AN NPZ MODEL WITH TWO PERIODIC DRIVING FORCES CAUSED BY SOLAR RADIATION AND SEA SURFACE TEMPERATURE CAN DESCRIBE PHYTOPLANKTON BLOOM EVENTS WITH BETTER TIMING.

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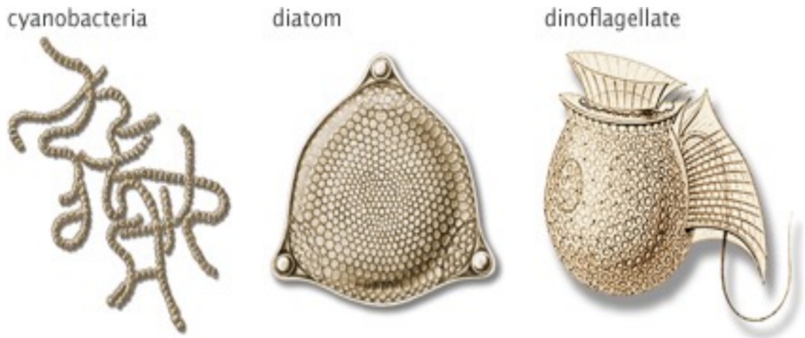
Introduction

- Phytoplankton bloom received considerable attention for many decades.

- **What is Phytoplankton?** Phytoplankton are the organisms, that photosynthesize and unable to swim actively against the current and usually found in marine and freshwater ecosystem. Phytoplankton includes diatoms, cyanobacteria and dinoflagellates etc.



Source : https://earthobservatory.nasa.gov/features/Phytoplankton

- **What is Phytoplankton Bloom?** When the phytoplankton population grows rapidly in water column, it creates bloom. Phytoplankton blooms, microalgal blooms, or red tides etc are different terms used to describe this phenomenon. Scientific community refers to these events as harmful algal bloom. Highly dense bloom can form steaks, foams or it can appear as thick mat on or just below the water surface and can take various colours like green, red, brown etc.

June 2016 Martin County, photo courtesy of James Breig

Source : https://www.surfrider.org/coastal-blog/entry/the-growing-problem-of-toxic-blue-green-algae-blooms-cyanobacteria

Source : https://serc.carleton.edu/microbelife/topics/redtide/general.html

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- **Effect of Phytoplankton Bloom :** During bloom, highly dense concentration of phytoplankton creates dead zone (hypoxic zone) by blocking sunlight and reducing oxygen level in water column which results in massive death of fish, plant and many other marine life.

- To study the bloom dynamics mathematically, many types of NPZ (nutrient – phytoplankton – zooplankton) models have been constructed and several hypotheses like critical depth hypothesis (CDH), critical turbulence hypothesis (CTH), disturbance-recovery hypothesis (DRH) etc. have been developed to explain bloom phenomena.

Source : https://i0.wp.com/chelsea.co.uk/wp-content/uploads/2021/11/algal-bloom-dead-fish.png?fit=2000,826&ssl=1

