What's Shaking in the Sandbox

particle localization
advection–diffusion–reaction
geophysical flows
thermodynamics of soft systems
chaos
complex systems
dynamical systems
geothermal energy

Guy Metcalfe

mixing
nonlinear dynamics
heat transfer
fluid mechanics
microfluidics
applied mechanics
granular dynamics
process design

MONASH University

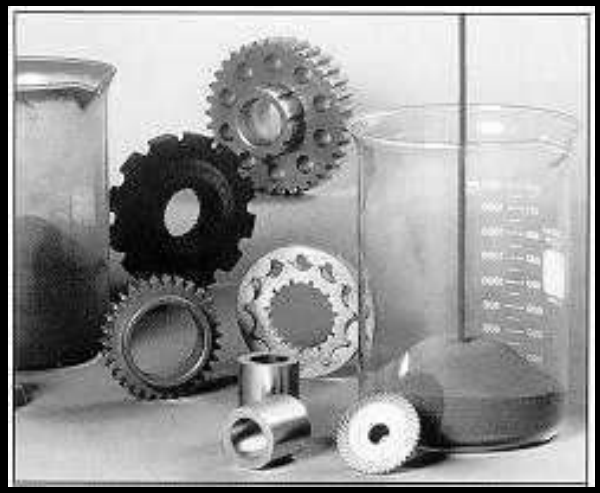
Cuts and Discontinuities:
Effects on Material Transport

Wedges

Piecewise Isometries

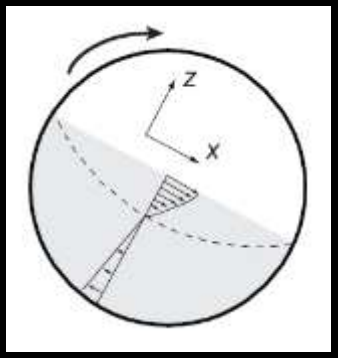
Web of Cutting Preimages

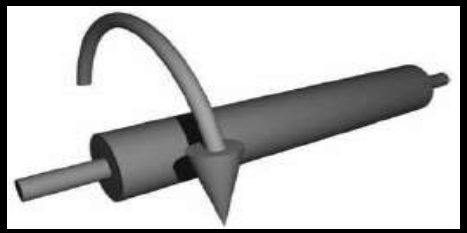
Industrial Granular Flows

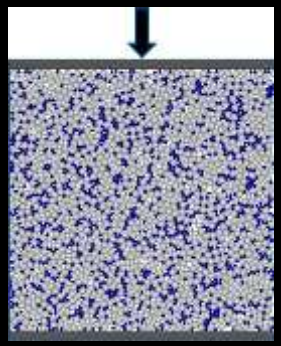


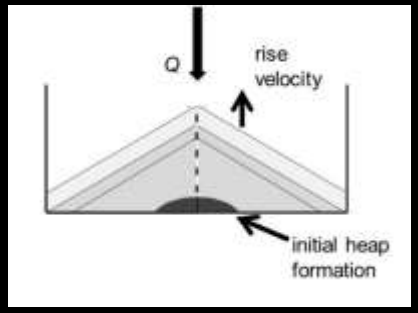
Natural Granular Flows

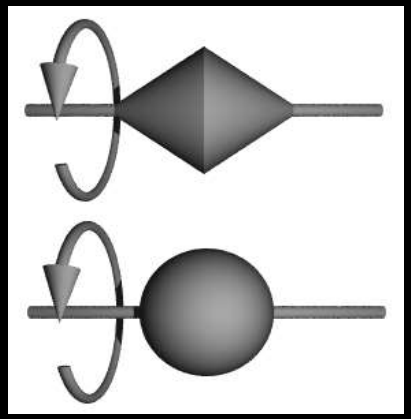
Typical granular experiments

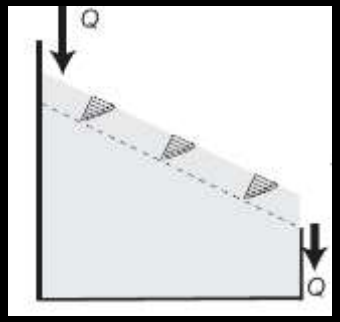


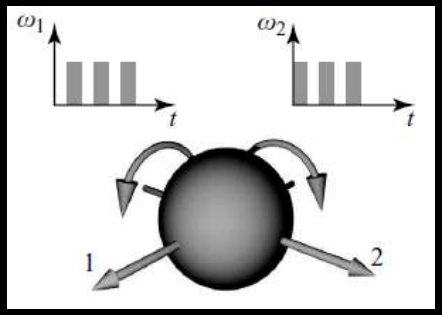


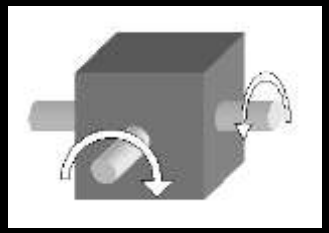




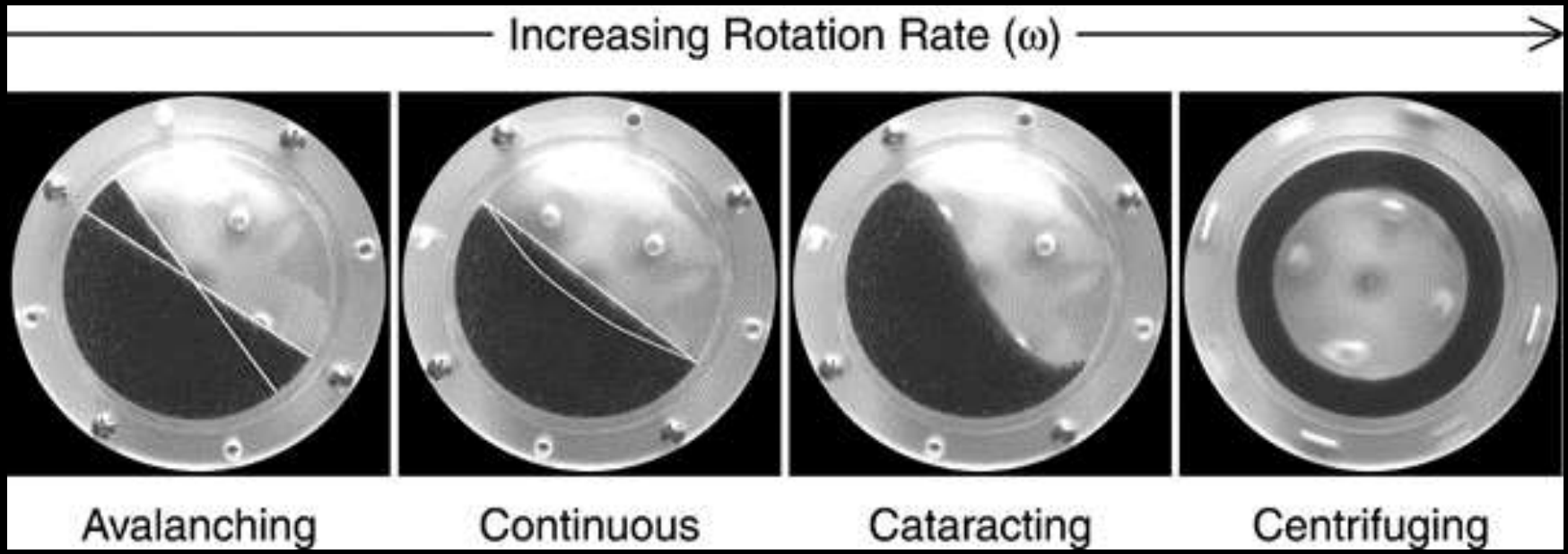


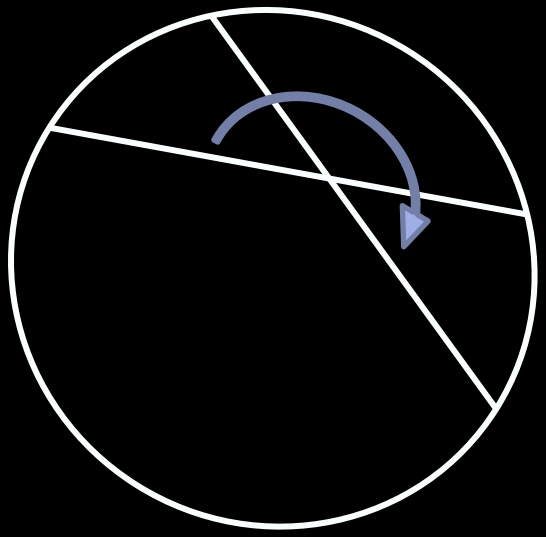






Flow in a Rotating Tumbler

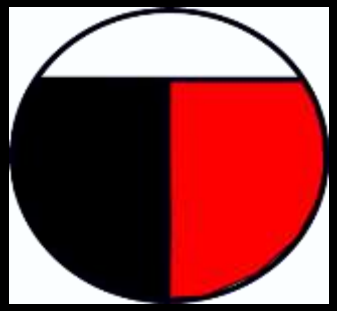




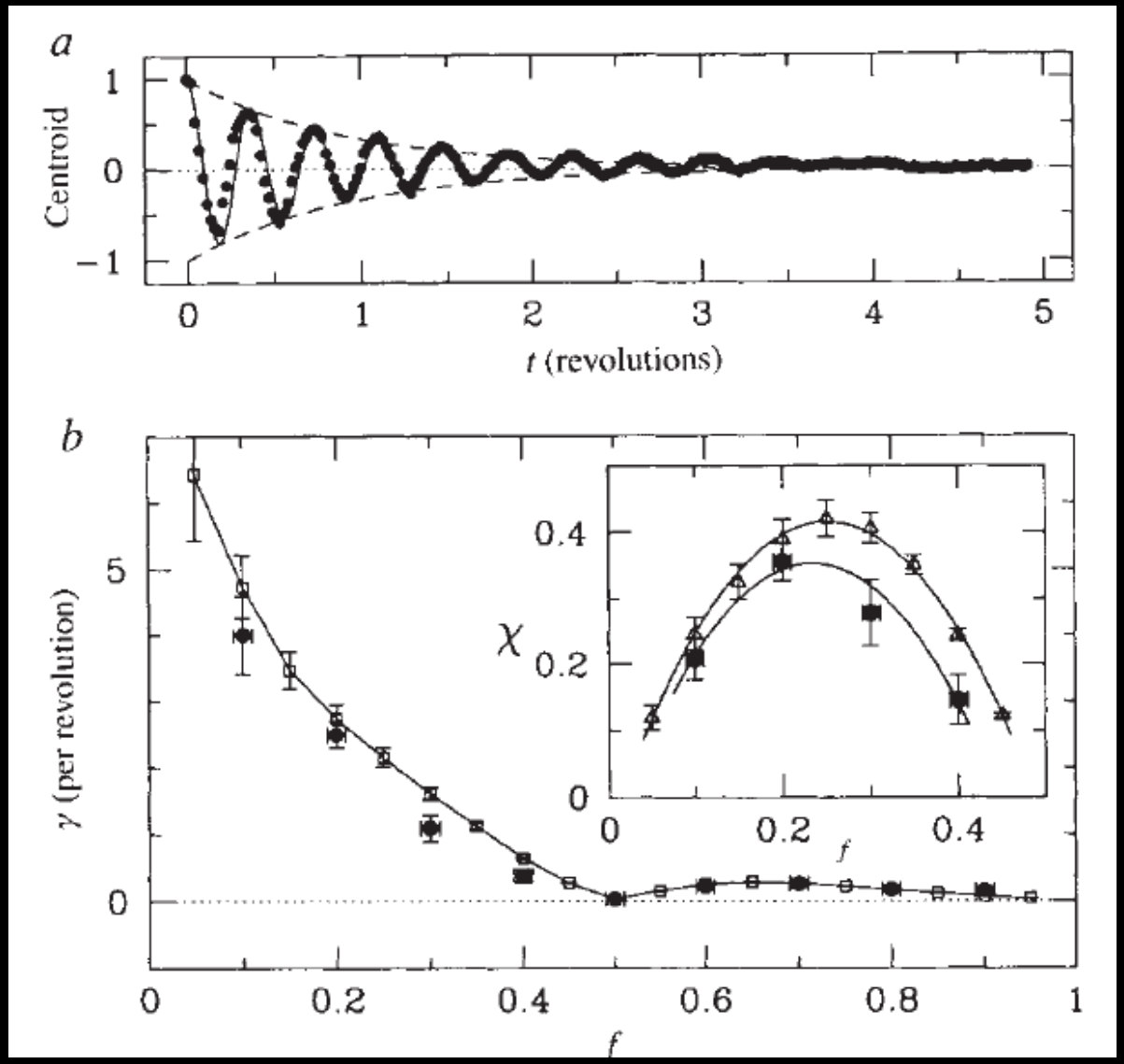
1. Maximum angle of repose to relaxed angle of repose

