The low-temperature spin-glass phase

Sample-dependent parallel tempering on the Janus computer

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http://teorica.fis.ucm.es/grupos/grupo-TEC.html

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[1] R. Alvarez Baños, A. Cruz, L.A. Fernandez, J.M. Gil-Narvion, A. Gordillo-Guerrero, M. Guidetti, A. Maiorano, F. Mantovani, E. Marinari, V. Martin-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, J.J. Ruiz-Lorenzo, S.F. Schifano, B. Seoane, A. Tarancon, R. Tripiccione and D. Yllanes

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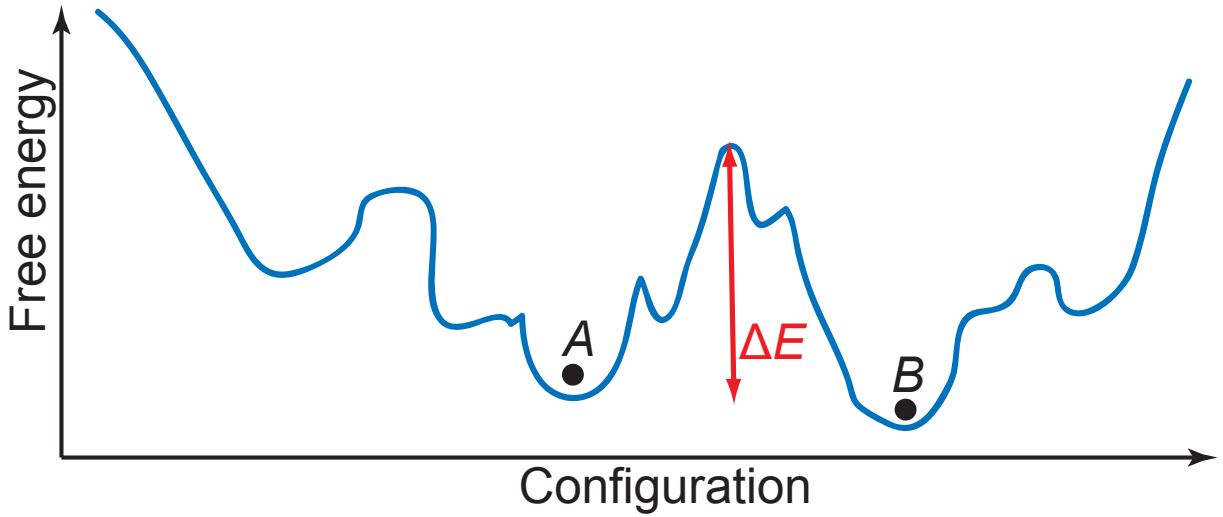
Introduction



Simulating disordered systems

- Rugged free-energy landscapes

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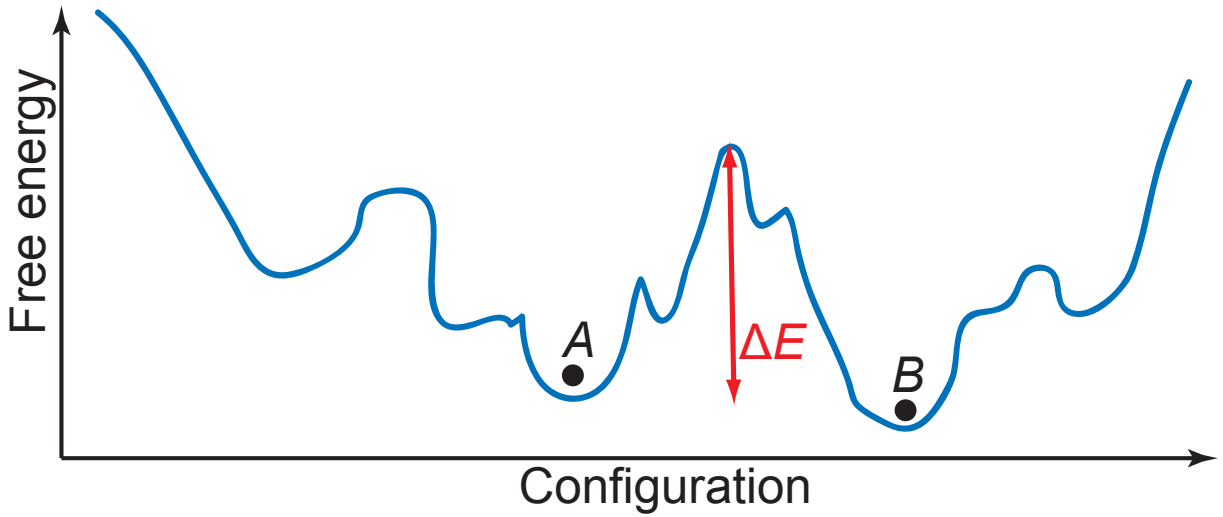

Simulating disordered systems

- Rugged free-energy landscapes
  - Many valleys, separated by large energy barriers.
  - The dynamics at low T is exceedingly slow.

