

## Keynote seminars

**Title:** Fluctuation relations and the foundations of statistical thermodynamics: a deterministic approach

**Presenter:** Debra J. Searles (Bernhardt)

**Affiliation:** The University of Queensland

**Authors:** Debra J. Searles (Bernhardt), James C. Reid, Stephen R. Williams, Lamberto Rondoni and Denis J. Evans.

**Abstract:** The fluctuation theorem and the work relation are exact nonequilibrium thermodynamic relations developed almost two decades ago. In the intervening time these relations have been applied to prove a number of new theorems including the dissipation theorem, the relaxation theorem, the maximum likelihood estimator and various phase function representations. They can also be applied to provide a proof of Boltzmann's postulate of equal a priori probability and a proof of the relationship between the phase space volume, the physical volume, the energy and the thermodynamic entropy and temperature for the equilibrium microcanonical ensemble. We will discuss these relationships and their application to some model systems.

**Title:** Topological order in spin systems

**Author:** Paul Fendley

**Affiliation:** University of Virginia

**Abstract:** Long ago the Ising model was solved by rewriting the spins in terms of Majorana fermionic variables. More recent studies have shown how the Ising ordered phase described in terms of the fermions is a simple example of topological order. I will explain what topological order is, and explain how this and other topological concepts are quite useful in studying spin systems. One example I will discuss is the role parafermionic zero modes play in chiral clock/Potts models. More generally, I will explain how two-dimensional integrable lattice models play a major role in understanding two-dimensional quantum models with topological order.

**Title:** Integrable dynamics: practical applications and abstract theory

**Author:** Jarmo Hietarinta

**Affiliation:** University of Turku

**Abstract:** Integrable systems were studied already in the late 1800's and their active development started again after the seminal paper of Zabusky and Kruskal in 1965. In this paper they demonstrated elastic scattering in the Korteweg-de Vries equation and coined the term "soliton". Subsequently many interesting mathematical properties of integrable systems were revealed, along with practical applications. I will present three different classes of equations and corresponding definitions of integrability specially suited for them. Finally I will demonstrate that even though different classes of integrable systems were originally defined by different criteria, there are in fact various connections between them.

**Title:** Quantum integrable systems and curve counting

**Author:** Andrei Okounkov

**Affiliation:** Columbia University

**Abstract:** Quantum cohomology is a deformation of the classical cohomology algebra of an algebraic variety  $X$  that takes into account enumerative geometry of rational curves in  $X$ . For many interesting varieties, it can be described in terms of certain quantum integrable systems. For example, Givental and Kim described the quantum cohomology of flag manifolds in terms

of Toda lattices. A general vision for a connection between quantum cohomology and quantum integrable systems recently emerged in supersymmetric gauge theories, in particular in the work of Nekrasov and Shatashvili. In this lecture, I will describe an approach to these problems developed in my joint work with Daveshe Maulik.

**Title:** The dynamics of compact bodies in General Relativity: recent progress and open problems

**Author:** Todd Oliynyk

**Affiliation:** Monash University

**Title:** Spectrum of states in gauge-string duality

**Author:** Arkady Tseytlin

**Affiliation:** Imperial College

## Contributed seminars

**Title:** Quasi-exact solvable double Morse potential and proton tunnelling in hydrogen-bonded crystals

**Author:** Davids Agboola

**Affiliation:** The University of Queensland

**Abstract:** We present exact solutions to the Schrödinger equation with the double Morse potential, consisting of two back-to-back Morse functions. The resulting differential equation is found to be quasi-exactly solvable and closed form energies and wavefunctions are obtained using the Bethe ansatz method. The results obtained are used as prototype description for the of hydrogen-bonded crystals COOH and KDP as well as their deuteriated cases.

**Title:** Integrability as a consequence of Discrete Holomorphicity

**Author:** Imam Tashdid ul Alam

**Affiliation:** Australian National University

**Abstract:** In the past few years, discretely holomorphic observables have been identified in several solvable lattice models at criticality. The existence of rhombic embeddings of general Baxter lattices allows us to consider the condition of discrete holomorphicity on these embeddings onto the complex plane. We show that, by considering the contour sum around adjacent rhombi, the integrability of the underlying model can be demonstrated. In particular, both the Yang-Baxter equations and the inversion relations, the main ingredients of the commuting transfer matrix method, can be derived from the contour sums.

**Title:** Is there a hidden 3-dimensional structure in the 2D Hubbard model?

**Author:** Vladimir Bazhanov

**Affiliation:** Australian National University

**Abstract:** The 1+1-dimensional Hubbard model in condensed matter physics describe two interacting XY-chains. The model was solved by Bethe Ansatz a long time ago, but the associated Yang-Baxter structure still remains obscure. In particular, an absence of the "difference property" in the model might be a manifestation of a hidden 3D structure. In this talk I will review various attempts to connect the Hubbard model with 3D integrable systems.