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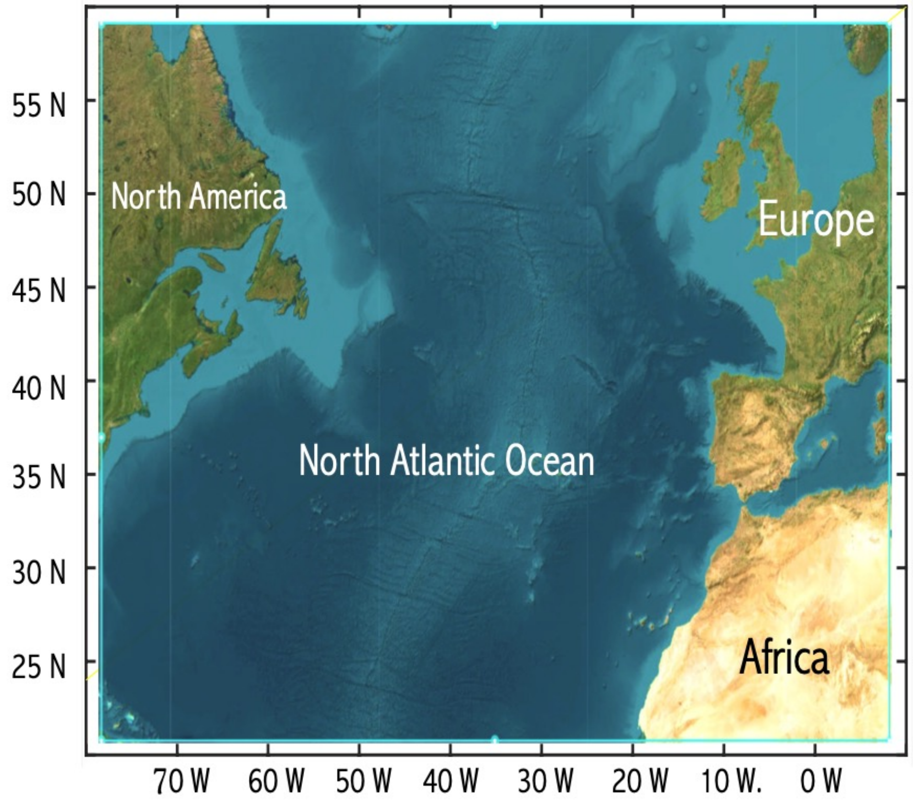
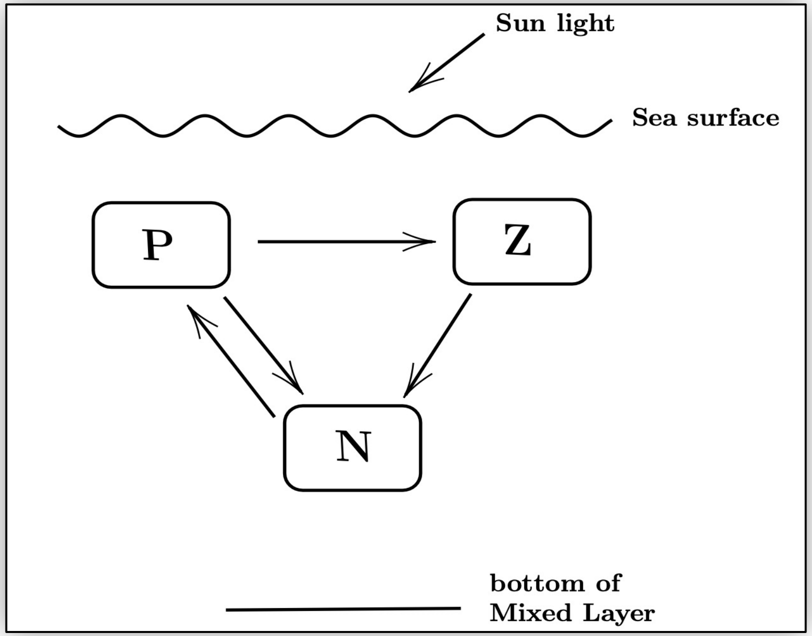
- In our study, we have studied the region 25-35^0 W, 40-45^0 N in the North Atlantic Ocean for phytoplankton bloom. In this region, It has been noticed that an increase in the chlorophyll concentration starts (i.e., bloom initiation) approximately around mid March (early spring) (also suggested by Kuhn et al. (2015)) and bloom lasts approximately on an average of 3.5 months.



Model Description

Model assumptions :

- The NPZ model is a two trophic level food chain model with the state variables : nutrients (N), phytoplankton (P), and zooplankton (Z).
- Nitrogen is the only limiting nutrient in the system.
- The processes of photosynthesis, nutrient intake, grazing, mortality, and excretion entirely control the concentrations of N, P, and Z.
- All zooplankton excretions and dead plankton are recycled into nutrients.
- The system is closed. Total nitrogen concentration of the system remains constant, i.e., $\frac{dN}{dt} + \frac{dP}{dt} + \frac{dZ}{dt} = 0$ i.e., $\mathrm{N} + \mathrm{P} + \mathrm{Z} = N_T$.



A two-trophic-level food chain model. The nutrient flow is depicted by arrows among the state variables P, Z and N. The figure is redrawn from the paper Wroblewski et al. (1988).

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Model is given by

$$\frac{dP}{dt} = \mu_0(Q_{10})^{\frac{T}{10}} \frac{I}{I_0} \frac{N}{K_N + N} P - h_{max}(1 - e^{-aP})Z - m_P P,$$

$$\frac{dZ}{dt} = \alpha h_{max}(1 - e^{-aP})Z - m_Z Z,$$

$$\frac{dN}{dt} = -\mu_0(Q_{10})^{\frac{T}{10}} \frac{I}{I_0} \frac{N}{K_N + N} P + (1-\alpha) h_{max}(1 - e^{-aP})Z + m_P P + m_Z Z.$$

This is inspired by Cowall et al. (2019). Here m_P, m_Z represents mortality rate of phytoplankton and zooplankton respectively, h_{max} represents maximum grazing rate by zooplankton, a is Ivlev constant, α denotes zooplankton's assimilation efficiency and K_N represents Michaelis-Menten half-saturation constant.