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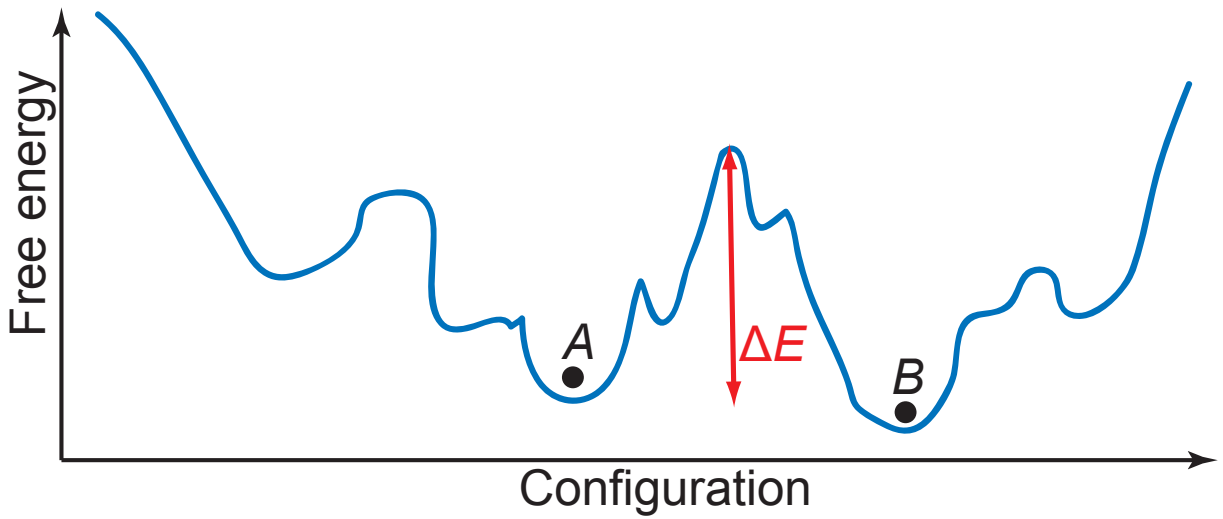

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  $\Longrightarrow$ long simulations to thermalise.

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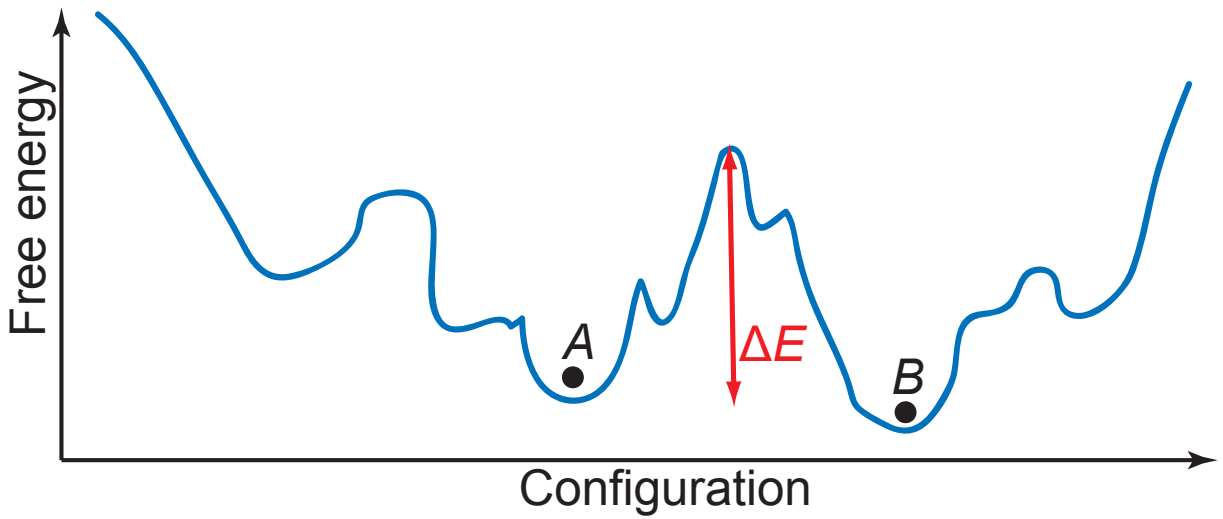

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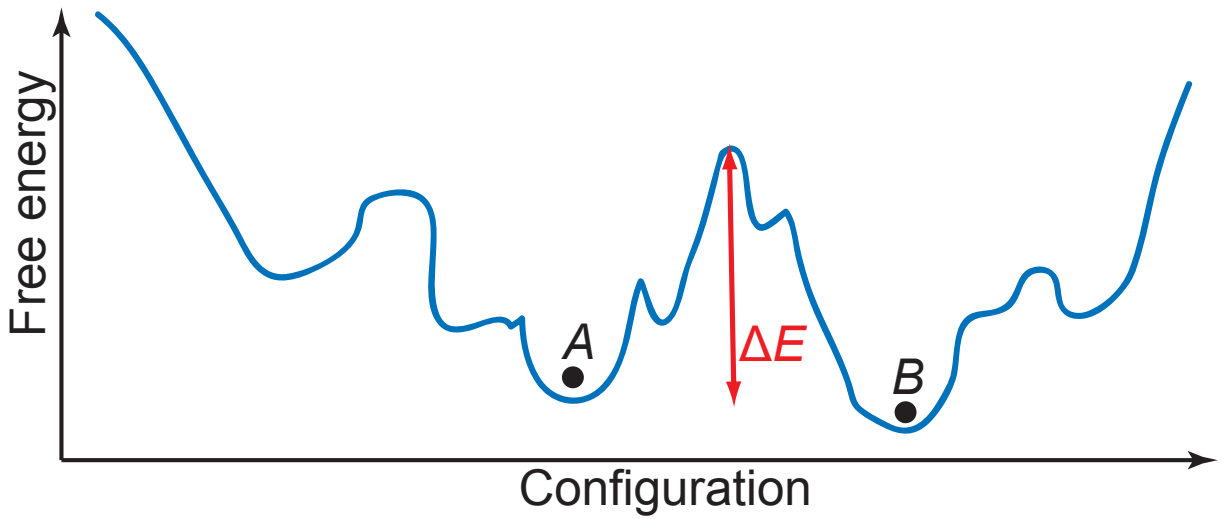