**Title:** Self-avoiding trails with nearest neighbour interactions on the square lattice

**Author:** Andrea Bedini

**Affiliation:** The University of Melbourne

**Abstract:** Self-avoiding walks and self-avoiding trails, two models of a polymer coil in dilute solution, have been shown to be governed by the same universality class. On the other hand, self-avoiding walks interacting via nearest-neighbour contacts (ISAW) and self-avoiding trails interacting via multiply-visited sites (ISAT) are two models of the collapse transition of a polymer in dilute solution and, on the square lattice, it has been established numerically that their collapse transitions lie in different universality classes. The two models differ in two substantial ways: in the types of subsets of random walk configurations utilised (site self-avoidance versus bond self-avoidance) and in the type of attractive interaction. It is therefore of some interest to consider self-avoiding trails interacting via nearest neighbour attraction (INNSAT) in order to ascertain the source

for the difference in the collapse universality class.

**Title:** A variational approach for a class of exactly solvable BEC-BCS crossover Hamiltonians

**Author:** Andrew Birrell

**Affiliation:** The University of Queensland

**Abstract:** By combining aspects of both the co-ordinate Bethe ansatz and algebraic Bethe ansatz we determine very general classes of exactly solvable Hamiltonians. This approach does not depend on any prior knowledge of a solution of the Yang-Baxter Equation, nor the conserved operators of the Hamiltonian it generates. By taking a variational form for the Hamiltonian and eigenstates we obtain general exact solvability conditions. The procedure is conducted in the framework of Hamiltonians describing the crossover between the low-temperature phenomena of superconductivity, in the Bardeen-Cooper-Schrieffer (BCS) theory, and Bose-Einstein condensation (BEC).

**Title:** On the K-theory classification of topological states of matter

**Author:** Peter Bouwknegt

**Affiliation:** Australian National University

**Abstract:** Topological insulators and superconductors are many-fermion systems possessing an unusual band structure that leads to a bulk band gap as well as topologically protected gapless extended surface modes (known as the 'bulk-boundary correspondence'). It was recently realised by Kitaev, that deformation classes of gapped Hamiltonians are naturally classified by K-theory. This classification parallels the classification of the 2 complex and 8 real symmetry classes of Hamiltonians (the '10-fold way') of Altland and Zirnbauer, and naturally leads to a periodic table of topological insulators. In this talk I will give a brief overview of these developments. I will also describe how the bulk-boundary correspondence fits in with the K-theory picture.

**Title:** Flux-quantization by Bäcklund transformations in a model of electrodiffusion based on Painlevé II

**Author:** Tony Bracken

**Affiliation:** Department of Mathematics, The University of Queensland

**Abstract:** An established model of steady one-dimensional two-ion electrodiffusion across a liquid junction involves three coupled first-order nonlinear ordinary differential equations, and has the second-order Painlevé II equation at its core. Solutions are grouped by Bäcklund transformations into infinite sequences, each characterized by evenly-spaced quantized fluxes of the two ionic species, and hence evenly-spaced quantization of the electric current-density. For the sequence generated from a particular exact solution, first constructed by Planck, this flux quantization is shown to reflect the underlying quantization of electric charge.

**Title:** A Bethe Ansatz Constant term solution to Osculating Lattice Paths

**Author:** Richard Brak

**Affiliation:** The University of Melbourne

**Abstract:** This talk will show how the Bethe Ansatz can be used to solve a set of coupled partial difference equations for osculating lattice paths. Osculating paths are sets of directed square lattice paths that are allowed to share vertices but not edges. The paths occur in the six vertex model and alternating

sign matrix enumeration.

**Title:** Constrained Separation of Variables in Schroedinger Equations via Incomplete Staeckel Matrices

**Author:** Philip Broadbridge

**Affiliation:** La Trobe University

**Title:** Invariant Classification of Second-Order Conformally-Superintegrable Systems

**Author:** Joshua Capel

**Affiliation:** University of New South Wales

**Title:** Hyperbolic Magnetic Monopoles

**Author:** Joseph Chan

**Affiliation:** The University of Melbourne

**Abstract:** Many Grand Unified Theories (GUTs) require the existence of magnetic monopoles. In Yang-Mills-Higgs gauge field theory, they arise as soliton solutions, tying together topology, algebraic geometry and integrable systems. The euclidean case has been studied extensively but the hyperbolic picture is still incomplete. I will speak about the spectral curve and holomorphic sphere associated to a hyperbolic monopole.