Temperature dependency of maximum growth rate of phytoplankton has been described by well-known Q_{10} formulation :

$$\mu_{max} = \mu_0(Q_{10})^{\frac{T}{10}}$$

where μ_0 is the maximum growth rate at 0^0C and T represents the sea surface temperature (SST). Q_{10} formulation asserts that the maximum growth rate (μ_{max}) increases by a factor Q_{10} when the temperature increases by 10^0C.

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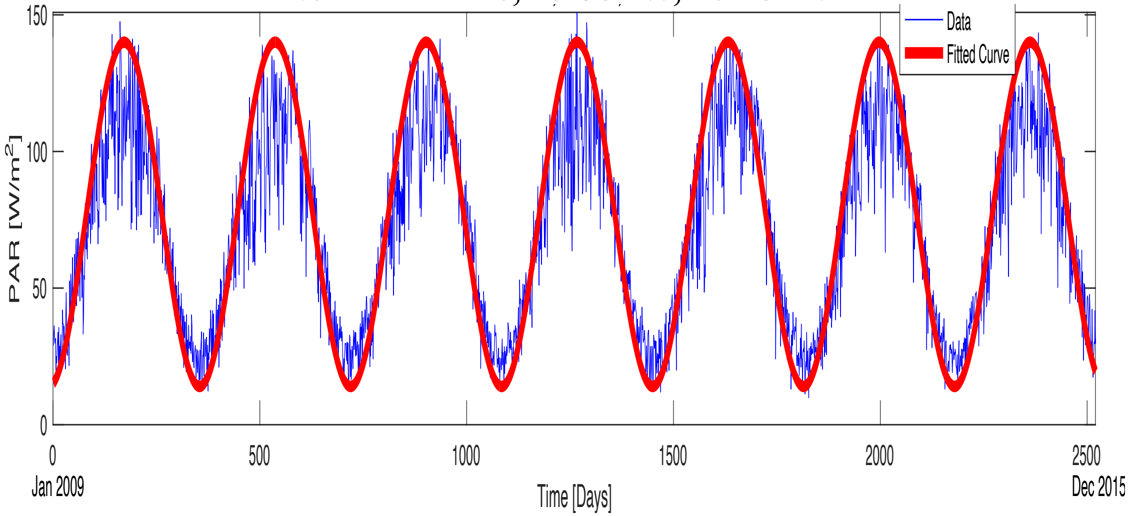
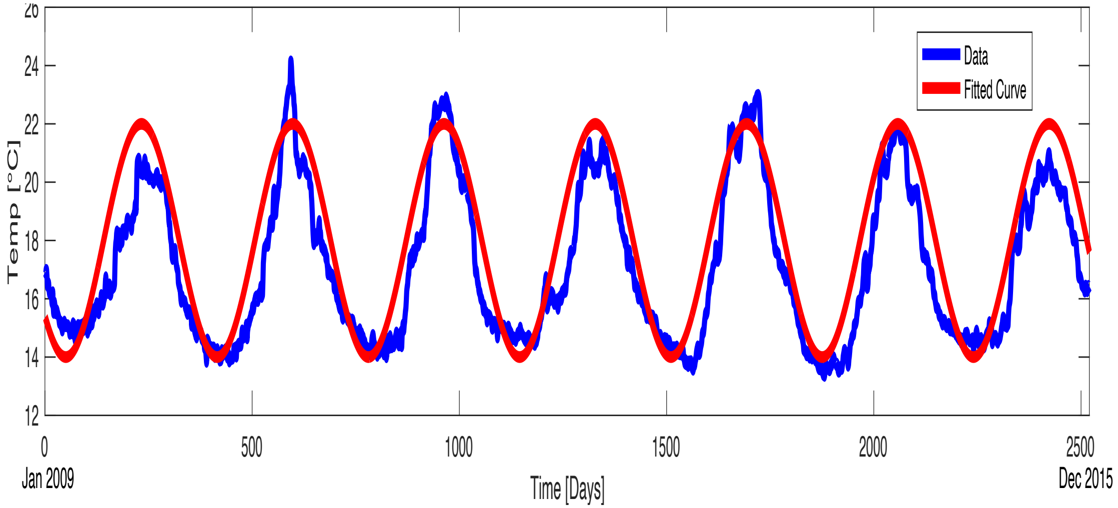
The two periodically driving forces of the model, namely solar radiation ($I(t)$) and sea surface temperature ($T(t)$) have been modelled by using sinusoidal function. They have been constructed by comparing with satellite data of the region 25-35^0 W, 40-45^0 N in the North Atlantic Ocean.

