
SU(2) Holomorphic Spheres, $q : \mathbb{P}^1 \to \mathbb{P}^n$

The spectral curve is exactly given by $\langle q(\hat{w}), q(z) \rangle$ and the curvature is the pullback of the Kähler form [Murray, Norbury, Singer].



SU(N) Spectral Curves

For SU(N), there is a curve for each vertex of the Dynkin diagram and for each edge, the intersection of the curves is the union of two components [Murray, Singer].



SU(N) Holomorphic Spheres?

Step 1: Compute examples of SU(N) spectral curves. No explicit examples in the literature.

References

- [Atiyah] Atiyah, *Instantons in Two and Four Dimensions*, 1984.
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- [Murray, Norbury, Singer] Murray, Norbury, Singer, *Hyperbolic Monopoles and Holomorphic Spheres*, 2003.
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