**Title:** Endless self-avoiding walks

**Author:** Nathan Clisby

**Affiliation:** The University of Melbourne

**Abstract:** We introduce a self-avoiding walk model for which end-effects are completely eliminated. We enumerate the number of these walks on the square and cubic lattices, and use these enumerations to study the properties of this model in two and three dimensions. The technique by which end-effects are eliminated may be generalised to other models of polymers such as interacting self-avoiding walks.

**Title:** Random rectangle-triangle tilings and Painleve VI

**Author:** Jan De Gier

**Affiliation:** The University of Melbourne

**Abstract:** Random rectangle-triangle tilings are non-free fermionic generalisations of rhombus tilings that were discovered in the late nineties. I will discuss three such tilings that are Bethe ansatz solvable. The Bethe integral equations have a non-trivial monodromy related to  $SL(2, \mathbb{Z})$  which reduces to a finite cyclic monodromy in special cases. It was recently pointed out that the general case has an interesting connection to Painleve VI.

**Title:** Diagrammatic Monte Carlo simulation of the Fermi-Hubbard model

**Author:** Youjin Deng

**Affiliation:** University of Science and Technology of China

**Abstract:** I will discuss the general scheme of the diagrammatic Monte Carlo method for correlated fermion system, and then discuss its application to the Fermi-Hubbard model.

**Title:** Unstable Klein-Gordon modes in an accelerating universe

**Author:** Kathryn Deutscher

**Affiliation:** La Trobe University

**Abstract:** Current observations show the universe to be made

up predominantly of “dark energy,” (energy that has no particle or radiation interpretation). It is known that when unstable modes are quantised, there is no Fock representation and therefore no particle or radiation interpretation. In this sense, these modes might be regarded as dark energy.

We develop a full solution of a scalar quantum field coupled to an accelerating universe. In the case of minimal coupling of massless fields, there exist modes that become unstable at a discrete sequence of times. After canonical quantisation, these unstable modes represent energy eigenstates that can be viewed as neither particle nor oscillatory radiation. We then partition the energy into stable and unstable components. As time increases, modes of increasing wave number cease to be oscillatory and become unstable. The number of unstable modes remains finite, guaranteeing a preferred physical representation for the system.

**Title:** Endings and Beginnings: The Story of Non-Intersecting Paths

**Author:** Paul Fijn

**Affiliation:** The University of Melbourne

**Title:** A domain wall theory for the prioritising exclusion process

**Author:** Caley Finn

**Affiliation:** The University of Melbourne

**Abstract:** The prioritising exclusion process is a model for the operation of hospital queues where customers (patients) are classified as high or low priority. Upon arrival, customers of both classes join the back of the queue, but a high priority customer can overtake a low priority customer immediately ahead of them, so that the order of service depends on priority class as well as time spent in the queue. The overtaking is analogous to the particle hopping in the asymmetric exclusion process (ASEP). However, as customers arrive and are served the queue grows and shrinks so, unlike the ASEP, the system does not have a fixed length. We show that this system is well described by a domain wall theory, and, in fact, this picture is exact for certain regions of the parameter space.

**Title:** Asymptotics of spacing distributions in random matrix theory

**Author:** Peter Forrester

**Affiliation:** The University of Melbourne

**Abstract:** It's now over 50 years since Dyson first used Coulomb gas methods to predict the asymptotics of the spacing distribution in random matrix ensembles. I'll review some of the subsequent progress.

**Title:** Numerical space-times near space-like and null infinity

**Author:** Joerg Frauendiener

**Affiliation:** University of Otago

**Title:** Complete solutions to a class of nonconvex variational/boundary value problems with applications in phase transitions of solids

**Author:** David Gao

**Affiliation:** University of Ballarat

**Title:** Novel Phase Transitions in XY Antiferromagnets on Plane Triangulations

**Presenter:** Tim Garoni

**Affiliation:** Monash University

**Authors:** Tim Garoni, Youjin Deng, Jian-Ping Lv

**Abstract:** Using Monte Carlo simulations and finite-size scaling, we investigate the XY antiferromagnet on the triangular, Union Jack and bisected-hexagonal lattices, and in each case find both Ising and Kosterlitz-Thouless transitions. As is well-known, on the triangular lattice, as the temperature decreases the system develops chiral order for temperatures  $T < T_c$ , and then quasi-long-range magnetic order on its sublattices when  $T < T_s$ , with  $T_s < T_c$ . On the Union Jack and bisected-hexagonal lattices, by contrast, we find that as  $T$  decreases the magnetizations on some of the sublattices become quasi-long-range ordered at a temperature  $T_s > T_c$ , before chiral order develops. In some cases, the sublattice spins then undergo a second transition, of Ising type, separating two quasi-long-range ordered phases. On the Union Jack lattice, the magnetization on the degree-4 sublattice remains disordered until  $T_c$  and then undergoes an Ising transition to a quasi-long-range ordered phase.