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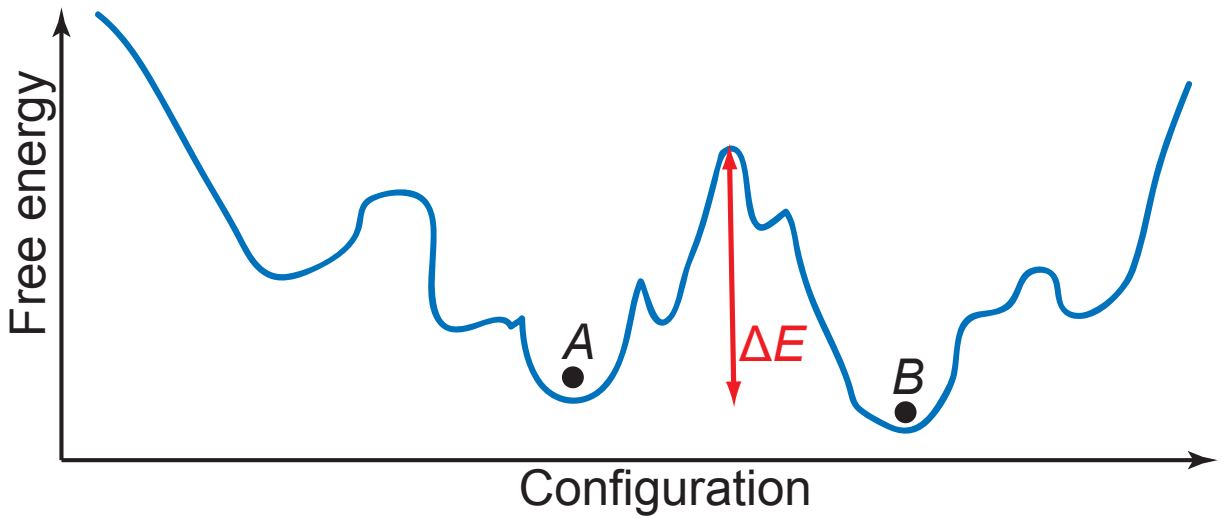

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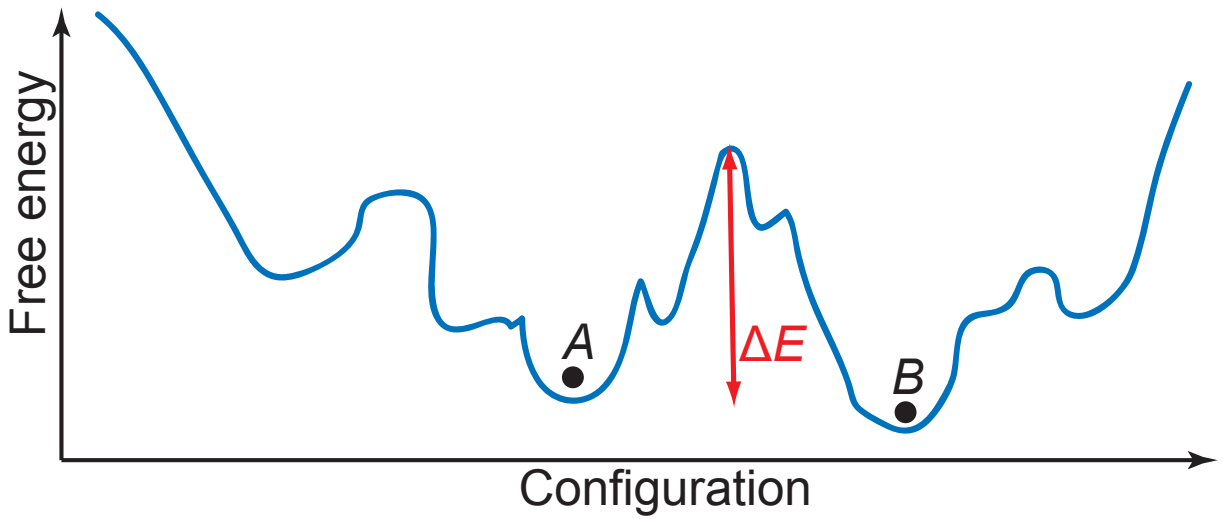

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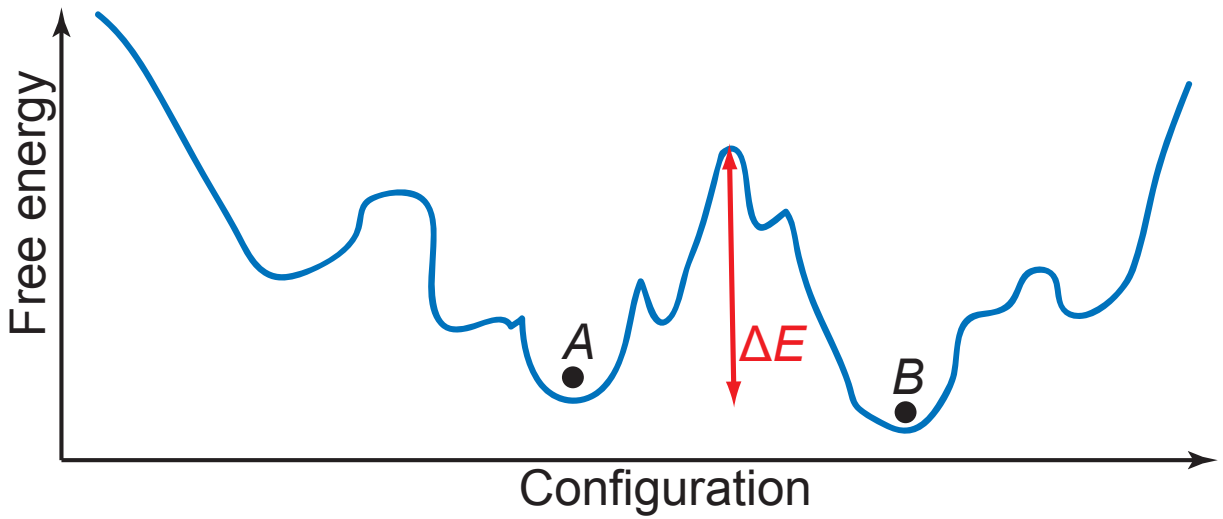