2. Transport wedge-to-wedge

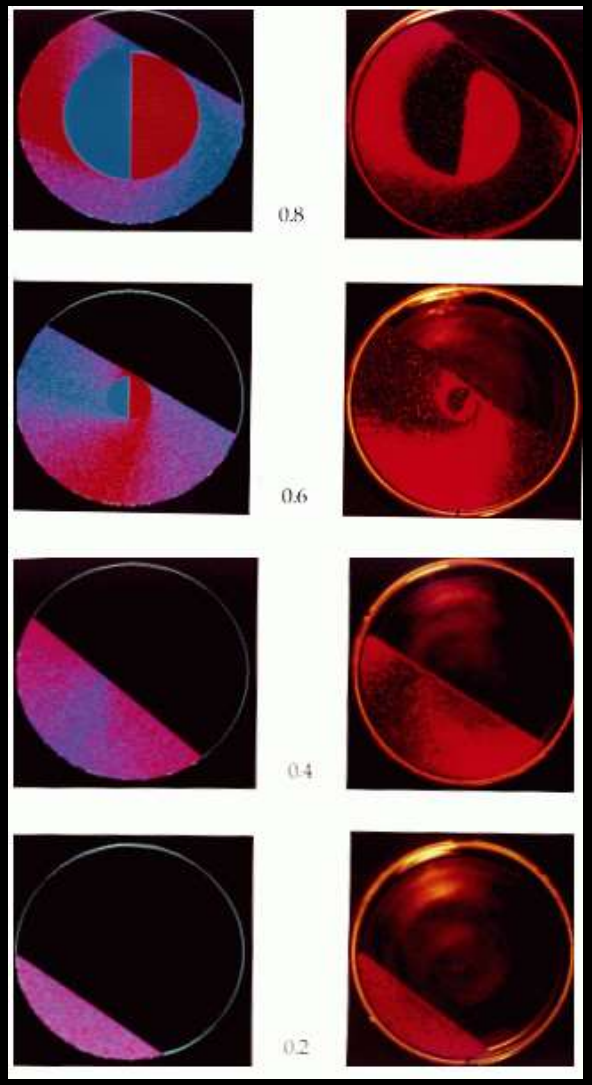
3. Random within wedges



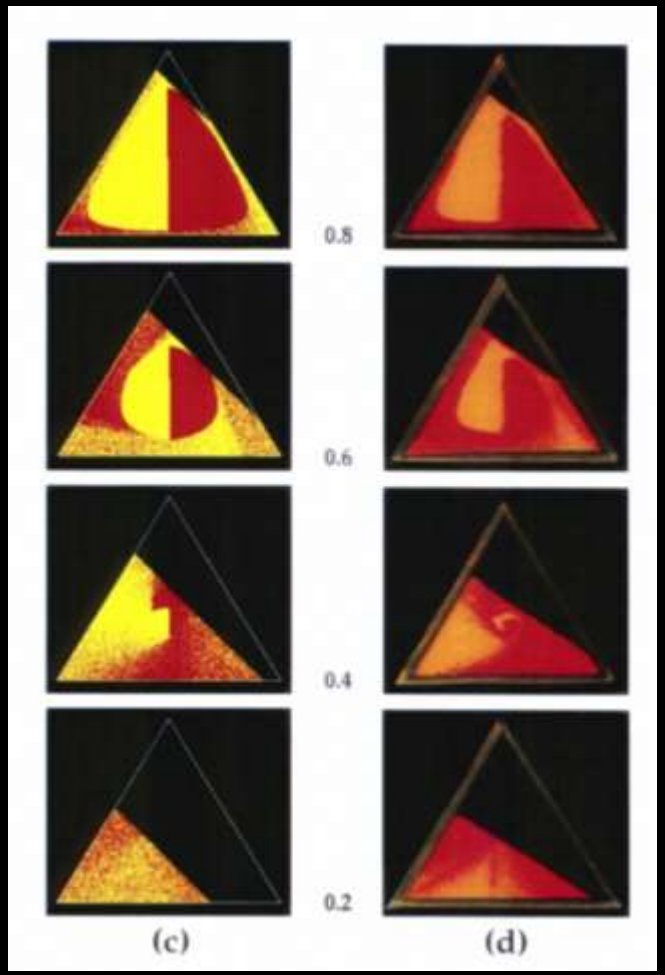
Initial Condition


a
Centroid
1
0
−1
0
1
2
3
4
5
t (revolutions)
b
γ (per revolution)
5
0
0
0.2
0.4
0.6
0.8
1
f
χ
0.4
0.2
0
0
0.2
0.4
f

Avalanche Mixing



Model Experiment



Model Experiment

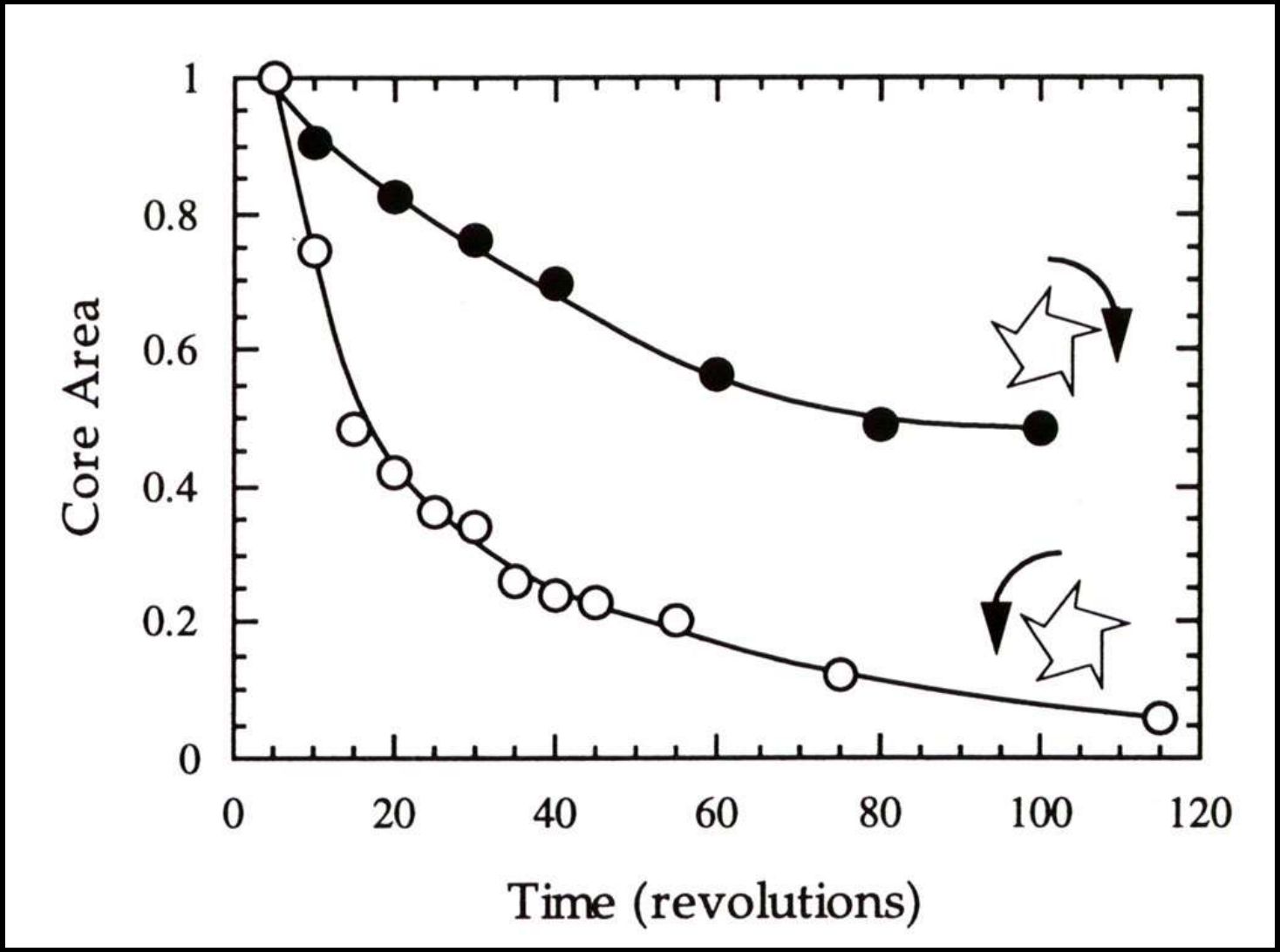
Model

Experiment

A Uniquely Australian Tumbler

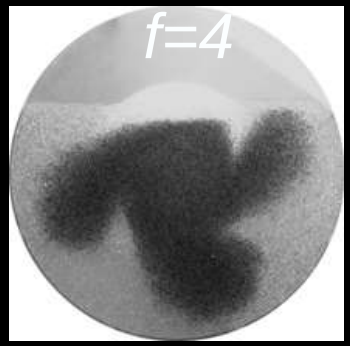
A Uniquely Australian Tumbler

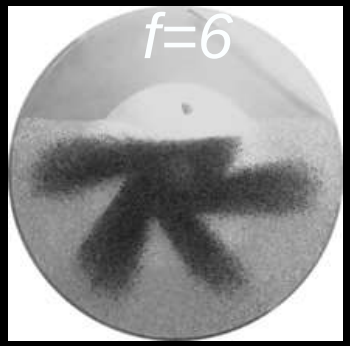
Cape Yorke Effect

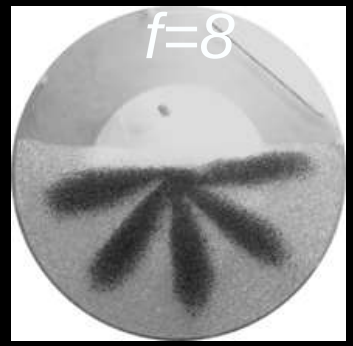

Core Area
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
120
Time (revolutions)

Mixing and Unmixing: *t*-periodic forcing

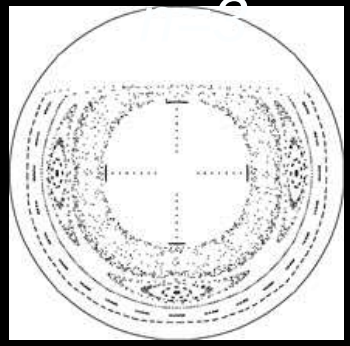
Dry

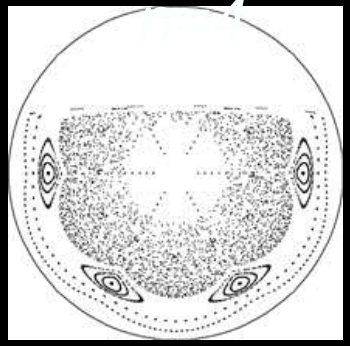


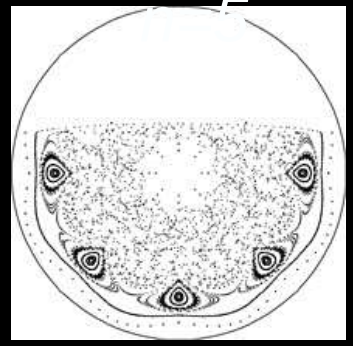




Liquid







Poincaré Sections

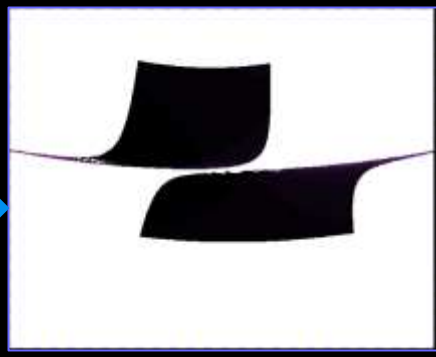
Fiedor & Ottino, *J. Fluid Mech*. (2005)