**Title:** Quantum critical matter in one dimension

**Author:** Xiwen Guan

**Affiliation:** 1. Wuhan Institute of Physics and Mathematics; 2. The Australian National University

**Abstract:** It has long been appreciated that exactly solved mathematical models describing the statistical mechanics of interacting particles have played a key role in the development of formerly unrelated areas of mathematics and theoretical physics, such as the study of knots, links and braids, quantum groups, combinatorics, conformal field theory and condensed matter physics. However, over the past few years striking experimental achievements in trapping and cooling atoms in one-dimensional optical waveguides have provided remarkable realisations of exactly solved models in the lab. More generally the study of cold atomic matter provides a unique environment to explore novel quantum many-body effects like quantum liquids, quantum correlations and quantum criticality. In this talk I will describe some of these fundamental mathematical models and their relevance to recent and future experiments on such exotic many-body physics.

**Title:** Self-avoiding walks in a rectangle

**Author:** Tony Guttmann

**Affiliation:** The University of Melbourne

**Abstract:** A decade ago the problem was posed of Brownian motion originating at the centre of a  $10 \times 1$  rectangle, and the problem posed was calculation of the ratio of probabilities of a Brownian path hitting the short end of the rectangle before hitting one of the long sides. Surprisingly, for Brownian motion this probability can be calculated exactly. Here we consider instead the problem of a self-avoiding walk in the scaling limit, and pose the same question. Assuming that the scaling limit of SAW is conformally invariant, we evaluate, asymptotically, the same ratio of probabilities. For the SAW case we find the probability ratio is approximately 200 times greater than for Brownian motion.

**Title:** Phants and surfaces

**Author:** Yi Huang

**Affiliation:** The University of Melbourne

**Abstract:** One way of studying a Riemann surface is to endow it with its canonical hyperbolic metric, and then to decompose

it into pairs of pants. This simple idea is the basis for much of Teichmüller space theory, and enables us to study the moduli space of Riemann surfaces from the hyperbolic perspective.

A modification on this theme is to introduce cone-point singularities to our hyperbolic metric. But this quickly runs into trouble as we lose pants-decomposition when cone-angles become greater than  $\pi$ . We introduce the idea of phants as a possible fix for this problem, and hence as a way to develop and understand the Teichmüller theory of cone-pointed surfaces.

**Title:** On highest weight representations of conformal Galilei algebras

**Author:** Phillip Isaac

**Affiliation:** The University of Queensland

**Abstract:** We discuss highest weight representations of a class of conformal Galilei algebras in one spatial dimension with central extension. This is done by explicitly constructing all singular vectors within the Verma modules and deducing irreducibility of the associated highest weight quotient modules. A resulting classification of infinite dimensional irreducible modules is presented. We deduce a formula for the Kac determinant from our construction, thus proving a conjecture of Dobrev, Doebner and Mrugalla for the case of the Schrodinger algebra.

**Title:** A multi-dimensional  ${}_1\psi_1$  sum and some related topics

**Author:** Masahiko Ito

**Affiliation:** Tokyo Denki University

**Abstract:** Ramanujan's  ${}_1\psi_1$  summation is a bilateral extension of the  $q$ -binomial theorem. We will present several multi-dimensional generalizations of the  ${}_1\psi_1$  summation. In these generalizations, we will mainly explain a multi-dimensional bilateral  $q$ -series extended from Evans's Dixon-Anderson integral, which is a multi-dimensional integral evaluation fundamental to the theory of the Selberg integral. The evaluation of the  $q$ -series is shown by a difference equation and a certain asymptotic behavior. This work was done jointly with P. J. Forrester.

**Title:** Algebraic structures from group character rings - an integrable model perspective

**Presenter:** Peter Jarvis

**Affiliation:** University of Tasmania

**Authors:** Bertfried Fauser (RHUL), Peter Jarvis, and Ronald C King (Southampton U)

**Abstract:** We study algebraic structures in the character rings of matrix groups, from a diagrammatic perspective. The primary object is the Hopf algebra of characters of the finite dimensional polynomial representations of the general linear group, realised as the ring of symmetric functions on variables  $(x_1, x_2, \dots)$ . Isomorphic as spaces are the character rings of the classical orthogonal and symplectic subgroups of the general linear group. We generalise these to study the formal character rings  $H_\pi$  of certain algebraic subgroups of the general linear group, comprised of matrix transformations leaving invariant a fixed tensor of Young symmetry type  $\pi$ . We present explicit constructions of these subgroup characters, and identify structural elements and verify the axioms to establish that each  $H_\pi$  ring is a ribbon Hopf algebra. The corresponding operators associated with knot projections are illustrated.