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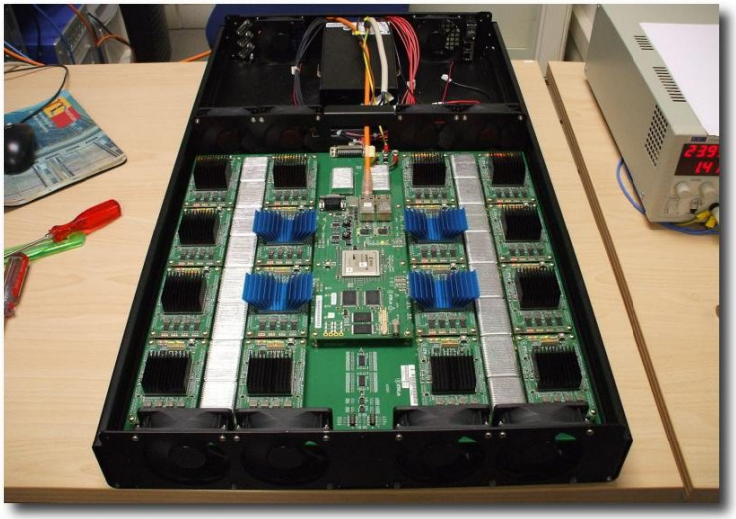
  - A reliable method to choose the simulation length.

The Janus computer



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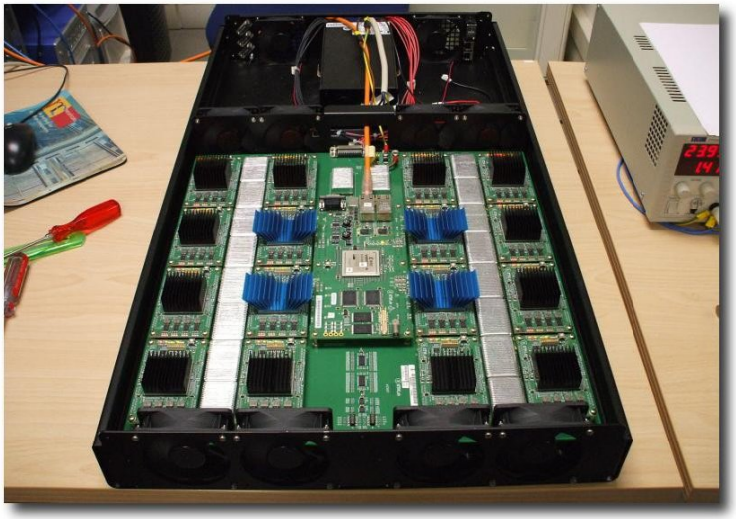

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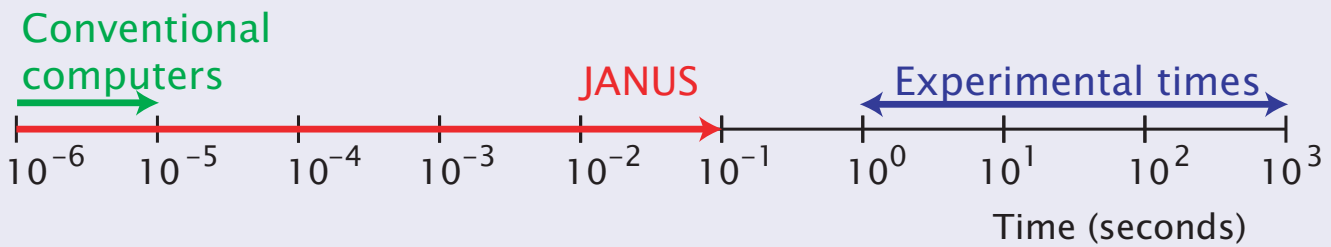
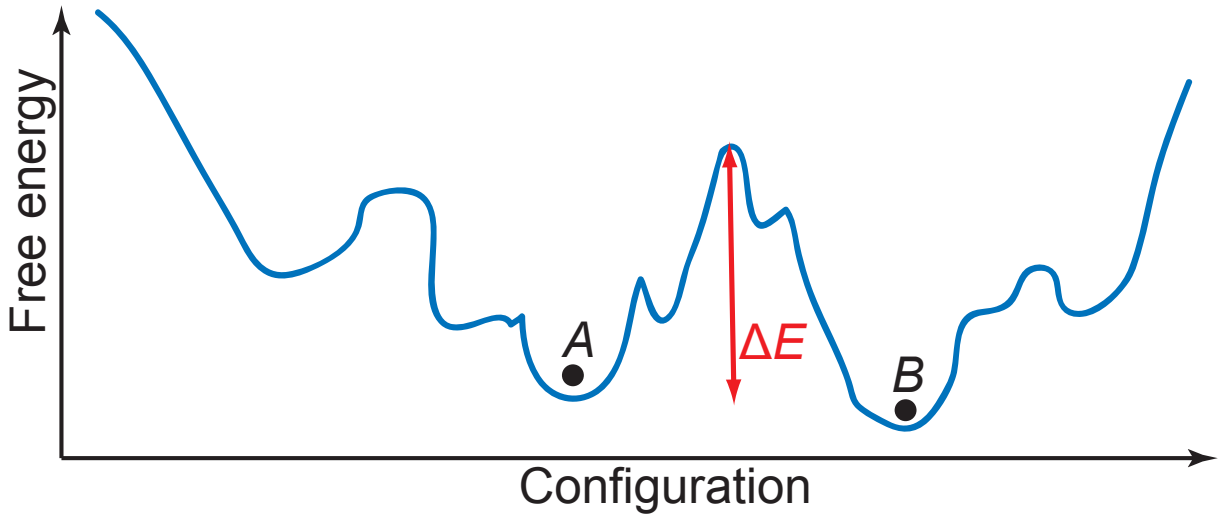
- We outperform conventional PCs by several orders of magnitude.



