The finite lattice method.

- here applied to enumeration of $d=2$

   SAWs   SAPs θ's

- also applicable to problems involving connected graph expansions e.g. Ising model

- mini-review — original work by Ian Enting, Tom de Neef, Tony Guttmann and Iwan Jensen

- goal: to extend count of c_n using
$$c_n = 2d\, c_{n-1} + \sum_{m=2}^{n} \pi_m c_{n-m}$$

$$\lambda^{d-1}$$

- For SAN $c_n \sim \mu^n n^{\gamma - 1}$
- For SAP $p_{2n} \sim \mu^{2n} n^{\alpha - 3}$
- $d = 2$ $\mu = 2.638....$
- i.e. exponential growth $\longrightarrow$ hard problem

Best approach:
- Find exact solution
  (or at least a polynomial time algorithm)
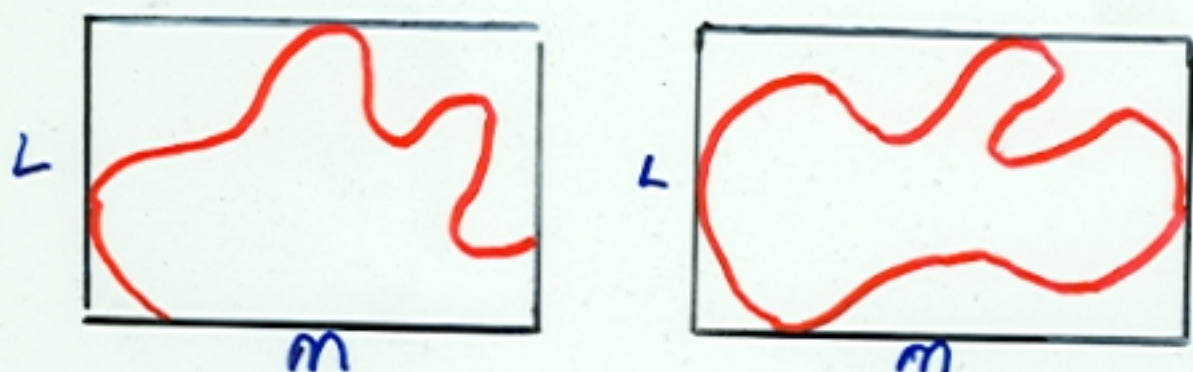
Simplest approach:
- Direct enumeration using backtracking
- time $O(c_n) = O(\mu^n)$
- Best known method for $d \geqslant 3$

Finite Lattice Method:
- still exponential $(O(\alpha^n))$ for $d = 2$
- BUT $\alpha < \mu$
- SAW $\alpha \simeq 1.3$, enumerated c_n to $n = 71$
- SAP $\alpha \simeq 1.20$, enumerated p_n to $n = 110$

Method - step 1

- Any SAW or SAP has an enclosing rectangle of smallest size

- C_n = # of SAWs of length n

$$= \sum_{\substack{L \times M \\ \text{rectangles}}} \left(\begin{array}{c} \text{# of SAWs of length } n \\ \text{which touch all boundaries} \\ \text{of an } L \times M \text{ rectangle} \end{array} \right)$$

- SAW of length n fits in a rectangle of perimeter $\leq 2n$. i.e $\underline{L+M \leq n}$
- SAP of length $2n$ fits in a rectangle of perimeter $\leq 2n$ i.e $L+M \leq n$

- In terms of generating functions

$$\chi_n(z) = \sum_{j=0}^{n} c_j \, z^j$$

$$= \sum_{\substack{L \times M \\ L+M \leq n}} \chi_n^{(L \times M)}(z)$$