**Title:** Perimeter polynomials and scaling analysis for percolation problems

**Author:** Iwan Jensen  
**Affiliation:** The University of Melbourne

**Title:** Quasi-Classical Expansion of the Star-Triangle Relation

**Author:** Andrew Kels

**Affiliation:** Australian National University

**Abstract:** The master solution of the star triangle relation introduced by Bazhanov and Sergeev, describes an exactly solvable lattice model of statistical mechanics with continuous spins and positive Boltzmann weights. Many different limits of this model can be taken which result in new integrable models. I will describe these limits and give two explicit examples. I will also show how the low temperature (quasi-classical) limit of these models are connected to the discrete integrable equations of Adler, Bobenko and Suris.

**Title:** Superintegrability in a non-conformally-flat space

**Author:** Jonathan Kress

**Affiliation:** University of New South Wales

**Abstract:** Until recently all known examples of superintegrable Hamiltonians were systems on constant curvature spaces, or by Staeckel transform, on conformally-flat spaces. Methods developed to investigate the superintegrability of the Tremblay-Turbiner-Winternitz system can be extended to a non-conformally-flat superintegrable system in 4 dimensions possessing irreducible higher rank Killing tensors. These Killing tensors give rise to higher order symmetries of a conformally covariant Laplacian that is not simply the usual one with a  $R/6$  but also includes a term constructed from the conformal curvature. Some recent results involving these systems will be discussed.

**Title:** Anti-de Sitter Supersymmetry and Hyperkaehler Geometry

**Author:** Sergei Kuzenko

**Affiliation:** The University of Western Australia

**Title:** Boundary conditions of discretely holomorphic observables and integrable loop models

**Author:** Alexander Lee

**Affiliation:** The University of Melbourne

**Abstract:** In two-dimensional lattice models, there exist certain observables that are discretely holomorphic in the sense that they satisfy a discrete analogue of the Cauchy-Riemann equations. Recent work by Cardy, Ikhlef and others has shown a surprising connection between discrete holomorphicity and integrability. In this talk I will be describing this connection in models where there is a boundary. By requiring that the observable satisfy certain boundary conditions, we are able to obtain new integrable boundary weights for several loop models.

**Title:** Generalised Hurwitz numbers and topological recursion

**Author:** Oliver Leigh

**Affiliation:** The University of Melbourne

**Abstract:** Hurwitz numbers count branched covers of the Riemann sphere with fixed genus and prescribed branching data. In the simplest case, there are known connections to the geometry of moduli spaces and the representation theory of the symmetric group. More recently, they have been linked to the topological recursion introduced by Eynard and Orantin. In this

talk, we define generalised Hurwitz numbers and discuss how they relate to geometry, and topological recursion.

**Title:** Internal Time and Quantum Action Principle in Relativistic Quantum Mechanics

**Author:** Inna Lukyanenko

**Affiliation:** The University of Queensland

**Abstract:** In my talk I will discuss a new form of one-particle relativistic quantum mechanics with a proper probabilistic interpretation, proposed by A. Lukyanenko and N. Gorobey (Saint Petersburg State Polytechnical University). The problem of probabilistic interpretation of the ordinary one-particle relativistic quantum mechanics comes from the hyperbolic structure of the Klein-Gordon equation, which does not conserve positive definite properties. It has been solved by secondary quantization of the theory (quantum field theory), but it is not the only possible solution. We propose a modification of the original theory in which the internal time of a particle is introduced. For this purpose we consider a quantum analogue of the classical action principle, where the role of a quantum action is played by the phase function of a solution of the Schrödinger equation. The internal time is determined by the condition of stationarity of the quantum action. Then, the stationary solution becomes a density of probability, in which time, equally to spatial coordinates, is a stochastic parameter. The correspondence of the new theory with non-relativistic quantum mechanics will be discussed.

**Title:** Elliptic parametrization of the Zamolodchikov model

**Author:** Vladimir Mangazeev

**Affiliation:** Australian National University

**Abstract:** We study the vertex form of the Zamolodchikov's solution of the tetrahedron equation in a special limit when one vertex of the tetrahedron goes to infinity. In this limit the weights can be parametrized by elliptic functions. A special projection of the tetrahedron equation produces the tetrahedral Zamolodchikov's algebra studied by Korepanov, Wadati and Shiroishi, and other authors.

