Laplace transform analysis of the coupled climate–carbon system

Nathan Clisby (MASCOS, University of Melbourne)
Ian G. Enting (MASCOS, University of Melbourne)

September 25, 2007
<table>
<thead>
<tr>
<th>Outline</th>
<th>Introduction</th>
<th>Climate</th>
<th>Feedbacks</th>
<th>Modelling</th>
<th>Laplace transform analysis</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Climate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feedbacks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modelling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laplace transform analysis</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
IPCC Fourth Assessment

- Warming of the climate system is unequivocal...
IPCC Fourth Assessment

- Warming of the climate system is unequivocal . . .
- Global atmospheric concentrations of carbon dioxide, methane and nitrous oxide have increased markedly as a result of human activities since 1950 . . .
• Warming of the climate system is unequivocal . . .
• Global atmospheric concentrations of carbon dioxide, methane and nitrous oxide have increased markedly as a result of human activities since 1950 . . .
• Most of the observed increase in global average temperatures since the mid-20th century is very likely due to the observed increase in anthropogenic greenhouse gas concentrations.
IPCC caveats

- The magnitude of the positive feedback between climate change and the carbon cycle is uncertain. (AR4: TS.5.5).
IPCC caveats

- The magnitude of the positive feedback between climate change and the carbon cycle is uncertain. (AR4: TS.5.5).
- Dynamical processes not included in current models but suggested by recent observations could increase the vulnerability of the ice sheets to warming, increasing future sea level rise. (AR4: TS.5.5).
• Climate: complex system. Difficult but important problem.
• Attack with a variety of approaches.
• Want conclusions to be robust: shouldn’t depend on a particular model or even kind of model.
• Laplace transform method can help frame appropriate questions.
• Also (potentially) useful numerically to understand feedbacks.
Data

- Direct measurements since 1958.
- Ice cores: air trapped as snow accumulates on glaciers and ice caps.
- High accumulation rates: excellent time resolution (about 11 years for Law Dome), but relatively short records (2000 years).
- Low accumulation rates: poor time resolution, but very long records (up to about 700,000 years).

Ice cores give information about
- \( \text{CO}_2 \) and other gas concentrations.
- Temperature (local to the ice) and global ice volume proxies.
Problem here! Missing image
Law Dome ice core and global temperature data (11-year smoothing of temperature matches smoothing of CO₂ in bubble trapping).
Concentrations of Greenhouse Gases from 0 to 2005

- **Carbon Dioxide (CO₂)**
- **Methane (CH₄)**
- **Nitrous Oxide (N₂O)**

Year Range:
- 0 to 2000

Units:
- CO₂, N₂O (ppm)
- CH₄ (ppb)
Greenhouse gases, temperature and ice volume proxies
Feedbacks

- The magnitude of the positive feedback between climate change and the carbon cycle is uncertain. (AR4: TS.5.5).
- Increasing temperature may amplify atmospheric CO$_2$. 
Greenhouse gases, temperature and ice volume proxies
• Forcing from changes in Earth’s orbit, northern hemisphere insolation.
• Leads to melting of polar ice.
• Positive feedback as albedo decreased, more radiation absorbed.
• CO$_2$ concentrations increase due to warming, additional positive feedback.
• Forcing from changes in Earth’s orbit, northern hemisphere insolation.
• Leads to melting of polar ice.
• Positive feedback as albedo decreased, more radiation absorbed.
• CO₂ concentrations increase due to warming, additional positive feedback.
• Will there be a corresponding positive climate–carbon feedback under present conditions?
Modelling Climate

- White box: built from basic physical and chemical processes.
- Advantages: can understand what is happening. May have predictive power.
- Disadvantages: hard, difficult to quantify missing processes.
- Black box: purely statistical models.
- Advantages: fewer assumptions.
- Disadvantages: difficult to extrapolate.
- Grey box: include some processes, exclude others.
Modelling Climate

- General Circulation Models: include physical and chemical processes.
  - Complex.
  - Computationally demanding.
- Simple models - useful for policy decisions, e.g. stabilisation of CO₂.
C$^4$MIP

- C$^4$MIP computer experiment: many coupled climate–carbon GCMS, forced with a standard IPCC scenario.
- Feedback amplification from 11 models range from 1.04 to 1.44, 20ppm to 200ppm.
- Estimate $1.18 \pm 0.11$. Not a genuine uncertainty - how do we know which model is better?
- Large uncertainty, and possibility that processes are missing despite ability to reconstruct 20th Century.
Linear response function, $R$, defines how concentrations, $C$, respond to source, $S$.

$$Q(t) = C(t) - C(t_0) = \int_{t_0}^{t} R(t - t') S(t') \, dt'$$

- **Forward modelling**: calculate $C$ given model response $R$ and sources, $S$.
- **Inverse problems**:
  - Deduce $R(t)$ from $C(t)$ and $S(t)$.  
  - Deduce $S(t)$ from $R(t)$ and $C(t)$.  