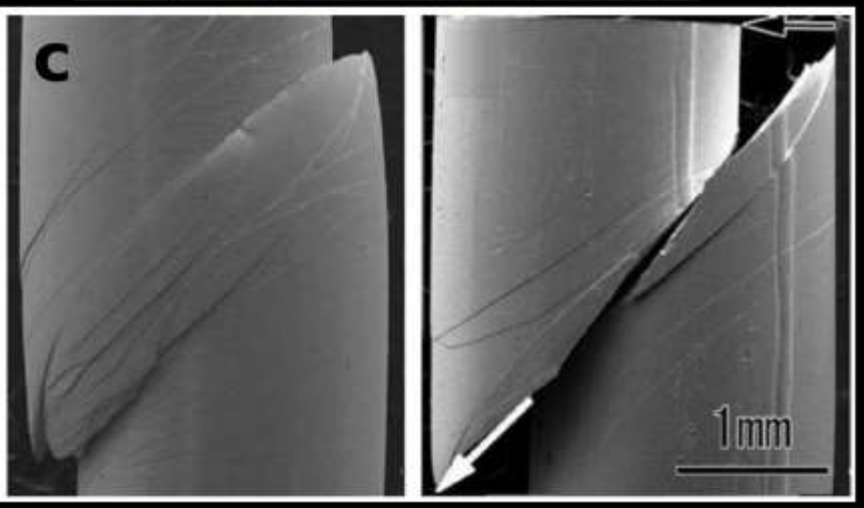
Discontinuous Deformations

Faults

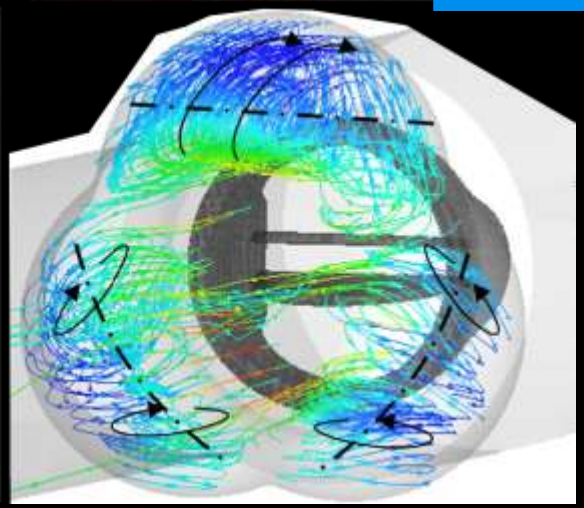
Granular

Lachlan Smith





Shear banding



Valved

MONASH University

Geophysical Mixing

Metcalfe et al. (2010a)
Phil Trans A 368, 217-230

Fig 1a from P[illegible] and Stuyfzand ES[illegible]

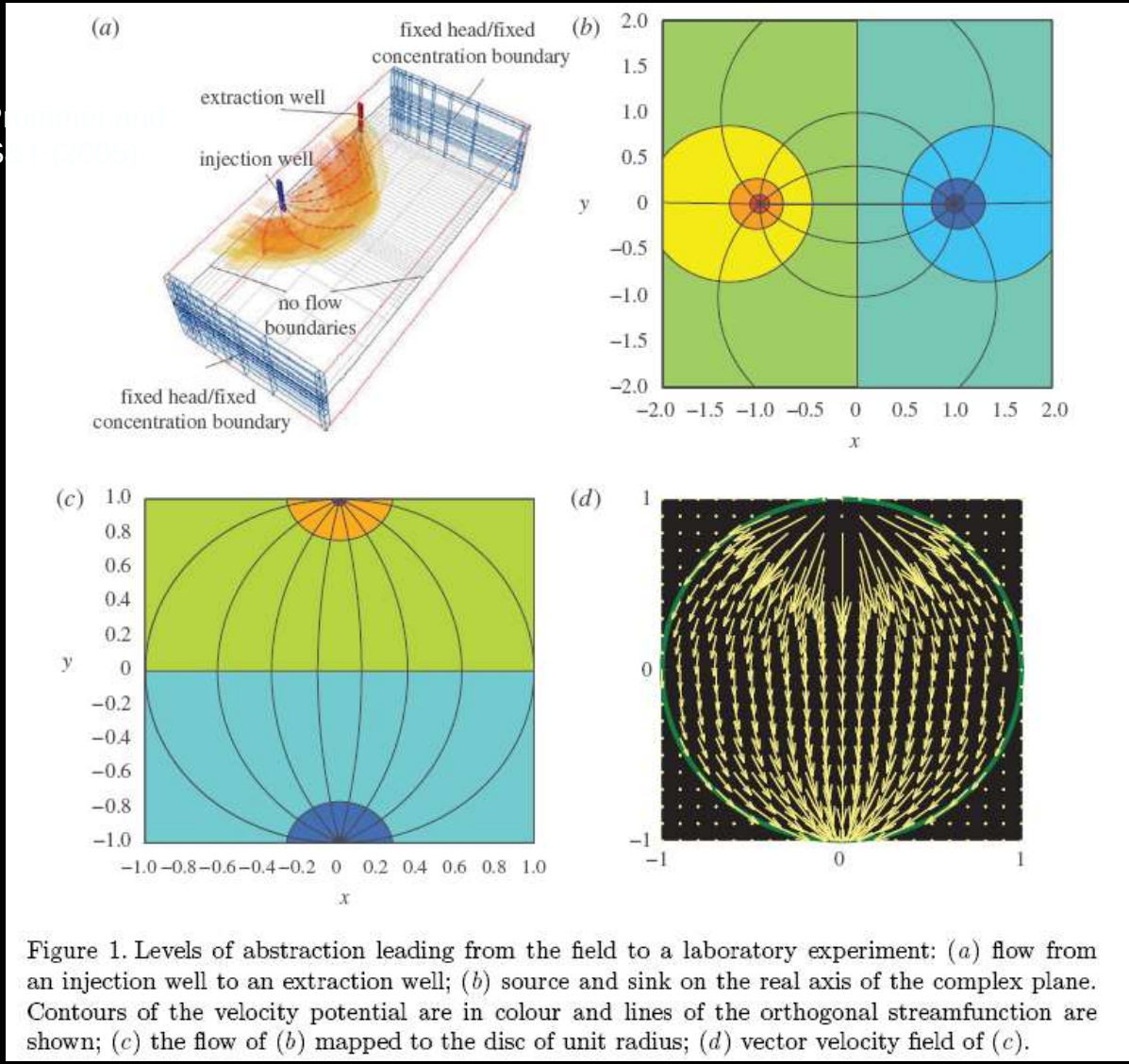
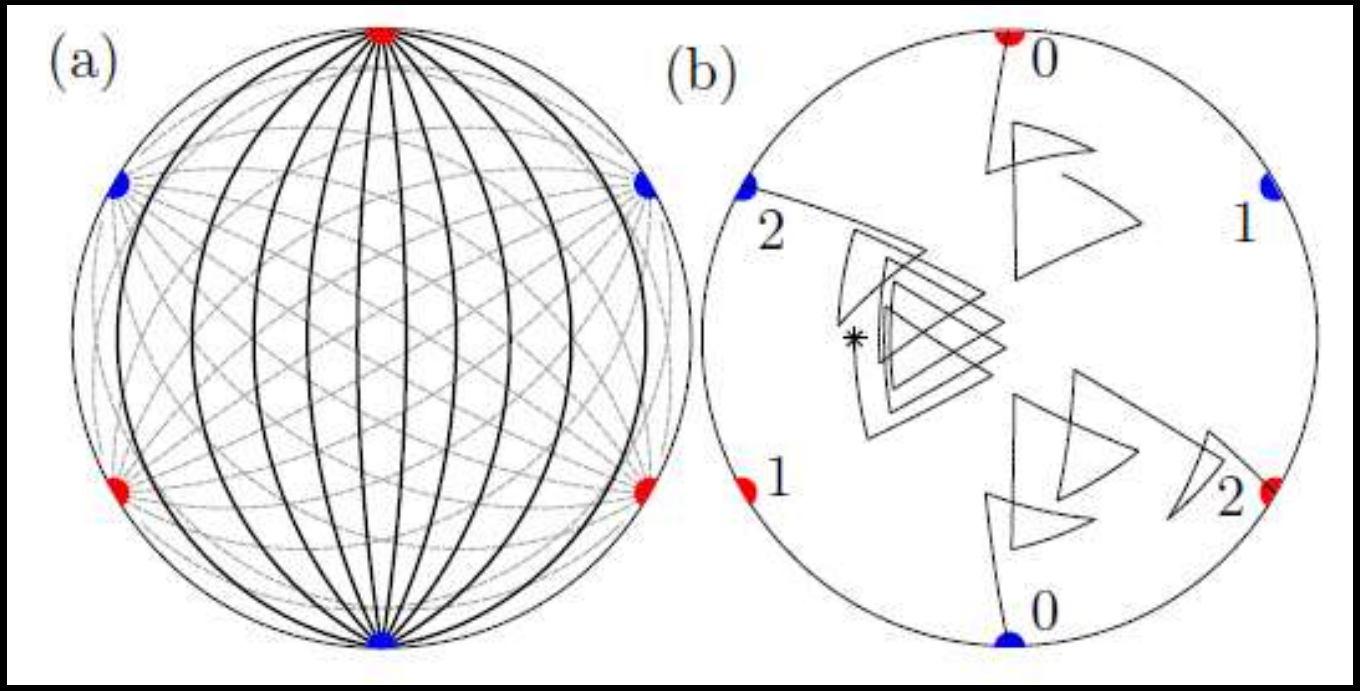
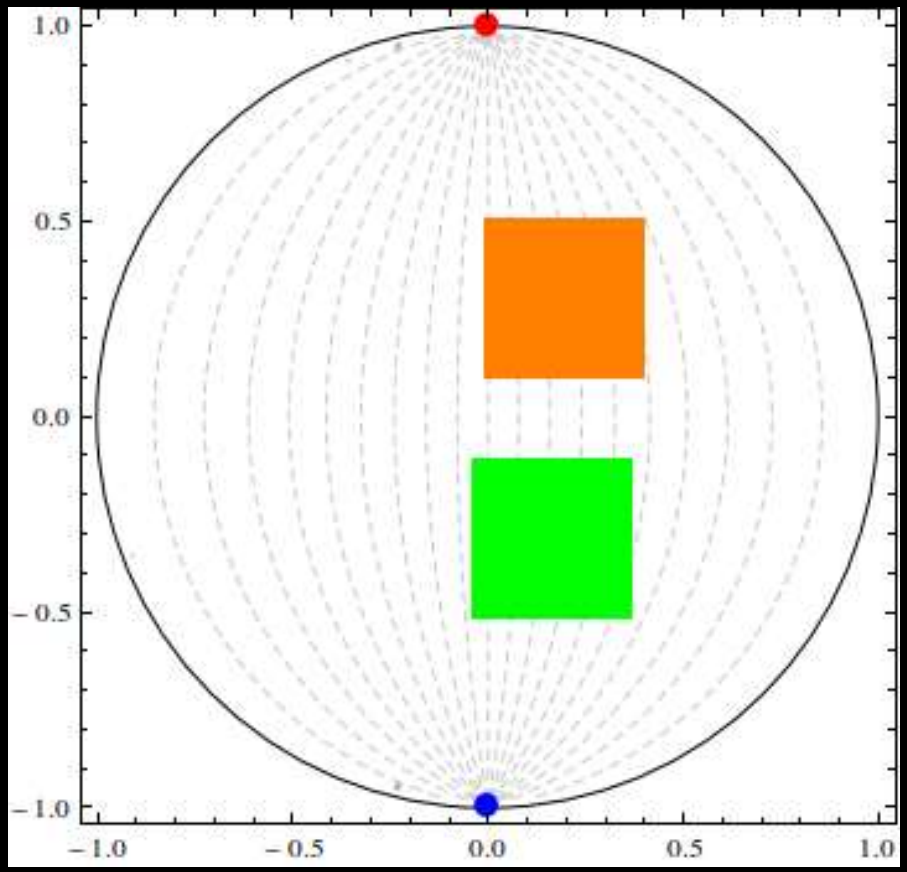


Figure 1. Levels of abstraction leading from the field to a laboratory experiment: (a) flow from an injection well to an extraction well; (b) source and sink on the real axis of the complex plane. Contours of the velocity potential are in colour and lines of the orthogonal streamfunction are shown; (c) the flow of (b) mapped to the disc of unit radius; (d) vector velocity field of (c).