Data : Surface Solar Radiation, North Atlantic, 25-35^0 W, 40-45^0 N



Data : Sea Surface Temperature, North Atlantic, 25-35^0 W, 40-45^0 N



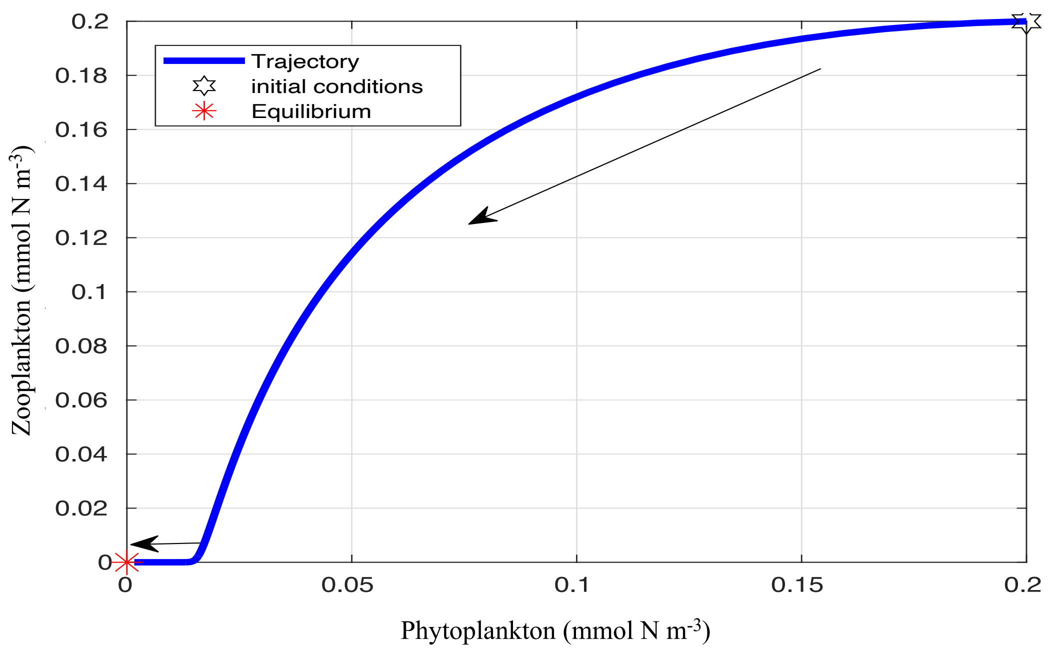
Time series of solar radiation and sea surface temperature data from January 2009 to December 2015 with sinusoidal best-fit curve. All data averaged over the region 25-35^0 W, 40-45^0 N . Blue line indicates data and red line indicates the fitted curve. These data are available at https://power.larc.nasa.gov/data-access-viewer/

Dynamics of Autonomous Model

Define $\rho_2 = \mu_0 (Q_{10})^{\frac{T(t)}{10}} \frac{I(t)}{I_0}$.

Treating ρ_2 as parameter, by linear stability analysis, all the stability conditions of all three equilibrium points of the system in terms of ρ_2 are given by

- The equilibrium point $(0,0,N_T)$ always exists and is stable if and only if $\rho_2 < \hat{P}_E$.



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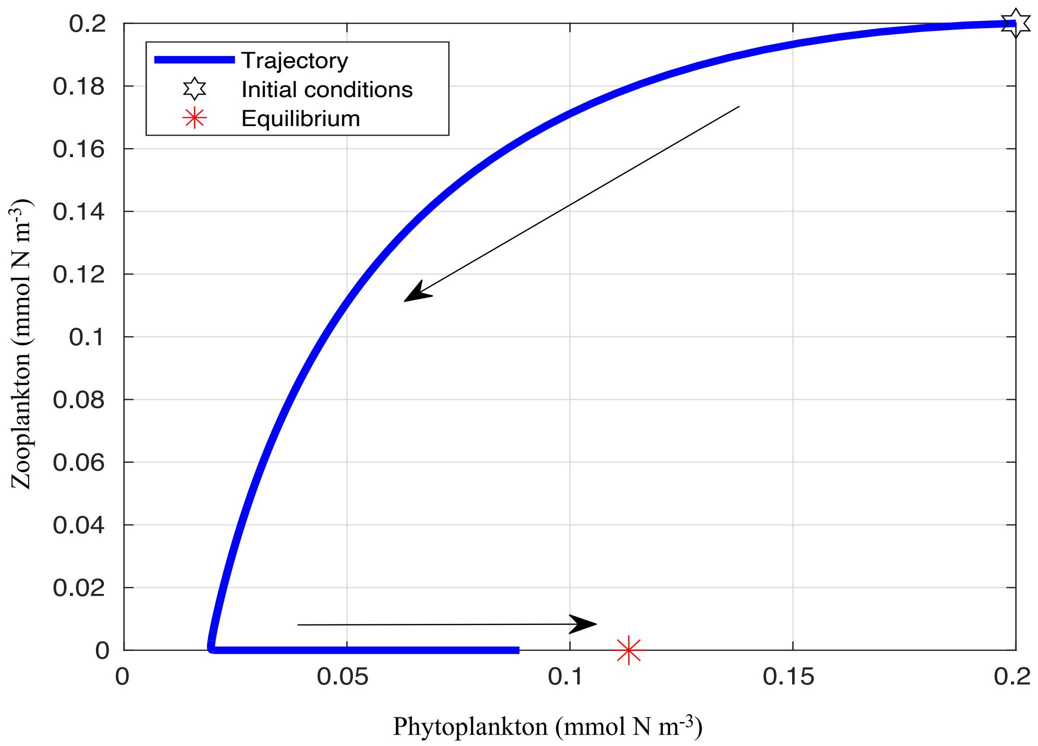
- If $\rho_2 > \hat{P}_E$, then the equilibrium point $(\bar{P}, 0, N_T - \bar{P})$ exists and is stable if and only if $\hat{P}_E < \rho_2 < \hat{P}_S$.