The parallel tempering algorithm



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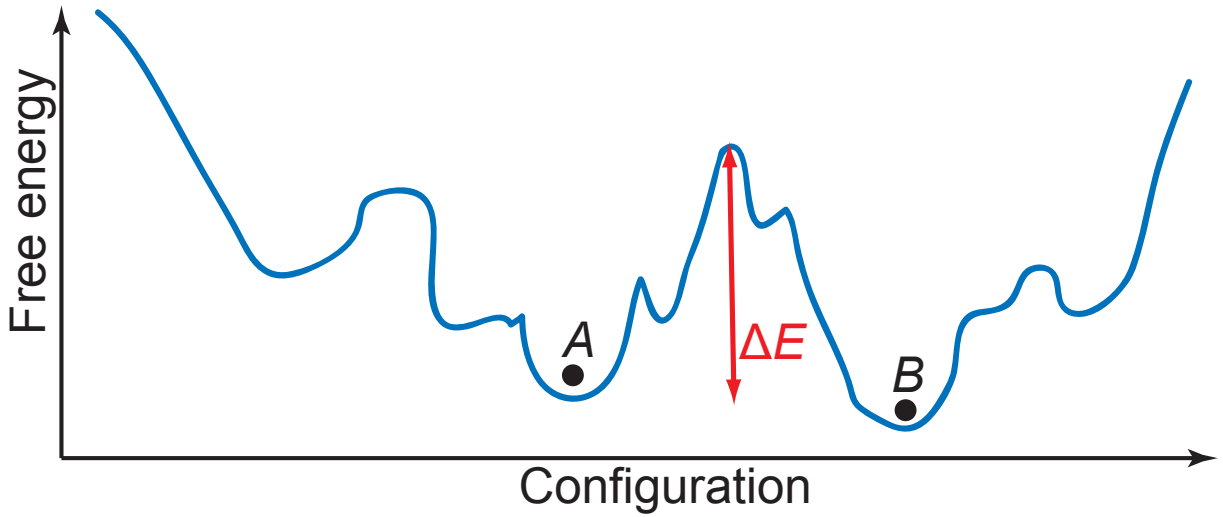

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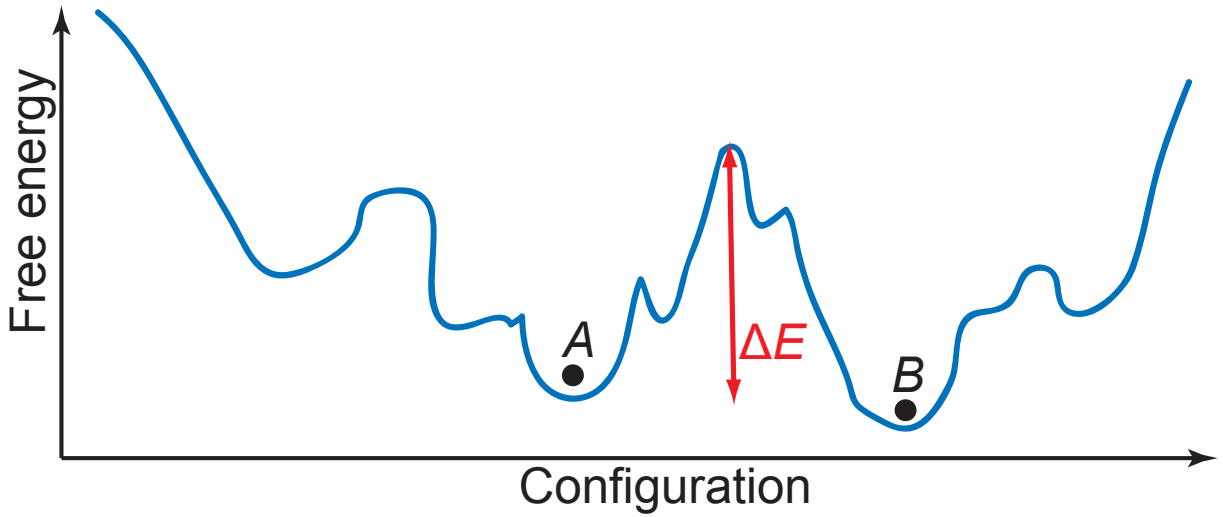

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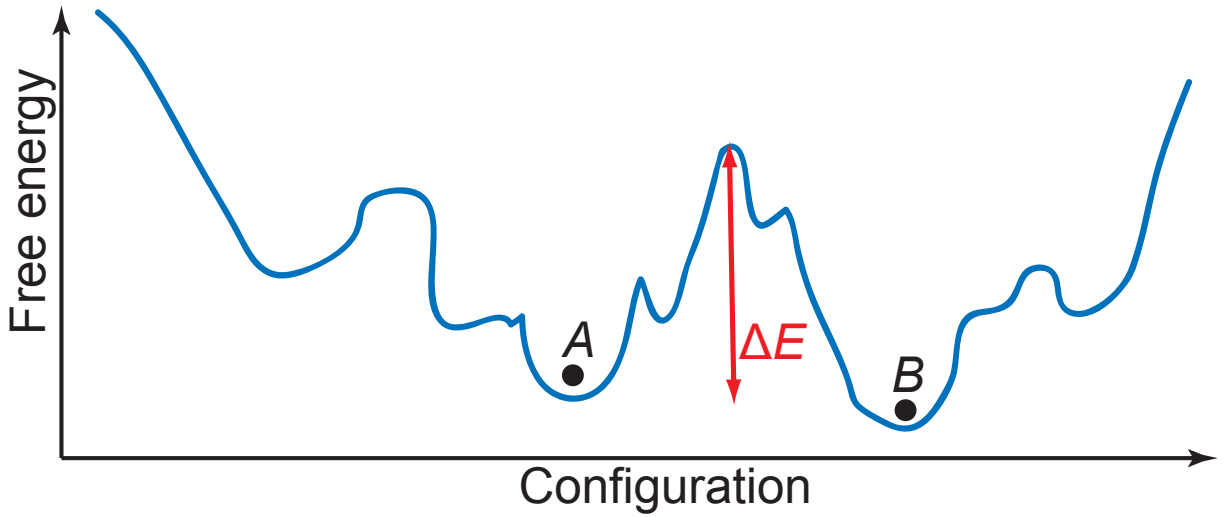

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- Simulate N_T copies of the system at several temperatures.
- Every N_{PT} heat-bath steps, try to exchange configurations at neighbouring temperatures with probability

$$p = \min\{1,\ \exp[-(\beta_{i+1} - \beta_i)(E_{i+1} - E_i)]\}$$

- The temperature of each copy performs a random walk in T space.