**Title:** Generalised Heine-Stieltjes and Van Vleck polynomials associated with integrable BCS models

**Author:** Ian Marquette

**Affiliation:** The University of Queensland

**Abstract:** We will present new results concerning numerical methods to study integrable systems based on the Bethe Ansatz/Ordinary Differential Equation (BA/ODE) correspondence. We will discuss how this approach can be applied to four cases of exactly solvable Bardeen-Cooper-Schrieffer (BCS) pairing models in their degenerate two-level limit. These are the s-wave pairing model, the p+ip-wave pairing model, the p+ip pairing model coupled to a bosonic molecular pair degree of freedom, and a d + id-wave pairing model with additional interactions. The zeros of the generalised Heine-Stieltjes polynomials provide solutions of the corresponding Bethe ansatz equations. We compare the roots of the ground states with curves obtained in the continuum limit.

**Title:** Some field-theoretic ideas out of contact geometry and elementary topology

**Author:** Daniel Mathews

**Affiliation:** Contextual Systems

**Abstract:** Sutured Floer homology and contact homology

are invariants of 3-manifolds developed through the ideas of holomorphic curves, symplectic geometry and contact geometry. They satisfy some of the properties of a topological quantum field theory. In various simple cases, these field-theoretic properties can be seen in terms of the elementary topology of curves on surfaces, and algebraic objects associated to them.

**Title:** A Non-Hermitian BCS Hamiltonian and Generalised Exclusion Statistics

**Author:** Amir Moghaddam

**Affiliation:** The University of Queensland

**Abstract:** The Bethe ansatz is a key tool in the area of quantum integrable and exactly solvable models. For each such model, understanding the nature of the roots of the Bethe ansatz equations is central to understanding the mathematical physics underpinning the model's behaviour. Here we analyse an exactly solvable, non-hermitian BCS pairing Hamiltonian dependent on a real-valued coupling parameter. The Hamiltonian displays a real spectrum for all values of this coupling parameter. The roots of the Bethe ansatz equations can be categorized into two classes, those which are dependent on the coupling parameter and those which are not. We will discuss how those roots which are independent of the coupling parameter can be associated to exotic quasi-particles obeying generalised exclusion statistics, in the sense proposed by Haldane in 1991.

**Title:** Feigin-Frenkel center and Yangian characters

**Author:** Alexander Molev

**Affiliation:** The University of Sydney

**Abstract:** For each simple Lie algebra  $\mathfrak{g}$  consider the vacuum module  $V(\mathfrak{g})$  at the critical level over the corresponding affine Kac-Moody algebra. The vacuum module has a vertex algebra structure. We construct explicit generators of the center of this vertex algebra for all Lie algebras  $\mathfrak{g}$  of classical types. This gives a new proof of the Feigin-Frenkel theorem (1992) and leads to explicit constructions of commutative subalgebras of the universal enveloping algebras  $U(\mathfrak{g}[t])$  and  $U(\mathfrak{g})$ , and to higher order Hamiltonians in the Gaudin model associated with  $\mathfrak{g}$ . Moreover, we use Yangian characters (or  $q$ -characters) to calculate the images of the central elements under an affine version of the Harish-Chandra isomorphism.

**Title:** The Jordan structure of periodic loop models

**Author:** Alexi Morin-Duchesne

**Affiliation:** The University of Queensland

**Abstract:** Non hermitian Hamiltonians play an essential role in the description of two dimensional statistical models such as the Fortuin-Kasteleyn model and the  $Q$ -state Potts model. The loop Hamiltonians, as elements of the periodic Temperley-Lieb algebra  $TLP(\beta)$ , are examples of such Hamiltonians: they have real eigenvalues and exhibit non-trivial Jordan structures for specific values of the parameter  $\beta$ . Loop Hamiltonians and XXZ Hamiltonians are known to be related. In this talk, the "twist" representations of the periodic Temperley-Lieb algebra will be introduced and it will be shown how one can study the non-diagonalizability of the loop Hamiltonian by using tools from an extended family of XXZ models.

**Title:** Eynard-Orantin invariants and Frobenius manifolds

**Author:** Paul Norbury

**Affiliation:** The University of Melbourne

**Abstract:** Many problems in enumerative geometry have been

brought together using methods from complex analysis developed by Eynard and Orantin. These include intersection theory and Weil-Petersson volumes on the moduli space of curves, various Hurwitz number problems and relative Gromov-Witten invariants of the two-sphere, each arising as the Eynard-Orantin invariants of a Riemann surface equipped with some extra structure. Another common context for these problems is Frobenius manifolds which can be used to give a rigorous notion of mirror symmetry in each case. I will describe a relation between Eynard-Orantin invariants and Frobenius manifolds.

**Title:** Lax representations of reductions of non-autonomous lattice equations

**Author:** Chris Ormerod

**Affiliation:** La Trobe University

**Title:** Exact solution of two friendly walks above a sticky wall with single and double interactions

**Author:** Aleks Owczarek

**Affiliation:** The University of Melbourne

**Abstract:** We find, and analyse, the exact solution of two friendly directed walks, modelling polymers, which interact with a wall via contact interactions. We specifically consider two walks that begin and end together so as to imitate a polygon. We examine a general model in which a separate interaction parameter is assigned to configurations where both polymers touch the wall simultaneously, and investigate the effect this parameter has on the integrability of the problem. We find an exact solution of the generating function of the model, and provide a full analysis of the phase diagram that admits three phases with one first-order and two second-order transition lines between these phases.