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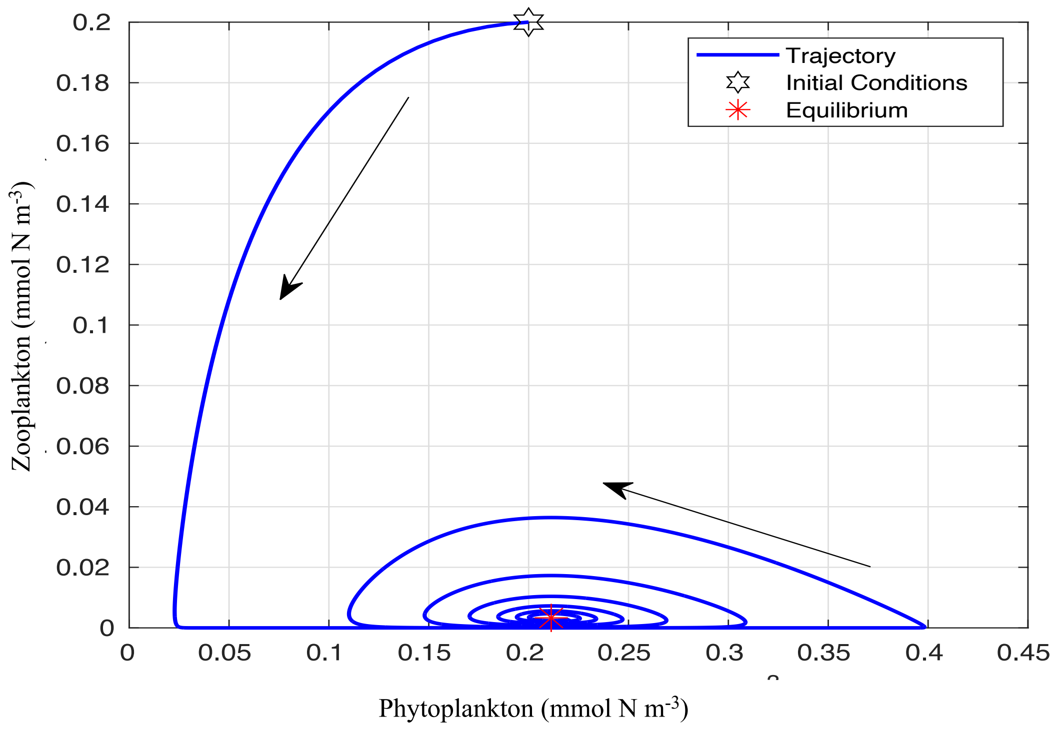
- If $\rho_2 > \hat{P}_S$, then the equilibrium point $(P^*, Z^*, N_T - P^* - Z^*)$ exists and is stable if and only if $\rho_2 > \hat{P}_S$ and $R_2 < 1$.