Our simulations

L	$T_{\min}$	$T_{\max}$	N_T	$N_{MC}^{\min}$	$N_{MC}^{\max}$
8	0.150	1.575	10	5×10^6	8.30×10^8
12	0.414	1.575	12	1×10^7	1.53×10^{10}
16	0.479	1.575	16	4×10^8	2.79×10^{11}
24	0.625	1.600	28	1×10^9	1.81×10^{12}
32	0.703	1.574	34	4×10^9	7.68×10^{11}

The model and our parameters

- $\mathcal{H} = -\sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} J_{\boldsymbol{xy}}\sigma_{\boldsymbol{x}}\sigma_{\boldsymbol{y}}, \quad P(J_{\boldsymbol{xy}}) = \delta(J_{\boldsymbol{xy}}^2 - 1).$
- We thermalise $L = 32$ down to $T = 0.703 \simeq 0.64\, T_c$.
- 4000 samples for $L \leq 24$ and 1000 samples for $L = 32$.
- Parallel tempering with sample-dependent simulation times.
- A total of 1.1×10^{20} spin updates.

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Assessing thermalisation: autocorrelation times

Autocorrelation times

- Robust thermalisation check $\longrightarrow$ compute autocorrelation times τ:

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- This requires very long simulations (orders of magnitude greater than τ).
- OK for ordered systems.
- Not practical for disordered systems (the main source of error is sample-to-sample fluctuation).

Assessing thermalisation in disordered systems

Traditional method

- Study the time evolution of disorder averages in a logarithmic scale.
- If the last few bins show no evolution, the system is thermalised.

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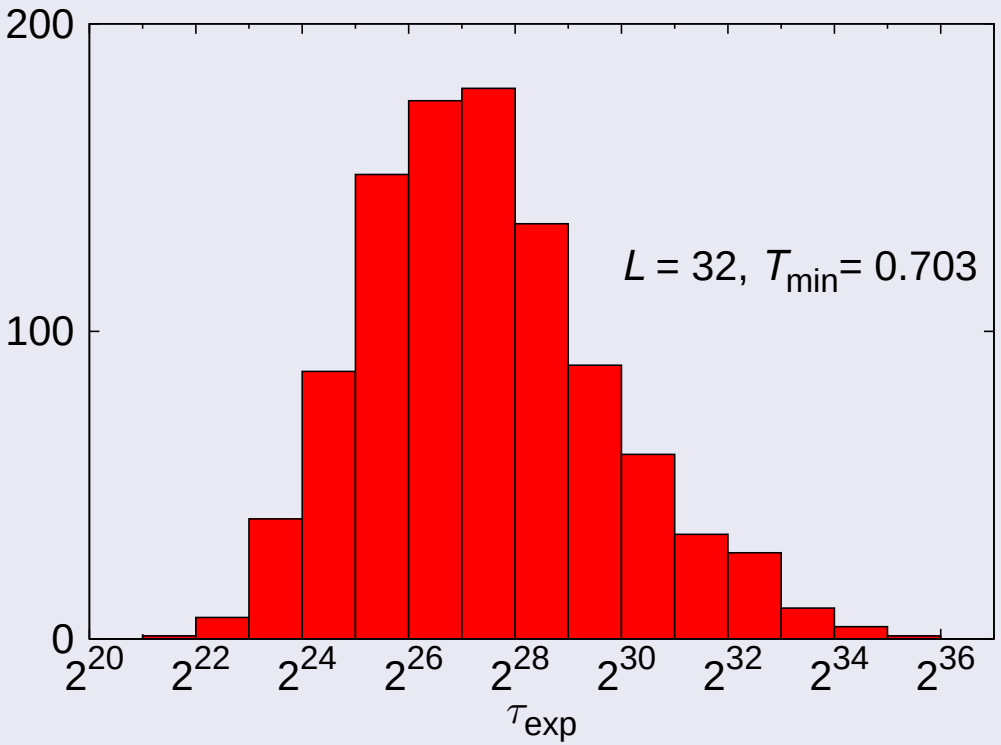

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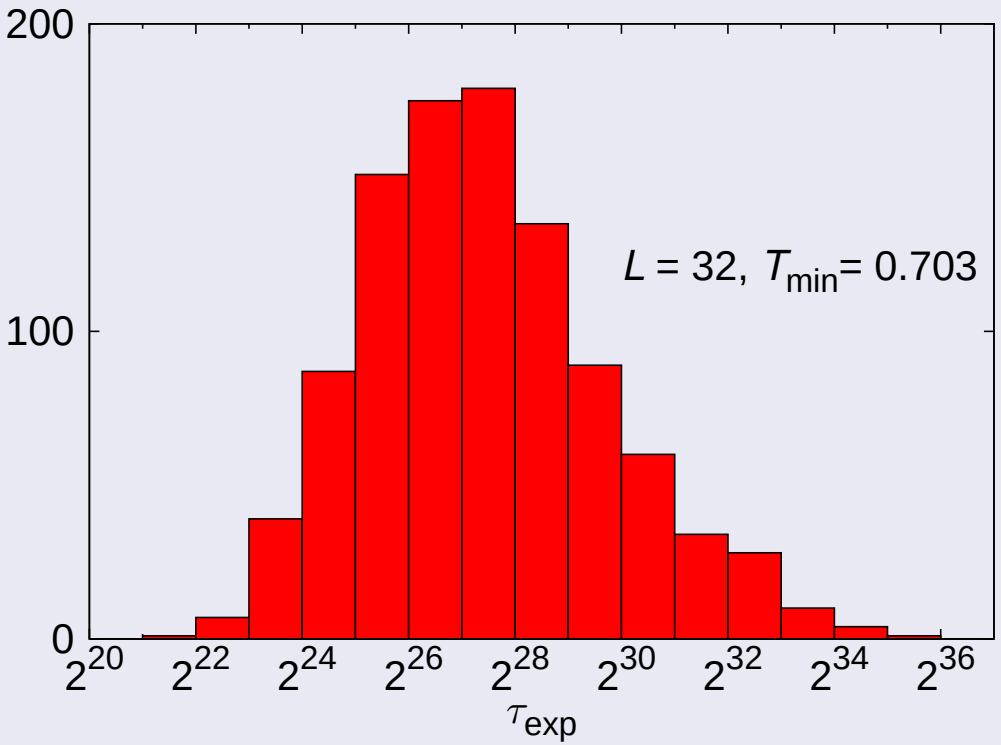

It is not efficient (or safe!)
to use the same
simulation time for all samples.

