

# The finite lattice method.

- here applied to enumeration of  $d=2$

SAWs



SAPs



$\mathcal{O}$ 's



- also applicable to problems involving connected graph expansions e.g. Ising model
- mini-review - original work by Ian Enting, Tom de Neef, Tony Guttmann and Iwan Jensen
- goal: to extend count of  $C_n$  using

$$C_n = 2d C_{n-1} + \sum_{m=2}^n \pi_m C_{n-m}$$

$\lambda^{d-1}$

- For SAN  $c_n \sim \mu^n n^{\alpha-1}$
- For SAP  $p_{2n} \sim \mu^{2n} n^{\alpha-3}$
- $d=2$   $\mu=2.638\dots$
- i.e. exponential growth  $\rightarrow$  hard problem

### Best approach:

- Find exact solution  
(or at least a polynomial time algorithm)

### Simplest approach:

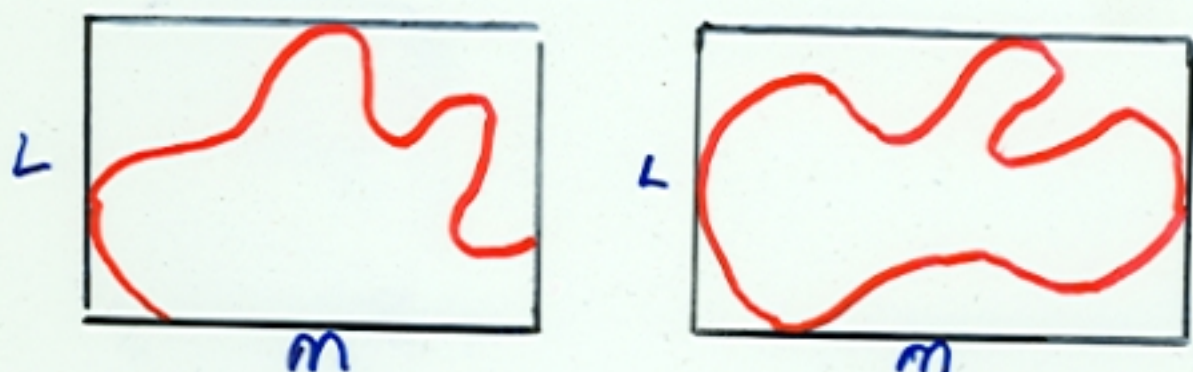
- Direct enumeration using backtracking
- time  $O(c_n) = O(\mu^n)$
- Best known method for  $d \geq 3$

### Finite Lattice Method:

- still exponential ( $O(\alpha^n)$ ) for  $d=2$
- BUT  $\alpha < \mu$
- SAW  $\alpha \approx 1.3$ , enumerated  $c_n$  to  $n=71$
- SAP  $\alpha \approx 1.20$ , enumerated  $p_n$  to  $n=110$

## Method - step 1

- Any SAW or SAP has an enclosing rectangle of smallest size



- $C_n = \#$  of SAWs of length  $n$   $\leftarrow$   
 $= \sum_{\substack{L \times M \\ \text{rectangles}}} \left( \# \text{ of SAWs of length } n \right.$   
 $\left. \text{which touch all boundaries of an } L \times M \text{ rectangle} \right)$
- SAW of length  $n$  fits in a rectangle of perimeter  $\leq 2n$ . i.e.  $L+M \leq n$
- SAP of length  $2n$  fits in a rectangle of perimeter  $\leq 2n$  i.e.  $L+M \leq n$
- In terms of generating functions

$$\chi_n(z) = \sum_{j=0}^n c_j z^j$$

$$= \sum_{\substack{L \times M \\ L+M \leq n}} \chi_n^{(L \times M)}(z)$$