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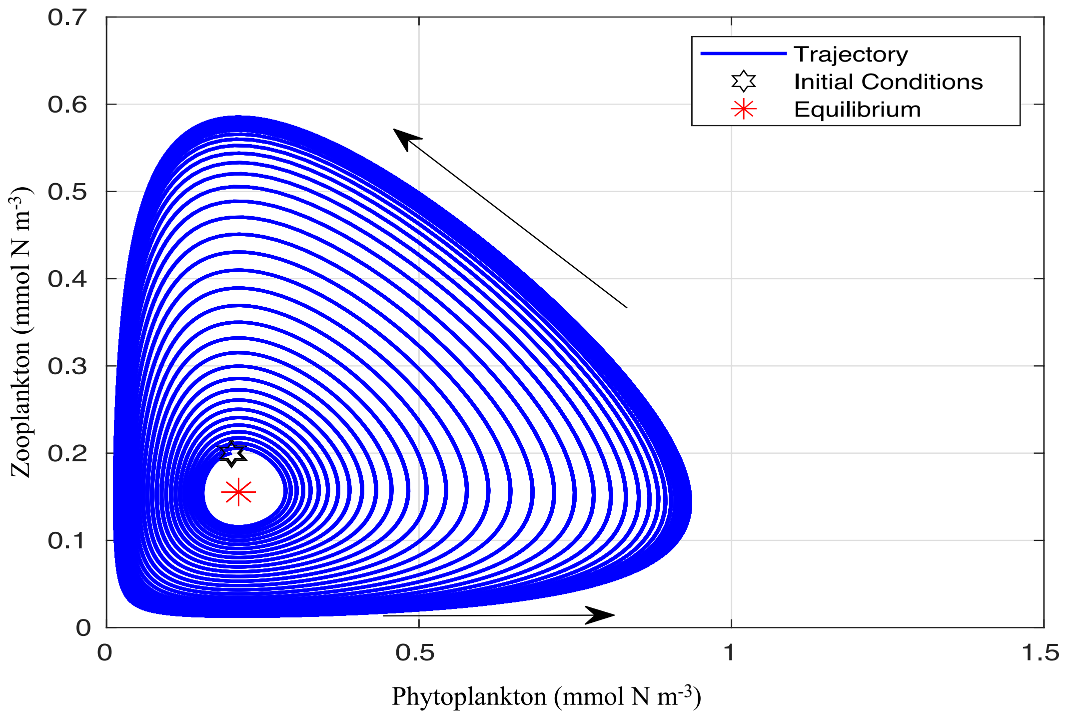
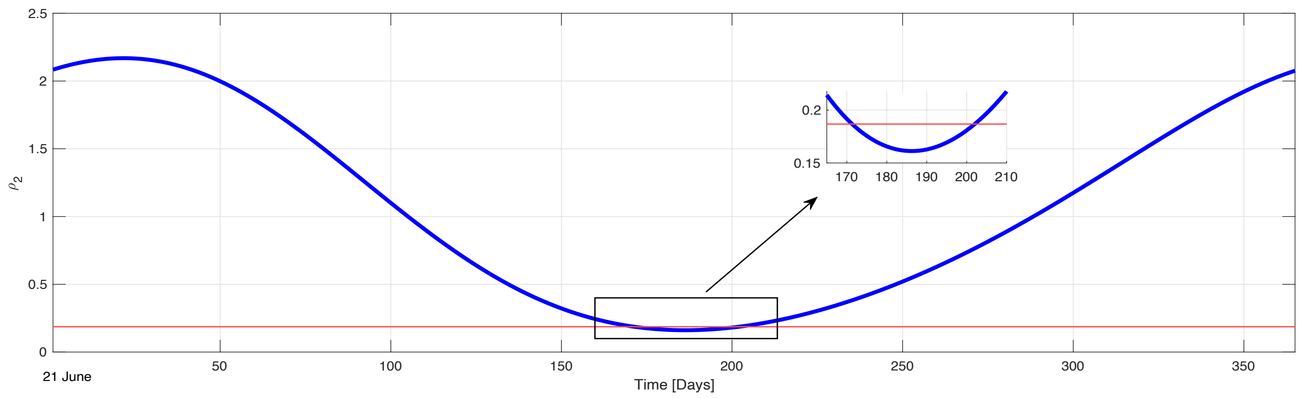
- If $R_2 > 1$ then the equilibrium point $(P^*, Z^*, N_T - P^* - Z^*)$ is unstable but is surrounded by at least one closed orbit.