(a)
(b)
0
1
2
1
2
0
*

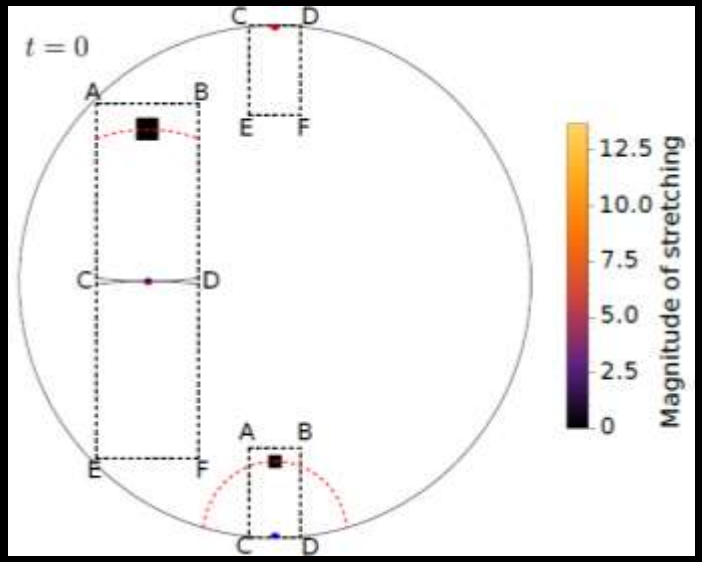
The 2D RPM Flow

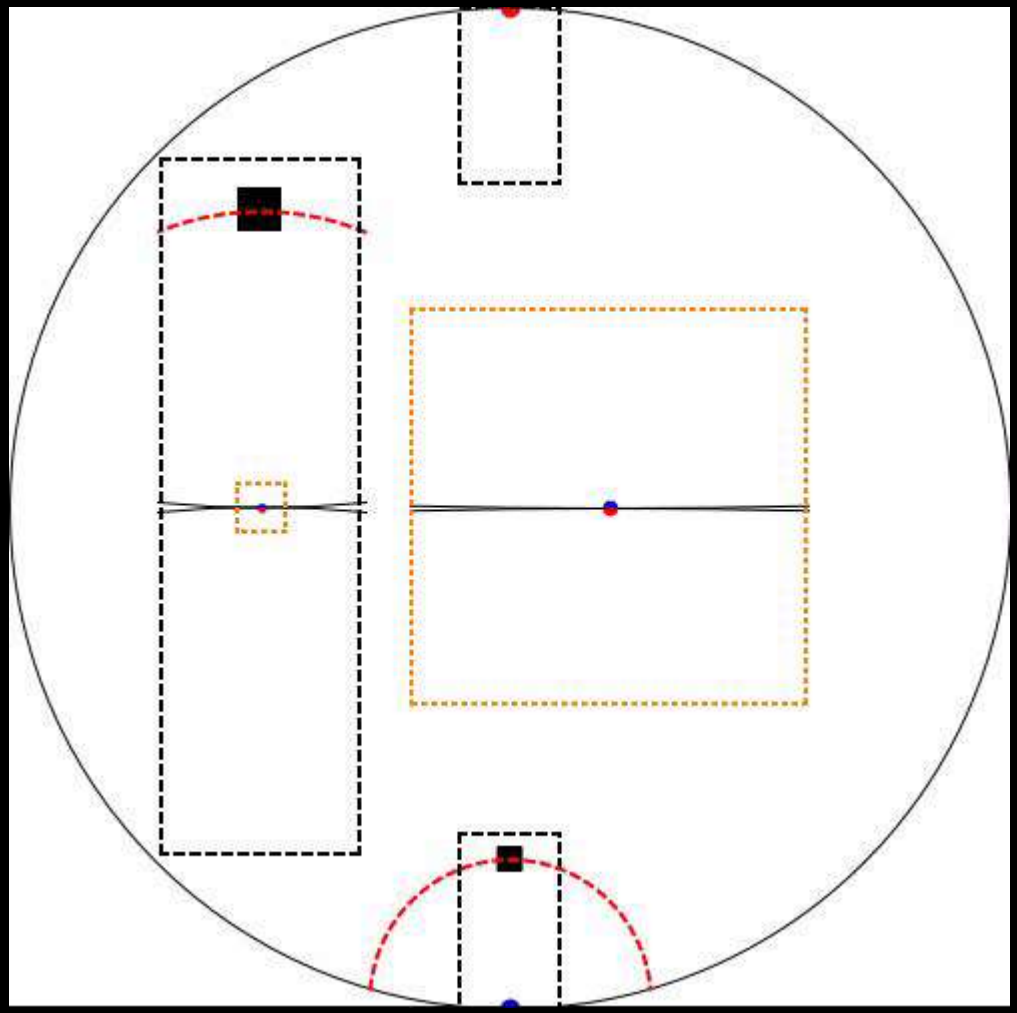


τ = switching period

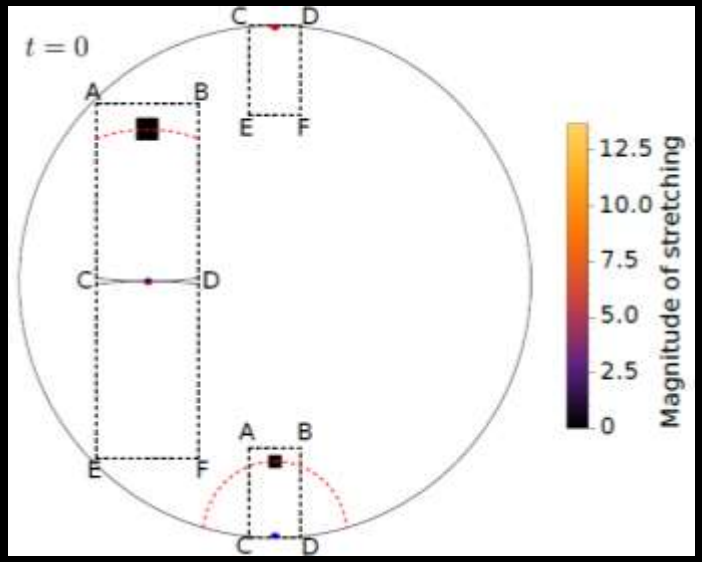
Θ = rotation angle

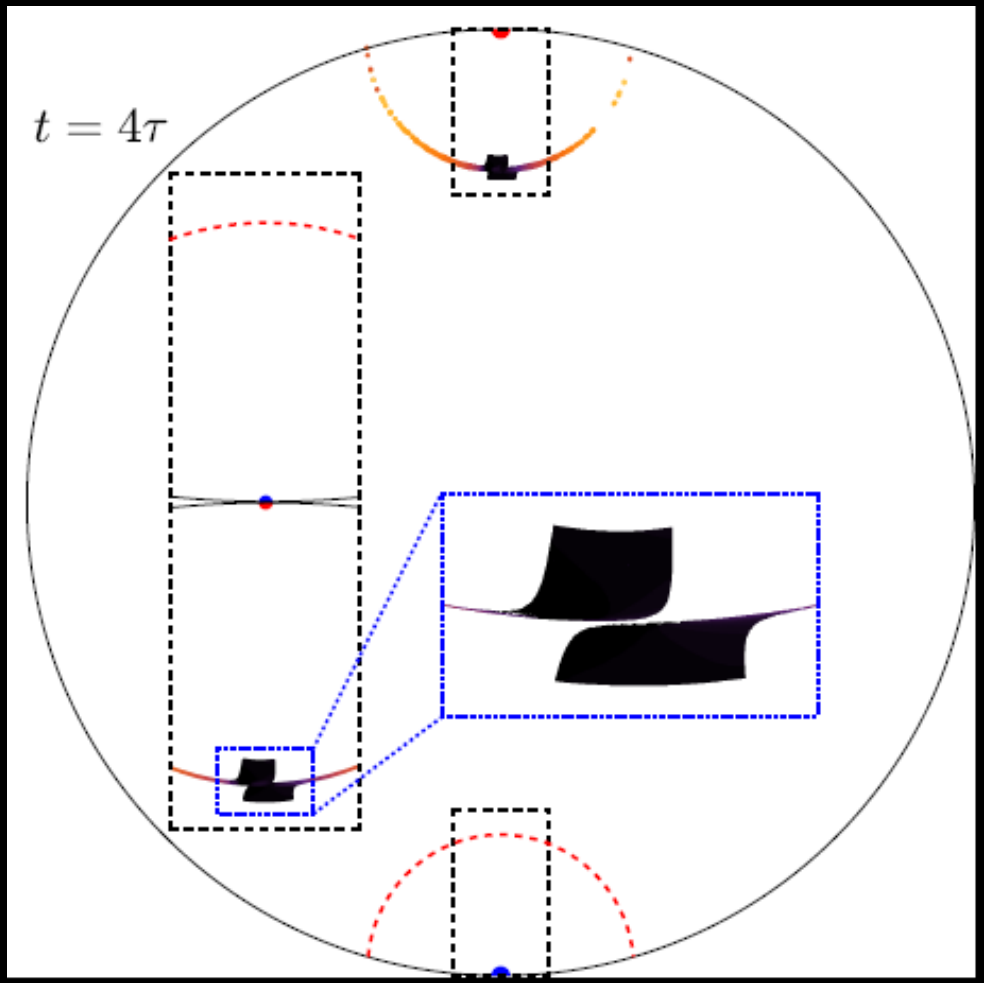
Fluid cutting produced by the dipole



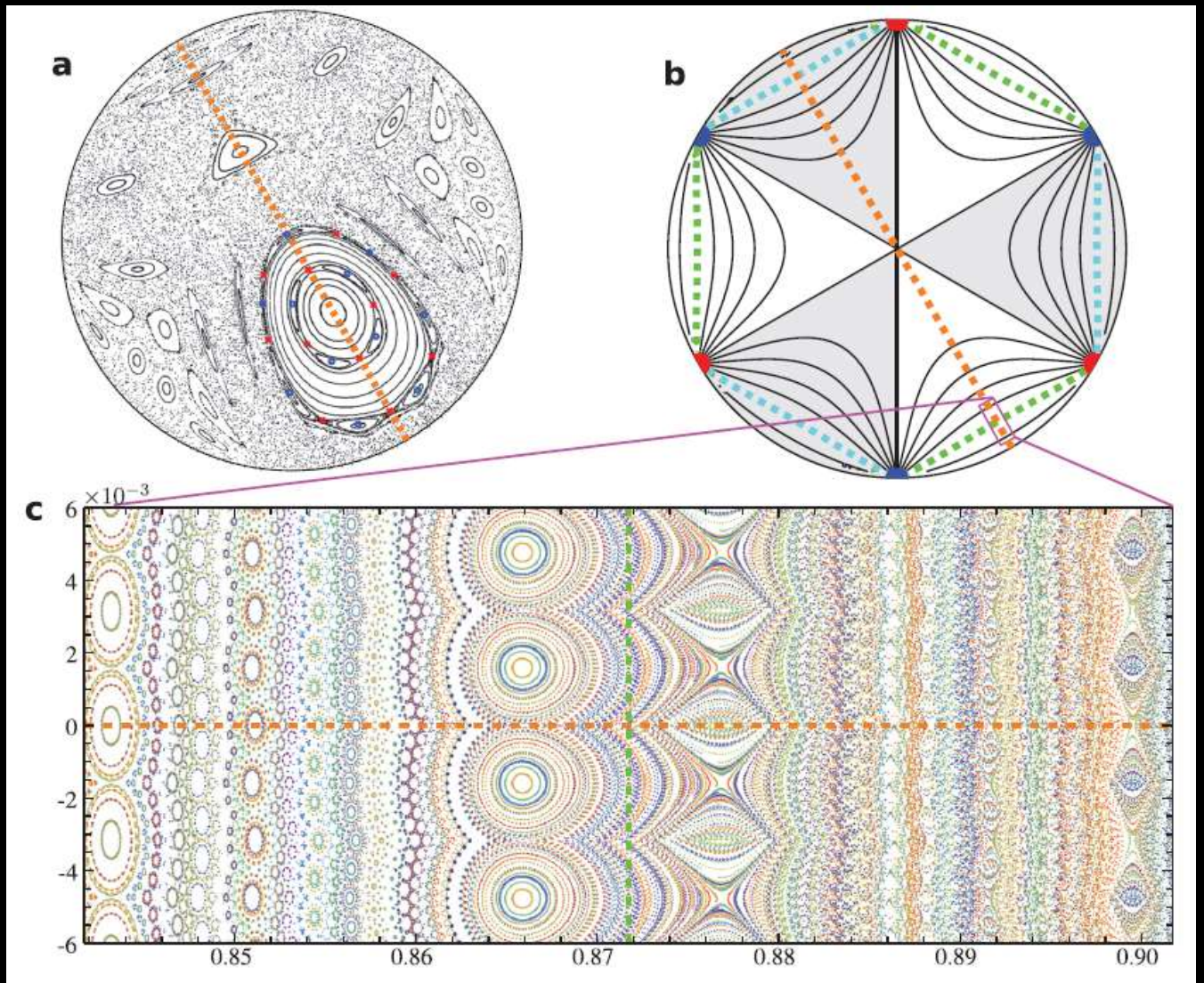


