

The finite lattice method.

- here applied to enumeration of $d=2$

SAWs



SAPs



\mathcal{O} 's



- also applicable to problems involving connected graph expansions e.g. Ising model
- mini-review - original work by Ian Enting, Tom de Neef, Tony Guttmann and Iwan Jensen
- goal: to extend count of C_n using

$$C_n = 2d C_{n-1} + \sum_{m=2}^n \pi_m C_{n-m}$$

λ^{d-1}

- For SAN $c_n \sim \mu^n n^{\alpha-1}$
- For SAP $p_{2n} \sim \mu^{2n} n^{\alpha-3}$
- $d=2$ $\mu=2.638\dots$
- i.e. exponential growth \rightarrow hard problem

Best approach:

- Find exact solution
(or at least a polynomial time algorithm)

Simplest approach:

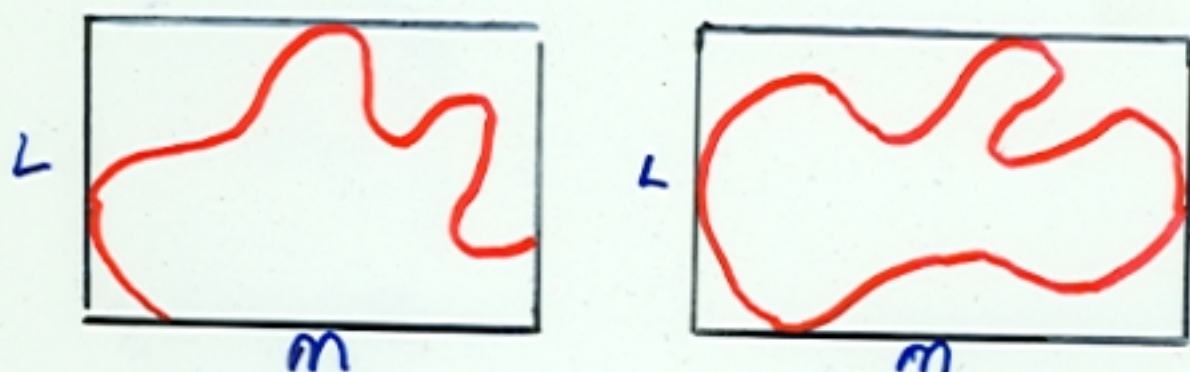
- Direct enumeration using backtracking
- time $O(c_n) = O(\mu^n)$
- Best known method for $d \geq 3$

Finite Lattice Method:

- still exponential ($O(\alpha^n)$) for $d=2$
- BUT $\alpha < \mu$
- SAW $\alpha \approx 1.3$, enumerated c_n to $n=71$
- SAP $\alpha \approx 1.20$, enumerated p_n to $n=110$

Method - step 1

- Any SAW or SAP has an enclosing rectangle of smallest size



- $C_n = \#$ of SAWs of length n \leftarrow
 $= \sum_{\substack{L \times M \\ \text{rectangles}}} \left(\# \text{ of SAWs of length } n \right.$
 $\left. \text{which touch all boundaries of an } L \times M \text{ rectangle} \right)$
- SAW of length n fits in a rectangle of perimeter $\leq 2n$. i.e. $L+M \leq n$
- SAP of length $2n$ fits in a rectangle of perimeter $\leq 2n$ i.e. $L+M \leq n$
- In terms of generating functions

$$\begin{aligned} \chi_n(z) &= \sum_{j=0}^n c_j z^j \\ &= \sum_{\substack{L \times M \\ L+M \leq n}} \chi_n^{(L \times M)}(z) \end{aligned}$$