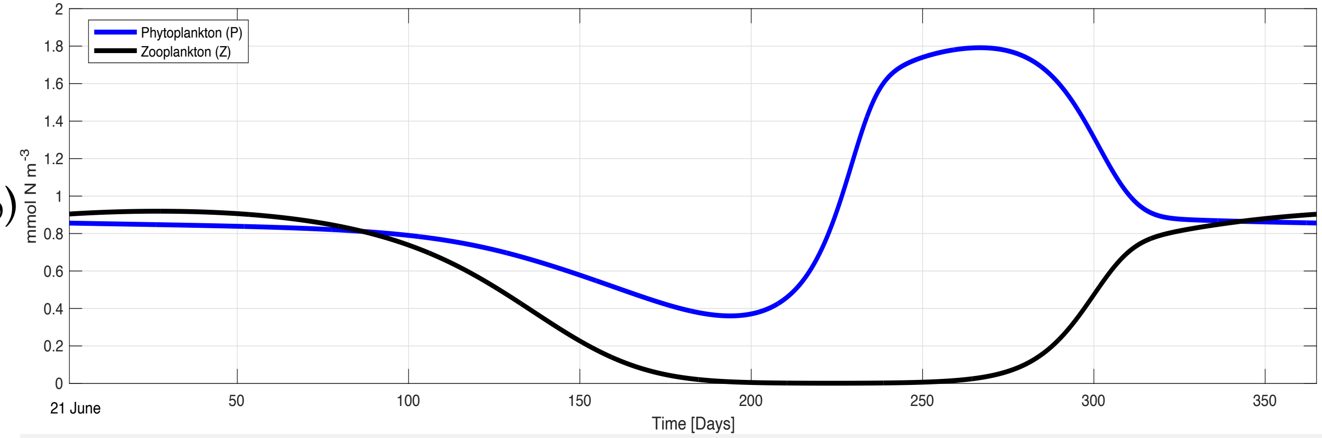
Results of Non-autonomous Model

By considering actual time dependent form of ρ_2 i.e., $\rho_2(t) = \mu_0 (Q_{10})^{\frac{T(t)}{10}} \frac{I(t)}{I_0}$, the non–autonomous model simulated results are given by

(a)



(b)

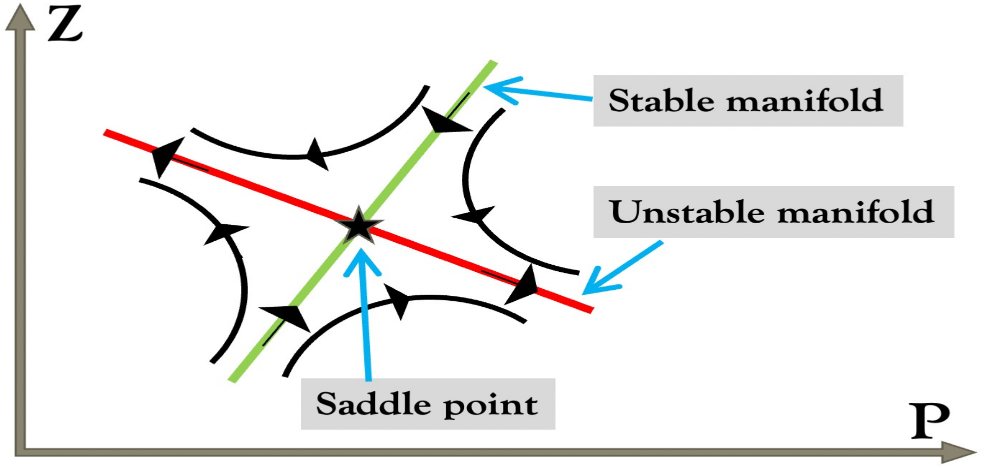


(a) ρ_2 vs time, where red line corresponding to the value $\hat{P}_S$. Embedded graph shows that $\rho_2 < \hat{P}_S$ for some period of time and (b) Time series for the solution of the non-autonomous system after 365 days.

- In numerical simulation, it has been considered that the year starting from summer solstice (21st June).
- With the decrease in ρ_2, phytoplankton population approaches towards the yearly low concentration.
- Thus, the food supply (as phytoplankton is only prey here) for zooplankton population decreases, consequently zooplankton population starts to decrease and attains its annual minimum just some days after phytoplankton's annual minimum.
- With the increase in light and temperature dependent growth rate function ($\rho_2(t)$) and low grazing pressure, phytoplankton population starts increasing indicating bloom initiation and the bloom stays up to mid march.
- Because of the availability of sufficient amount of food, zooplankton population starts increasing, so grazing pressure on phytoplankton starts increasing which results in the termination of bloom.

Mathematics Behind Bloom Dynamics

Description of bloom dynamics mathematically using saddle point bloom mechanism following the work of Cowall et al. (2019).



Three results which are useful behind the explanation :

- When $\rho_2 < \hat{P}_S$, the plankton-free equilibrium is a saddle point with the stable manifold $\{(P, Z): P = 0, 0 < Z \leq N_T\}$, the phytoplankton-only equilibrium is stable and coexisting equilibrium doesn't exist.
- When $\rho_2 > \hat{P}_S$, the phytoplankton-only equilibrium is a saddle point with the stable manifold $\{(P, Z): Z = 0, 0 < P \leq N_T\}$, coexisting equilibrium exists. And co-existing equilibrium is stable if $R_2 < 1$.
- $\bar{P} > P^*$, when co-existing equilibrium exists.