**Title:** Boundary Conditions for Infinitely Extended Kac Table of Critical Dense Polymers

**Author:** Paul Pearce

**Affiliation:** The University of Melbourne

**Abstract:** Solvable critical dense polymers is a Yang-Baxter integrable model of polymers on the square lattice. It is the first member  $\mathcal{LM}(1, 2)$  of the family of logarithmic minimal models  $\mathcal{LM}(p, p')$ . The associated logarithmic conformal field theory admits an infinite family of Kac representations labelled by the Kac labels  $r, s = 1, 2, \dots$ . Here we explicitly construct the conjugate boundary conditions on the strip and solve exactly for the transfer matrix eigenvalues on arbitrary finite-width strips to obtain the conformal spectra using the Euler-Maclaurin formula. The key to the solution is an inversion identity satisfied by the commuting double-row transfer matrices. The analysis of physical combinatorics involves generalized  $q$ -Catalan polynomials. In the scaling limit, the central charge  $c = -2$  and the Kac formula  $\Delta_{r,s} = \frac{(2r-s)^2-1}{8}$  with  $r, s = 1, 2, 3, \dots$  for the conformal weights in the infinitely extended Kac table is confirmed.

**Title:** Is Kahan's 'unconventional' method conventional?

**Author:** Reinout Quispel

**Affiliation:** La Trobe University

**Abstract:** We show that Kahan's discretization of quadratic vector fields is equivalent to a Runge-Kutta method. When the vector field is Hamiltonian, with constant Poisson structure, the map determined by this discretization has a conserved modified Hamiltonian and an invariant measure. This produces large classes of integrable rational mappings, explaining some of the

integrable cases that were previously known, as well as yielding many new ones.

**Title:** How to arrange dominoes using quantum numbers

**Author:** Jorgen Rasmussen

**Affiliation:** The University of Queensland

**Abstract:** We consider domino tilings of square lattices on rectangles, cylinders and tori. Working with Lieb's transfer matrix, we point out that the full set of domino configurations may be partitioned into disjoint subsets (sectors) closed under the action of the transfer matrix. These sectors are labelled by an integer or half-integer quantum number we call the variation index. In the continuum scaling limit, each sector gives rise to a representation of the Virasoro algebra. We discuss the corresponding partition functions and how similarities with the critical dense polymer model yield natural finitizations by means of physical combinatorics.

**Title:** Modular Properties of Fractional Level WZW Models

**Author:** David Ridout

**Affiliation:** Australian National University

**Abstract:** The modular properties of fractional level WZW models and, in particular, the application of the Verlinde formula, have a long and checkered history in conformal field theory. Such theories were postulated in the late eighties as a means of generalising the GKO-coset construction of the unitary minimal models to their non-unitary cousins. Unfortunately, while their modular properties appeared satisfactory, the Verlinde formula (which is supposed to give the dimensions of certain vector spaces) always gave a few negative integers in addition to the expected non-negative ones. This notorious problem is referred to in textbooks as suggesting that fractional level theories are "intrinsically sick".

Luckily, the formalism of logarithmic conformal field theory has led to a radically new approach to this issue. We will survey the shift in paradigm that has recently been shown to cure the fractional level theories based on affine  $sl(2)$  of all sickness. If time permits, we will then discuss (with examples) a selection of the beautiful mathematics that has been used to finally resolve this long-standing problem.

**Title:** From conformal invariance to quasistationary states

**Author:** Vladimir Rittenberg

**Affiliation:** Bonn University

**Abstract:** In a conformal invariant one-dimensional stochastic model, a certain non-local perturbation takes the system to a new massless phase of a special kind. The ground-state of the system is an adsorptive state. Part of the finite-size scaling spectrum of the evolution Hamiltonian stays unchanged but some levels go exponentially to zero for large lattice sizes becoming degenerate with the ground-state. As a consequence one observes the appearance of quasistationary states which have a relaxation time which grows exponentially with the size of the system. Several initial conditions have singled out a quasistationary state which has in the finite-size scaling limit the same properties as the stationary state of the conformal invariant model.