Fluid cutting produced by the dipole

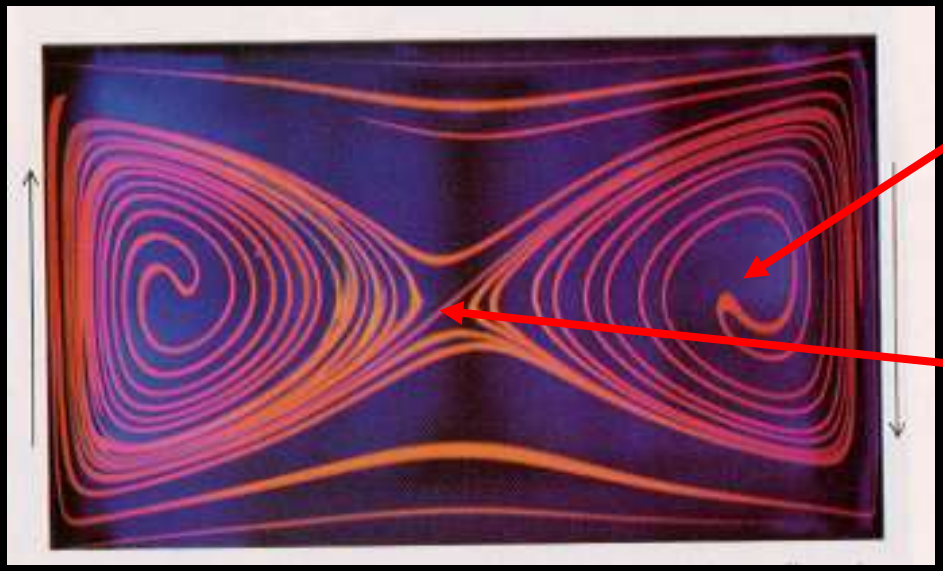


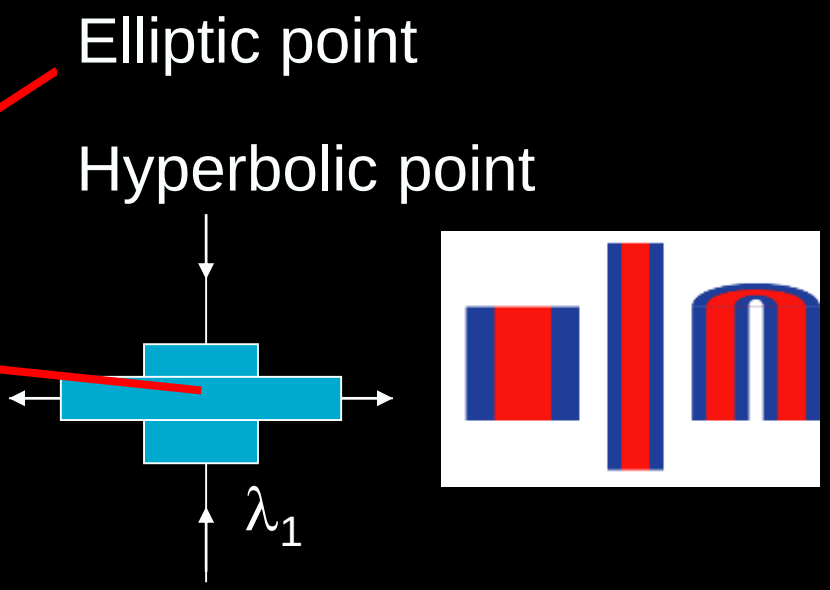


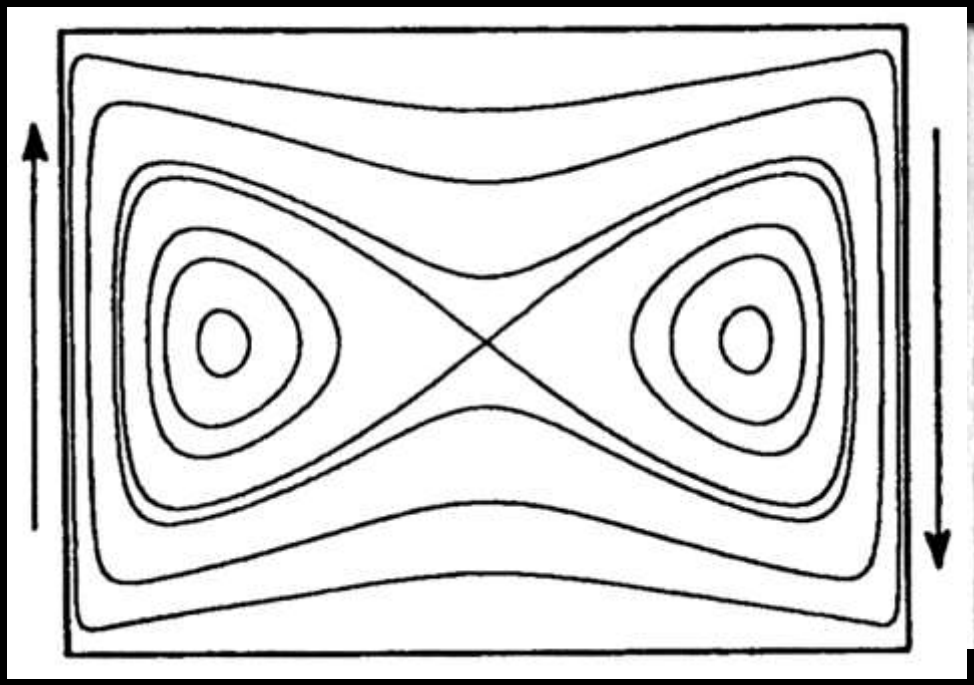
Non-Hamiltonian structures



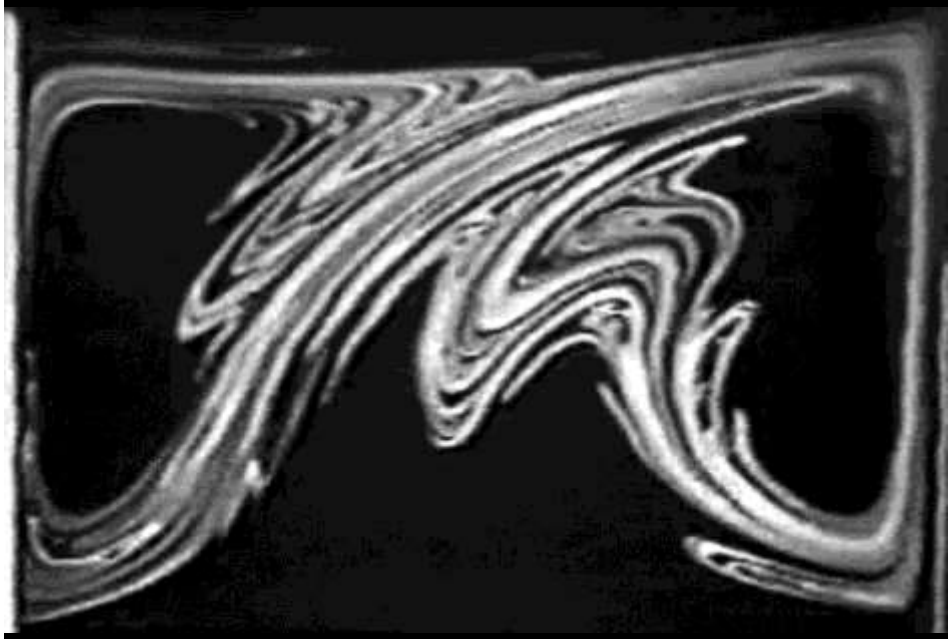
Dynamical Systems Theory: invariant sets, manifolds