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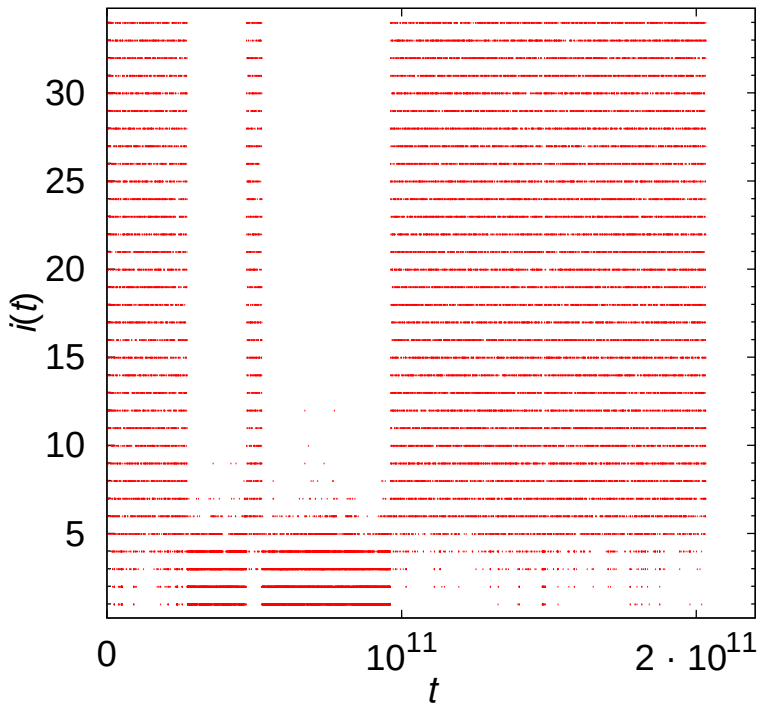

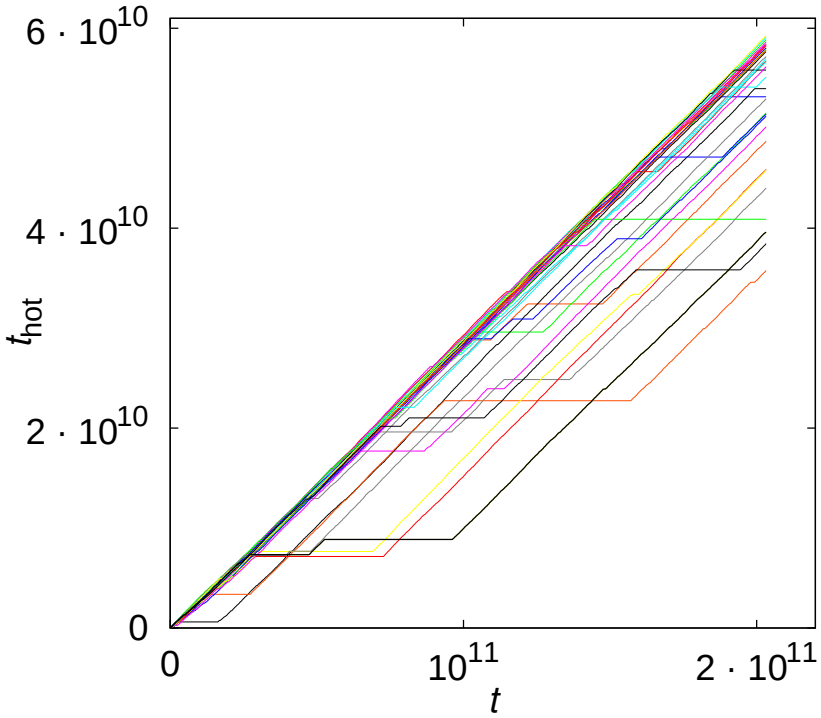

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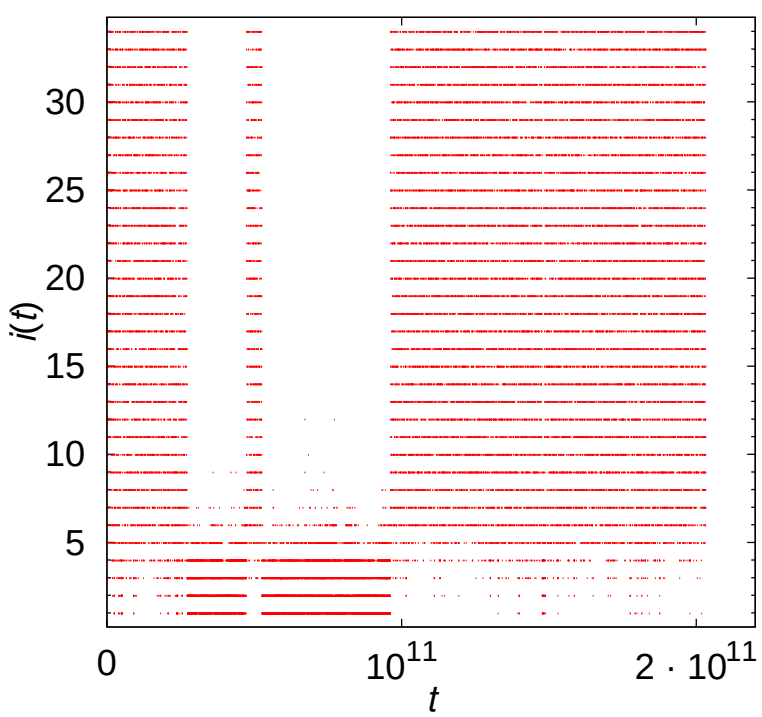