- When solar radiation and sea surface temperature dependent growth rate ρ_2 decrease and falls below $\hat{P}_S$, the system temporarily approaches towards plankton-free equilibrium (0,0) (a saddle point), which implies P approaches 0 but it will not attain the zero concentration. So, phytoplankton population attains its annual minimum at this point .
- After this ρ_2 starts increasing and goes above $\hat{P}_S$, the system temporarily approaches a phytoplankton-only equilibrium $(\bar{P}, 0)$ (a saddle point), which implies P approaches $\bar{P}$.
- We know that $\bar{P} > P^*$, which implies that when the system is close to a phytoplankton-only equilibrium, the phytoplankton concentration will be larger than P^*. At this stage, the bloom reaches its peak.
- The system then shifts away from the phytoplankton-only equilibrium (saddle point) along unstable manifold, bringing the bloom to an end.

Improvement in Bloom Timing

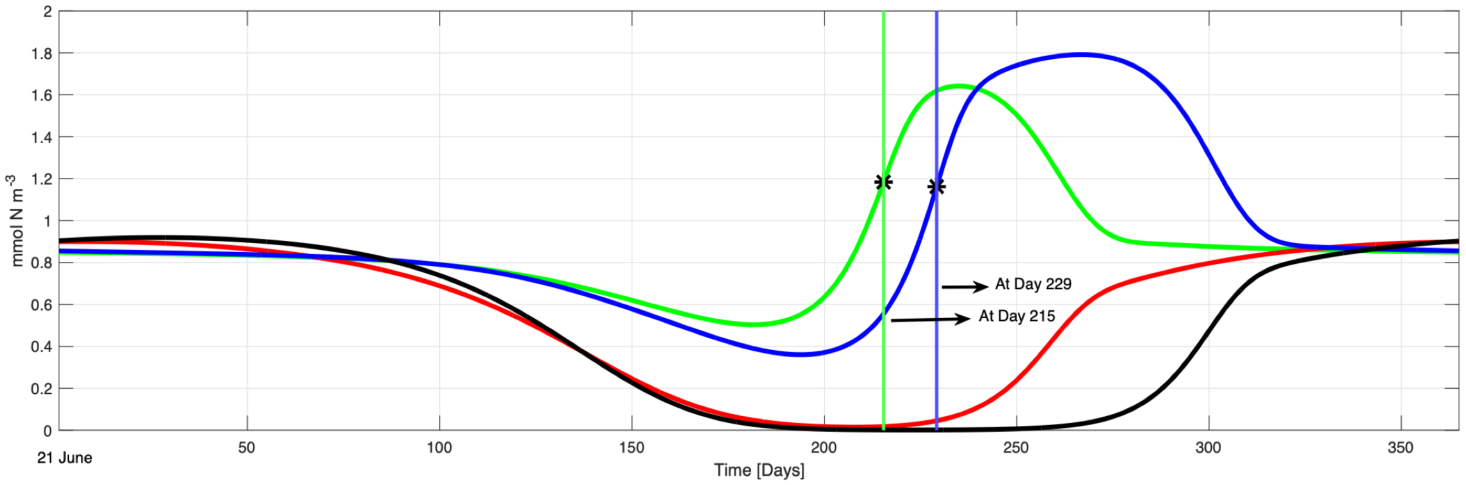
Comparison of our model simulated results with an existing model result :

- Cowall et al. (2019) studied a NPZ model with one periodically driving force (solar radiation). Model simulated results showed that the rapid increase in the phytoplankton concentration (i.e., bloom initiation) starts after winter solstices in North Atlantic.

- But in reality, actual bloom initiation starts in North Atlantic in mid March which is far later than the predicted one.

- We have applied the rate of change method (ROC method) to predict when the bloom initiates for both the models (i.e., our model and the model used by Cowall et al. (2019)).

- It has been observed that bloom initiates around 21st January for the model used by Cowall et al. (2019) and around 4th February for our model. Thus, an improvement of 14 days has been seen in the bloom initiation.



Green and red lines represent phytoplankton and zooplankton concentrations obtained from the numerical solution of the model used by Cowall et al. (2019) . Blue and black lines represent phytoplankton and zooplankton concentrations obtained by simulating our model. The vertical lines mark the day of bloom initiation according to the ROC method

Conclusion

- Usually in saddle point bloom mechanism, coexisting equilibrium always exists and the unstable nature of phytoplankton only equilibrium (saddle point) is used to explain bloom dynamics (see Cowall et al. (2019)). But we are able to use saddle point bloom mechanism even when coexisting equilibrium doesn't exist for some value of ρ_2 by involving unstable nature of plankton-free equilibrium (saddle point).

- Cowall et al. (2019) considered only one periodically driving force in their model and the model predicted bloom initiation timing was far earlier than the actual bloom initiation time in North Atlantic. This discrepancy suggests us to include another periodically driving force (sea surface temperature) into the model.

- Our model simulated result describes the phytoplankton bloom in region 25-35^0 W, 40-45^0 N in the North Atlantic Ocean with better timing.

References

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