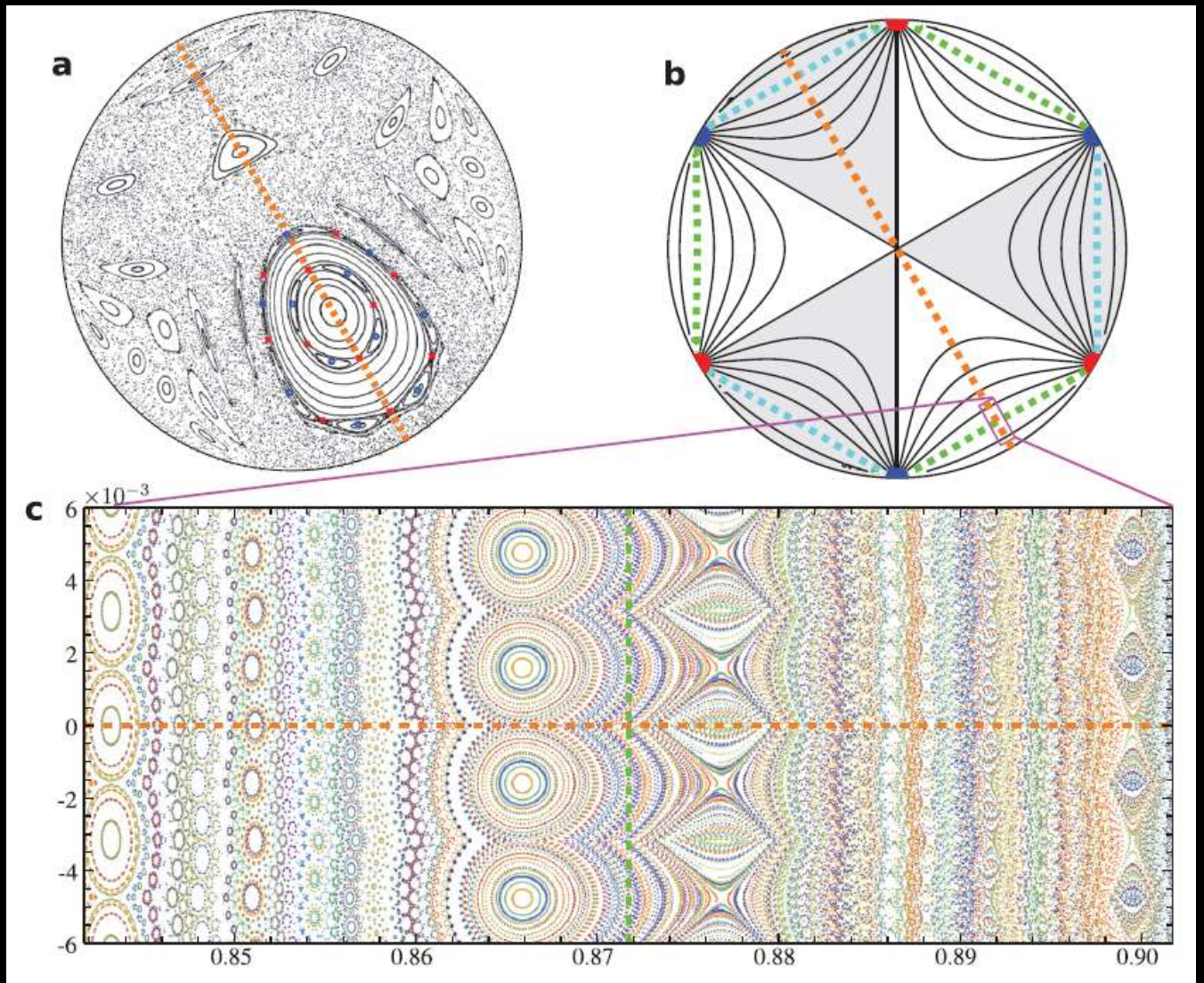


Simple flow



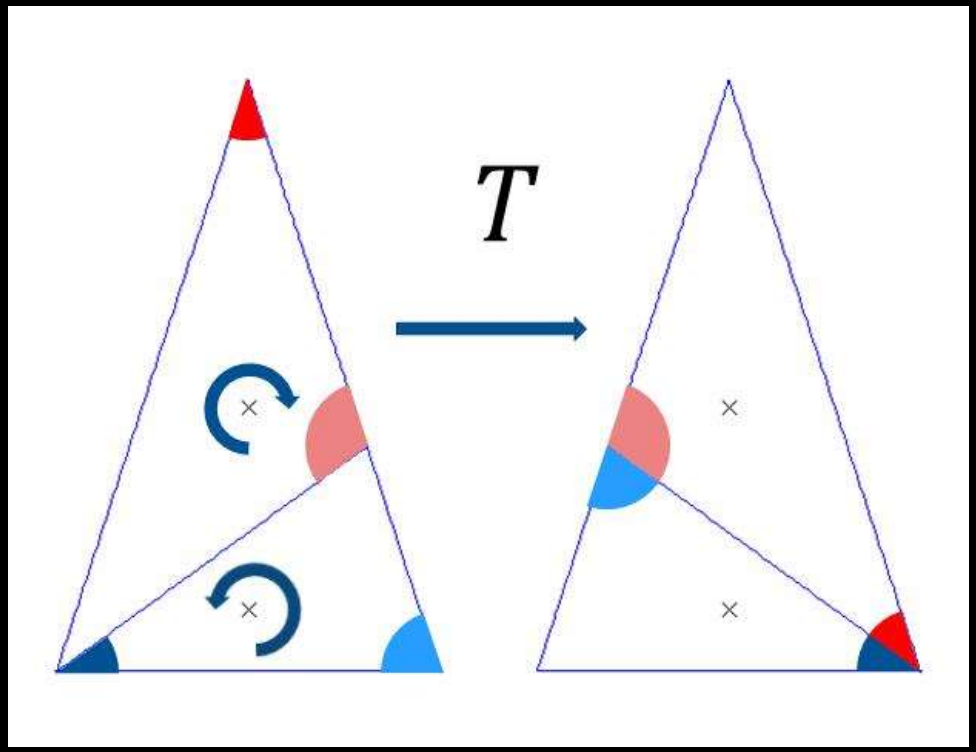
Complex trajectories

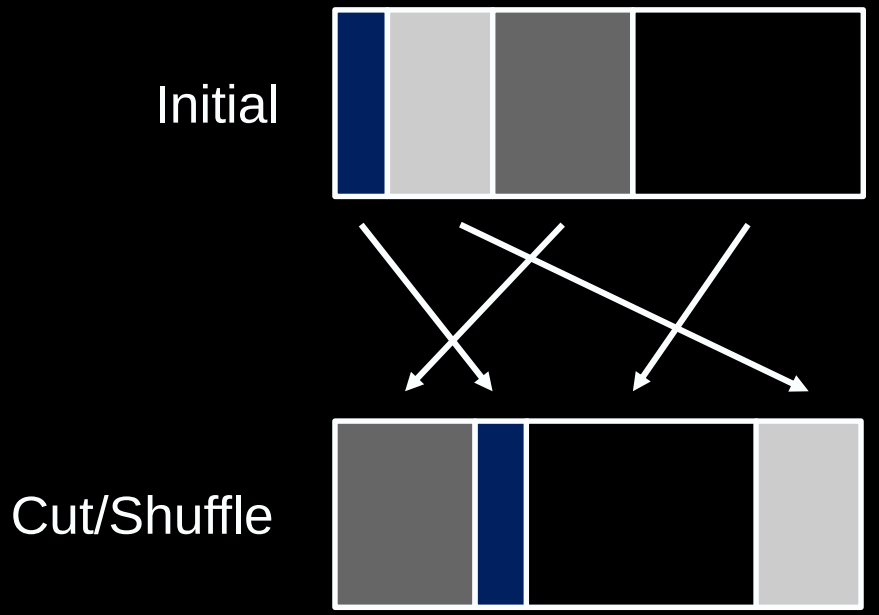
Non-Hamiltonian structures



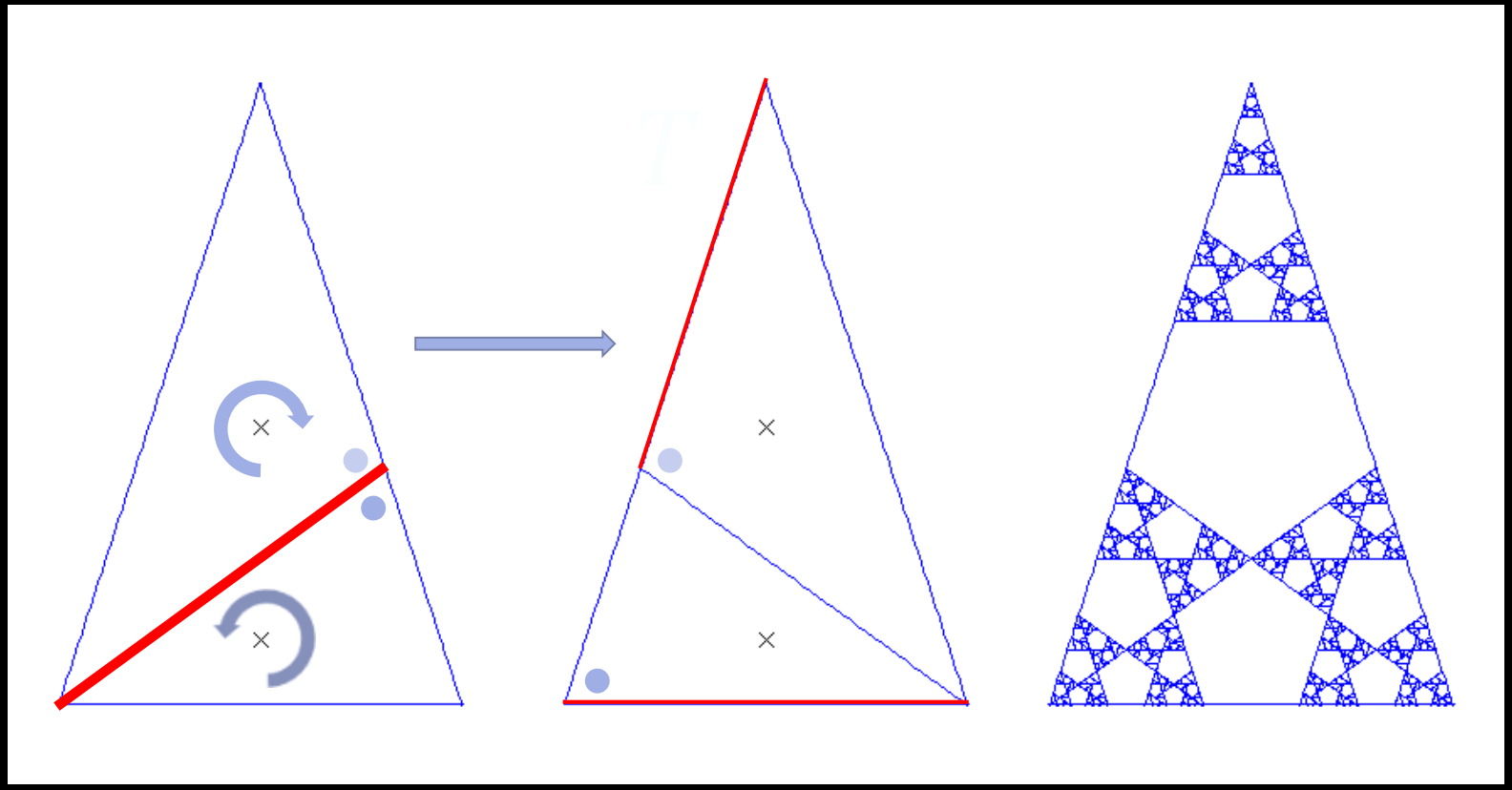
Piecewise Isometry

break object into finite pieces, rearrange into original shape

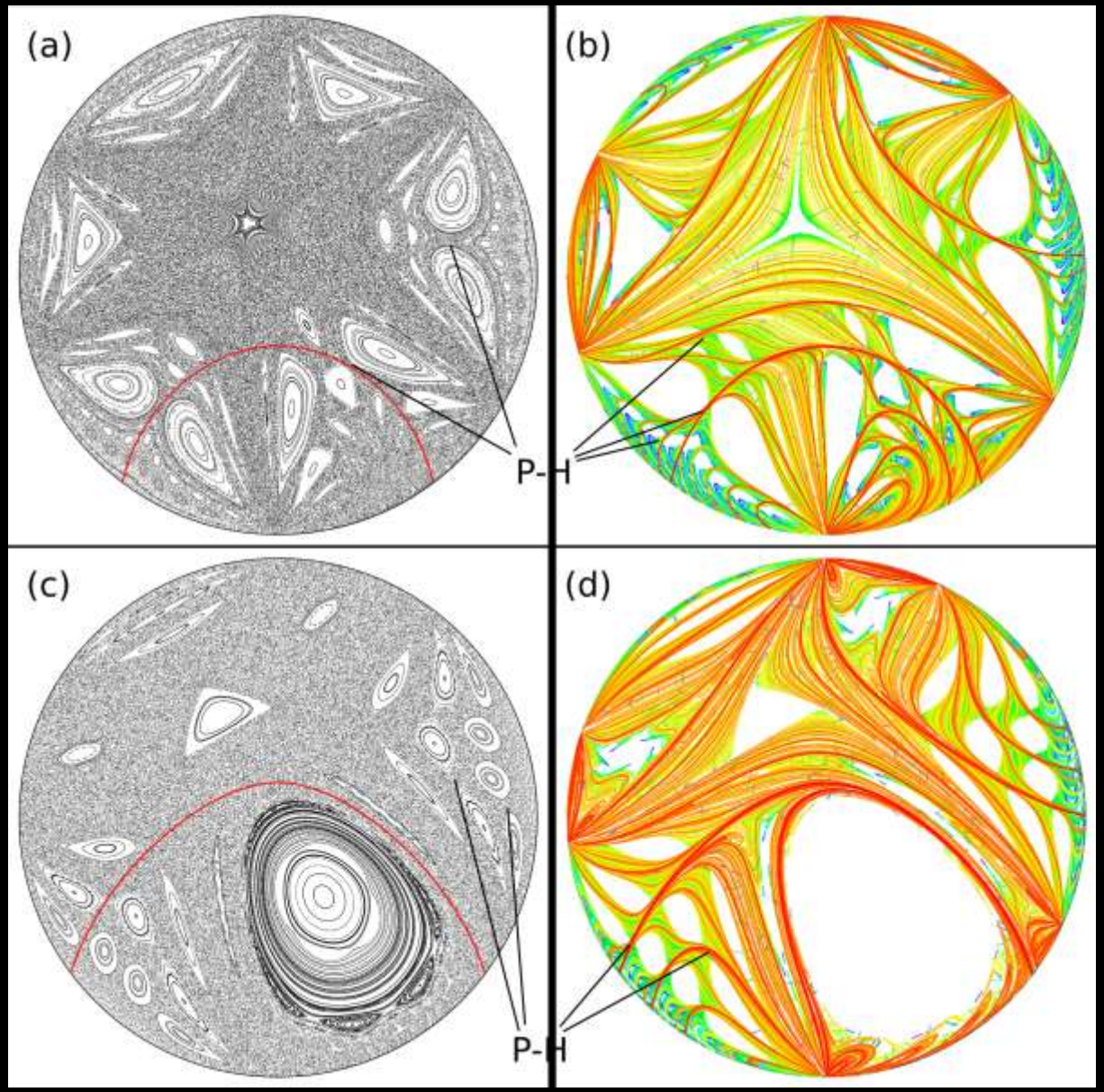




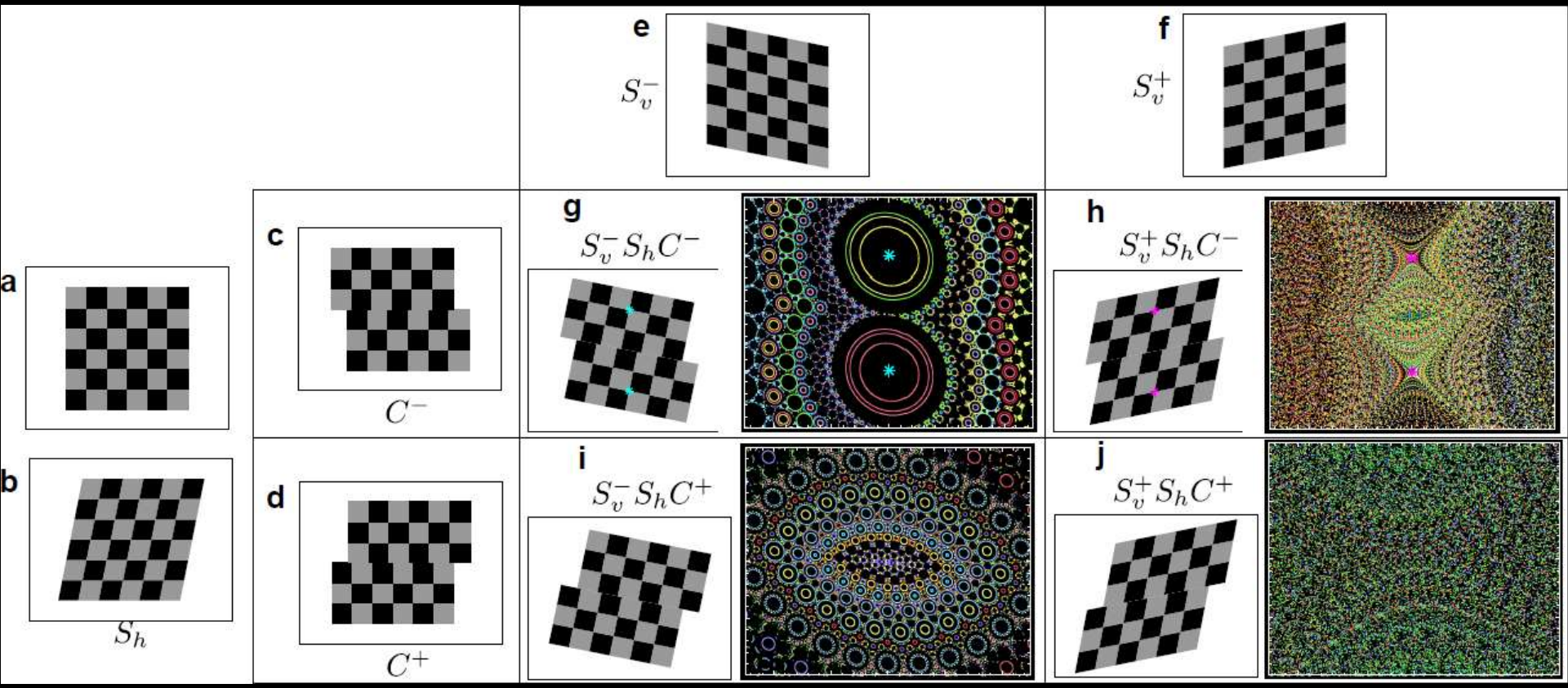
PWI and Exceptional Set (E)