**Title:** A new look at nonlinear dynamical systems

**Author:** Ning Ruan

**Affiliation:** University of Ballarat

**Abstract:** Duality is a fundamental concept that underlies

almost all natural phenomena. The inner beauty in general systems are bound up with duality. In this talk, the speaker will present a potentially useful canonical duality theory for solving discrete dynamical systems governed by logistic equation. In nonlinear dynamics, the chaotic behaviour is mainly due to non-convexity of the objective function. How to identify the global stability and extremality of the critical solutions is a challenge task. The speaker will first show that by using the finite difference and least squares, nonlinear differential equation can be converted into a non-convex optimization with unknown parameter, which is equivalent to a concave maximization problem by using canonical dual transformation. Applications will be illustrated by a study on the population dynamics of fisheries. The method can be applied to many NP-hard problems in global optimization and computational science, such as integer programming, network optimization and travelling salesman problem (TSP), etc. This talk should bring some new insights into non-convex analysis, nonlinear dynamics and computational methods.

**Title:** Solvable off-critical logarithmic models

**Presenter:** Katherine Seaton

**Affiliation:** La Trobe University

**Authors:** Paul Pearce, Katherine Seaton

**Abstract:** The logarithmic limit of the Forrester-Baxter models has yielded generalised order parameters with critical exponents corresponding to the conformal weights of the non-unitary minimal series. We conclude that generalized models of polymers and percolation are exactly solvable both at criticality and off-criticality.

**Title:** Exact solution of a simple adsorption model of denaturing DNA

**Author:** Rami Tabbara

**Affiliation:** The University of Melbourne

**Abstract:** We consider a DNA strand in a solvent near an attractive surface modelled as two interacting (*friendly*) directed walks along the square lattice. We establish a functional equation for the corresponding generating function, which is further refined by means of the *obstinate kernel method*. Specifically, we utilise the kernel method in a novel way to express the exact-solution of the model in terms of two simpler generating functions for the same underlying combinatorial class. The Zeilberger-Gosper algorithm is then utilised to computationally determine linear homogeneous differential equations solved by these simpler generating functions, thereby allowing us to analyse the singularity structure and thus critical behaviour of the model. We deduce the phase diagram for this model, interestingly finding that the system exhibits four phases along with a *quad*-critical point.

**Title:** Logarithmic Superconformal Minimal Models

**Author:** Elena Tartaglia

**Affiliation:** The University of Melbourne

**Abstract:** We introduce new two-dimensional exactly solvable superconformal loop models generalising the logarithmic minimal models. These models include superconformal polymers and superconformal percolation as the first members. The superconformal loop models are constructed using the generators of a one-parameter specialisation of the Birman-Wenzl-Murakami (BMW) algebra arising from fusion of the Temperley-Lieb algebra. The BMW algebra extends the Temperley-Lieb algebra by allowing over- and under-crossings of loop segments. In the

continuum scaling limit, the new models describe logarithmic superconformal minimal Conformal Field Theories (CFTs). The link states for these models are constructed, and we find they are counted by Riordan and Motzkin numbers. Finite-size corrections are studied to obtain the central charges, conformal dimensions and finitised conformal characters associated with the corresponding superconformal logarithmic minimal models. The analytic and numerical findings are in agreement with general theory giving these logarithmic CFTs as a “logarithmic limit” of the rational superconformal minimal models.

**Title:** An integrable random tiling model

**Author:** Maria Tsarenko

**Affiliation:** The University of Melbourne

**Title:** Exploring the corner: Numerical evolution of Spin-2 fields at space-like infinity in Minkowski space-time.

**Author:** Ben Whale

**Affiliation:** University of Otago

**Title:** Crossover from isotropic to directed percolation

**Author:** Zongzheng Zhou

**Affiliation:** Monash University

**Abstract:** We generalize the directed percolation (DP) model by relaxing the strict directionality of DP such that propagation can occur in either direction but with anisotropic probabilities. We denote the probabilities as  $p_{\downarrow} = p \cdot p_d$  and  $p_{\uparrow} = p \cdot (1 - p_d)$ , with  $p$  representing the average occupation probability and  $p_d$  controlling the anisotropy. The Leath-Alexandrowicz method is used to grow a cluster from an active seed site. We call this model with two main growth directions *biased directed percolation* (BDP). Standard isotropic percolation (IP) and DP are the two limiting cases of the BDP model, corresponding to  $p_d = 1/2$  and  $p_d = 0, 1$  respectively.

In this work, besides IP and DP, we also consider the  $1/2 < p_d < 1$  region. Extensive Monte Carlo simulations are carried out on the square and the simple-cubic lattices, and the numerical data are analyzed by finite-size scaling. We locate the percolation thresholds of the BDP model for  $p_d = 0.6$  and  $0.8$ , and determine various critical exponents. These exponents are found to be consistent with those for standard DP. We also determine the renormalization exponent associated with the asymmetric perturbation due to  $p_d - 1/2 \neq 0$  near IP, and confirm that such an asymmetric scaling field is relevant at IP.