**Model calibration** 
**Deconvolution**
The Laplace transform

Laplace transforms have similar properties to Fourier transforms in analysing linear systems.

- Transforms from time, $t$ to inverse time variable, $p$.
- Convolution relations transform to products.
- Integration multiples transform by $1/p$.
- More appropriate for one-sided causal relationships than Fourier transforms.

- Notation:

\[
f(p) = \mathcal{L} [F(t)] = \int_{0}^{\infty} F(t) e^{-pt} \, dt
\]

- Carbon relations are $q(p) = r(p) s(p)$ whence:
  
  $r(p) = q(p)/s(p)$ and $s(p) = q(p)/r(p)$
Information in CO$_2$ data

For exponentially growing emissions

\[ Q(t) = \int_{-\infty}^{t} R(t - t') A \exp(\beta t') \, dt' \]

\[ = A \exp(\beta t) \int_{0}^{\infty} R(\tau) A \exp(-\beta \tau) \, d\tau \]

and \( s(p) = \mathcal{L} [A \exp(\beta t)] = A/(p - \beta) \), so

\[ q(p) = Ar(p)/(p - \beta) \]
Information in CO₂ data

For exponentially growing emissions

\[ Q(t) = \int_{-\infty}^{t} R(t - t') A \exp(\beta t') \, dt' \]

\[ = A \exp(\beta t) \int_{0}^{\infty} R(\tau) A \exp(-\beta \tau) \, d\tau \]

and \( s(p) = \mathcal{L}[A \exp(\beta t)] = A/(p - \beta) \), so

\[ q(p) = Ar(p)/(p - \beta) \]

- For 20th century, \( \beta \approx 0.02 \).
- Concentrations determined by behaviour of \( r(p) \) in vicinity of \( p = \beta \).
- Almost no information about \( r(p) \) elsewhere.
Linearised model of the coupled climate–carbon system

Laplace transformed equations

\[ q(p) = r(p)[s(p) + h(p)w(p)] \]
\[ w(p) = u(p)[f(p) + \alpha q(p)] \]
\[ \Rightarrow q(p) = \frac{r(p)[s(p) + h(p)u(p)f(p)]}{1 - \alpha u(p)r(p)h(p)} \]
\[ \Rightarrow w(p) = \frac{u(p)[f(p) + \alpha r(p)s(p)]}{1 - \alpha u(p)r(p)h(p)} \]
• Atmospheric response $r(p)$ to emissions amplified by
  $1/(1 - \alpha u(p)r(p)h(p))$.
• Warming response to forcings amplified by
  $1/(1 - \alpha u(p)r(p)h(p))$.
• Require $\alpha u(p)r(p)h(p) < 1$ for all $p$ for stability.
• Caveat: any predictive power obtained from determination of $h(p)$ depends on dynamics remaining unchanged.
• Atmospheric response $r(p)$ to emissions amplified by 
  $1/(1 - \alpha u(p)r(p)h(p))$.

• Warming response to forcings amplified by 
  $1/(1 - \alpha u(p)r(p)h(p))$.

• Require $\alpha u(p)r(p)h(p) < 1$ for all $p$ for stability.

• Caveat: any predictive power obtained from determination 
  of $h(p)$ depends on dynamics remaining unchanged.

• Is calibration using $C(t)$ and $S(t)$ giving models with $r(p)$ 
  or $r(p)/(1 - \alpha u(p)r(p)h(p))$?

• If yes, need to avoid double counting feedback when 
  making 21st century predictions.
• Scheffer et al., assume a linear model and instantaneous response.

\[
\frac{\Delta T_{\text{feedback}}}{\Delta T_0} = \frac{1}{1 - \delta \alpha}
\]

• From data of little ice age,
  - Moberg gives 1.18, range 1.07-1.25
  - Mann and Jones gives 1.78, range 1.28-2.93

• Positive feedback, but very large uncertainties!

• Can improve handling of time dependence with Laplace method, but limiting factor is reconstruction quality.
High quality global instrumental data for temperature available from around 1880.

CO$_2$ response to Pinatubo may tell us about feedback $H(t)$ for timescales of years.

1940s dip in CO$_2$, temperature may give information about decadal time scale.

Little ice age, $H(t)$ for timescales of centuries (large uncertainties due to temperature reconstructions).

Improved data from high resolution Law Dome ice cores. Will also need improved temperature reconstructions.

Glacial-interglacial data from Vostok ice core (difficult to quantify relevance for current climate).
• Laplace transform formalism clarifies some issues regarding information content of measurements.
• We do not yet know if feedbacks are typically included via calibration with 20th Century data.
• Use recently obtained high resolution data from Law Dome ice core to obtain quantitative understanding of feedbacks over last 2000 years (work in progress).
Twisted: The Distorted Mathematics of Greenhouse Denial.
Ian G. Enting
October 2007