The exceptional set of all images of the cut; physically, it keeps track of where the material has been split


(a)
(b)
P-H
(c)
(d)
P-H

The Cut-Shear-Shear Map



Key Results

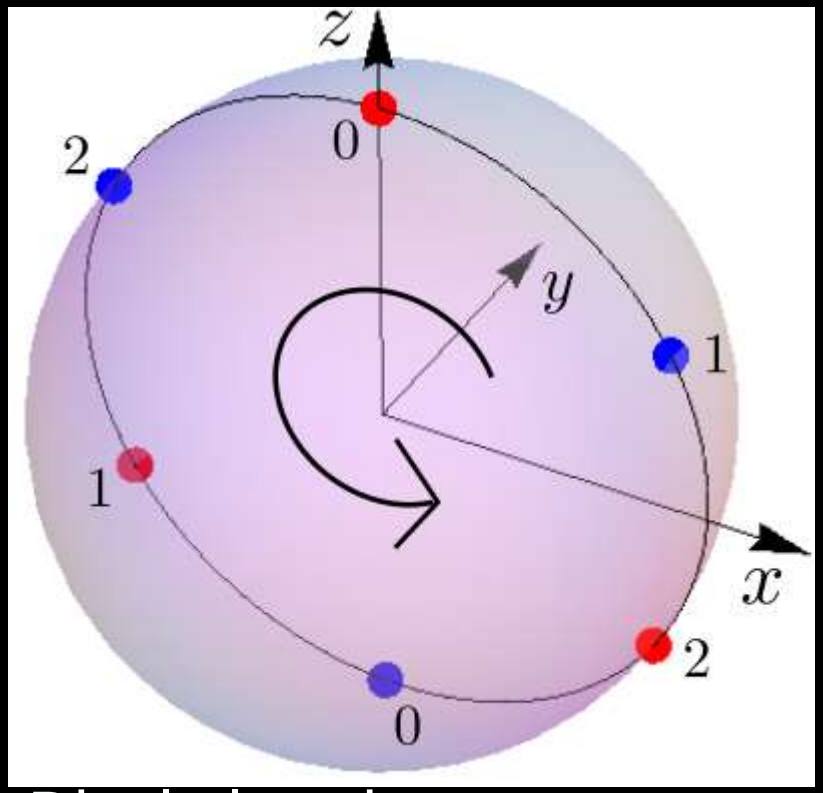
Non-Hamiltonian particle transport even though the base flow is Hamiltonian

The “webs of images and preimages” form a kinematic template, similar to that of periodic points, but also includes transport structures created by discontinuous deformations.

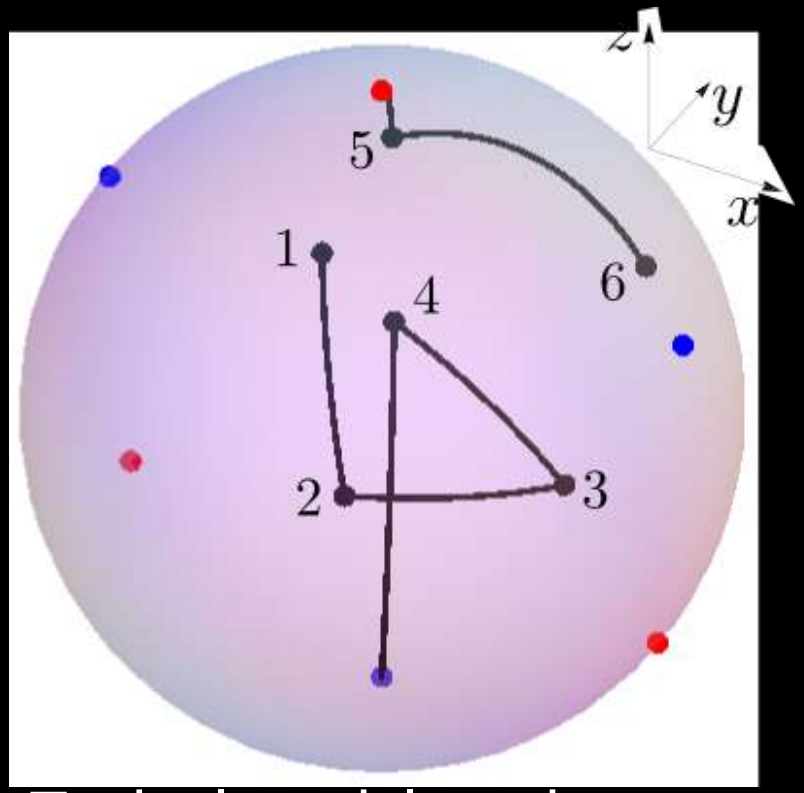
Cutting replaces folding

Discontinuous deformations can either enhance or impede mixing

The 3D RPM Flow

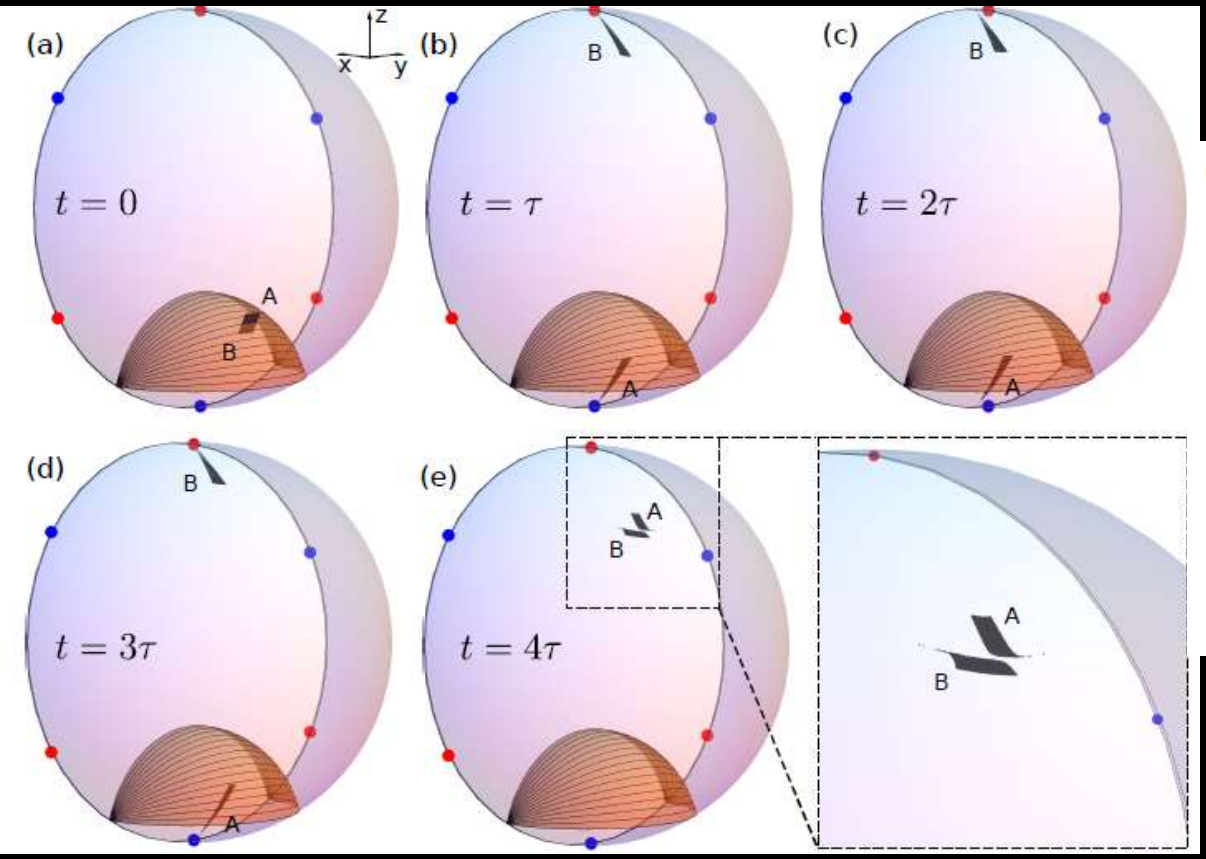


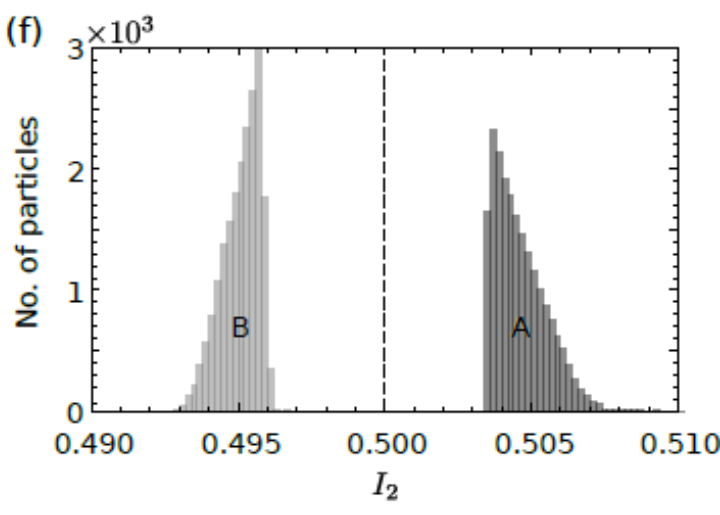
Dipole locations

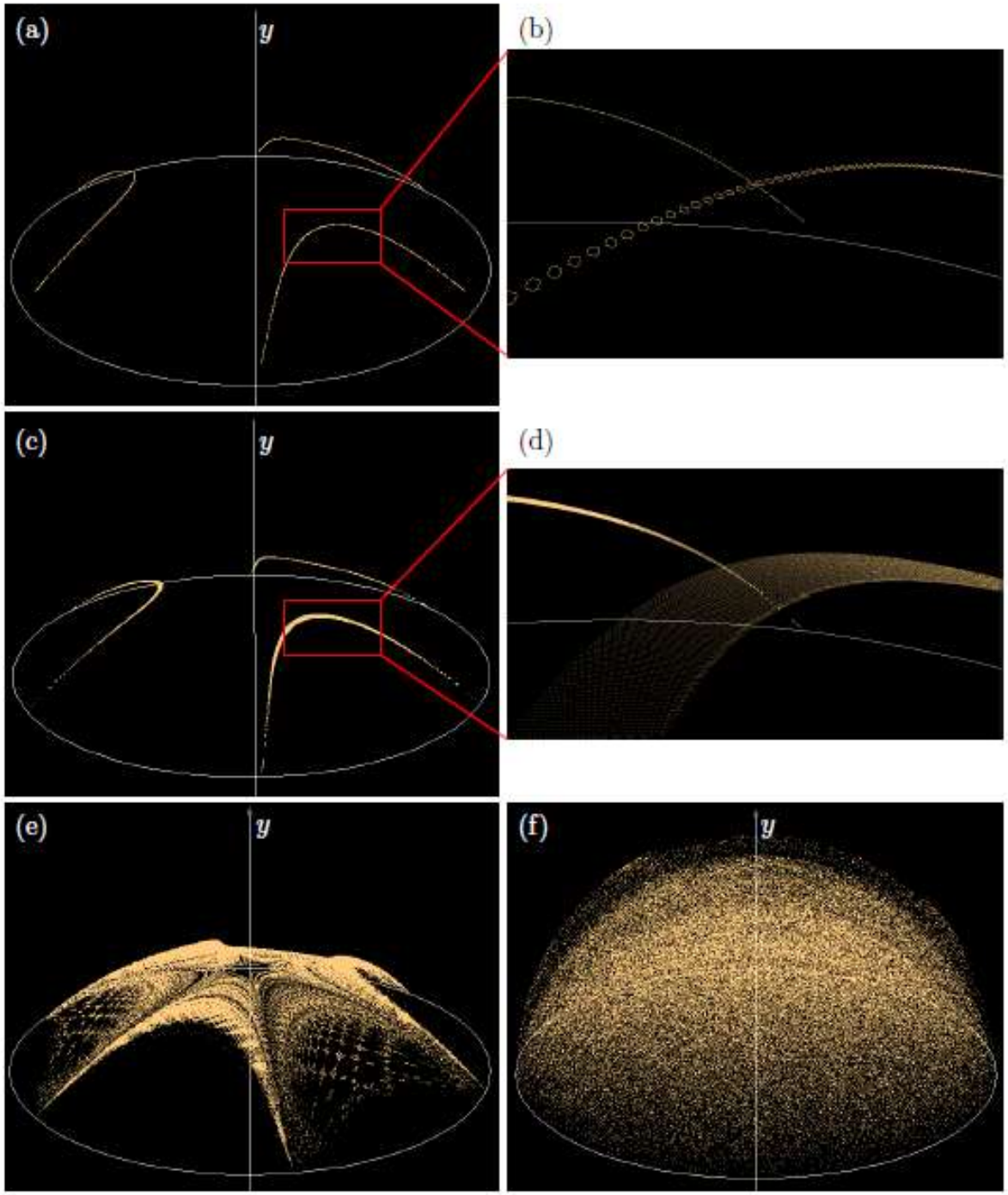


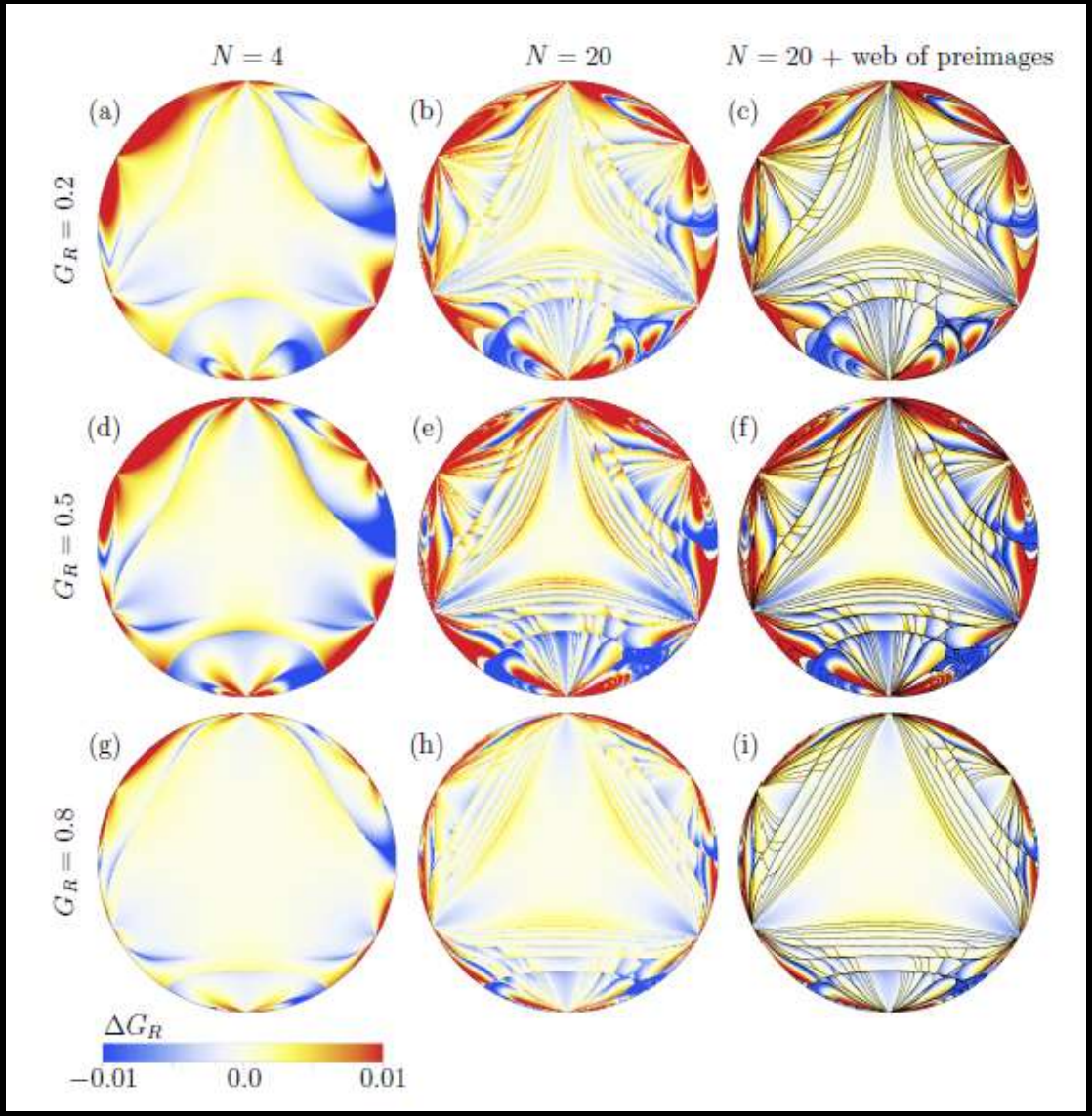
Typical particle trajectory

What can generate in the 3DRPM?





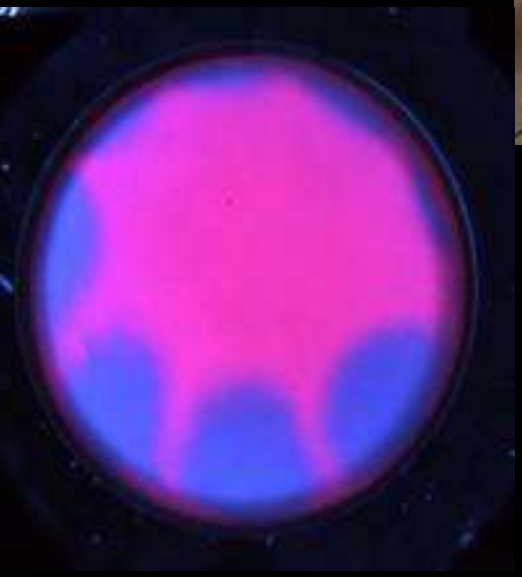

(a)
y
(b)
(c)
y
(d)
(e)
y
(f)
y


$N = 4$
$N = 20$
$N = 20$ + web of preimages
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
$G_R = 0.2$
$G_R = 0.5$
$G_R = 0.8$
ΔG_R
-0.01
0.0
0.01

Messages

- Discontinuities effect deformation
- Experiments
- Simple models capture much
- Wedges, Cut-Line Preimages

Experiments



Messages

- Discontinuities effect deformation
- Experiments
- Simple models capture much
- Wedges, Cut-Line Preimages

Acknowledgements and Collaborators

Julio Ottino, Northwestern University
Lachlan Smith, Northwestern University
Murray Rudman, Monash University
Dan Lester, RMIT
Kurt Liffman, Swinburne University
Joe McCarthy, University of Pittsburgh
Troy Shinbrot, Rutgers University
Bob Behringer, Duke University
Mike Trefry, CSIRO
Paul Cleary, CSIRO
Neil Thomson, University of Ottowa
Michelle Cho, University of Ottowa

Thank You