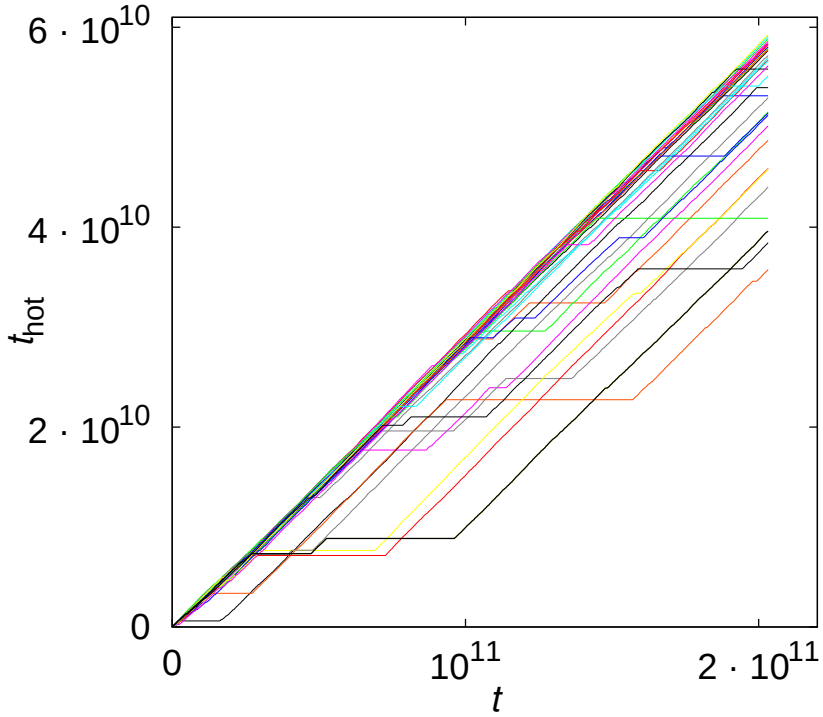

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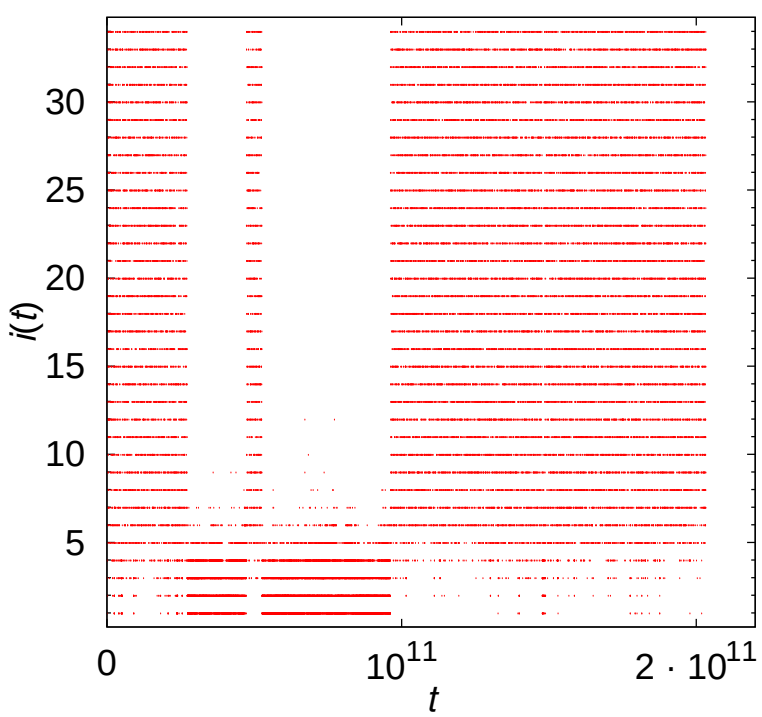

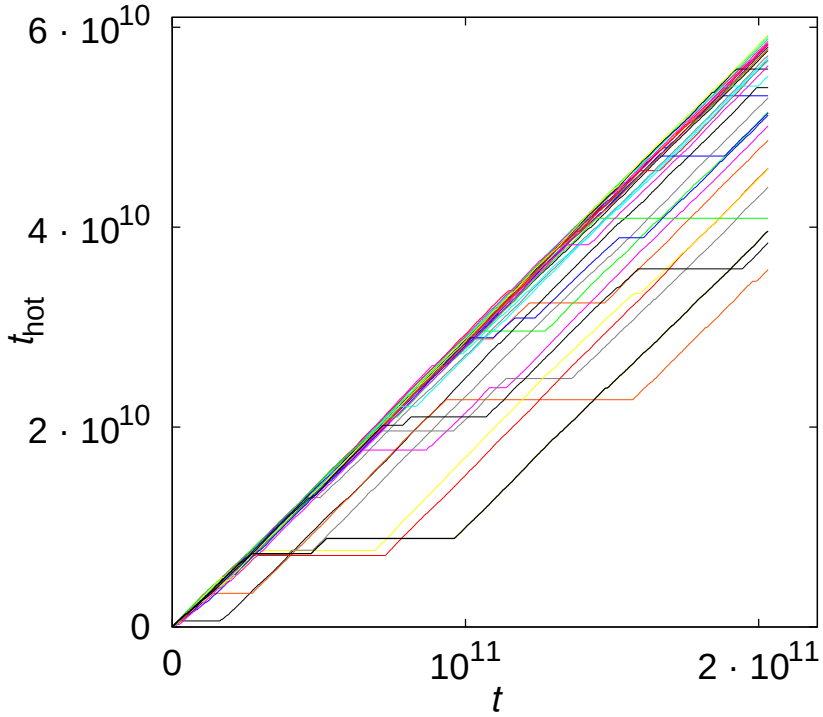

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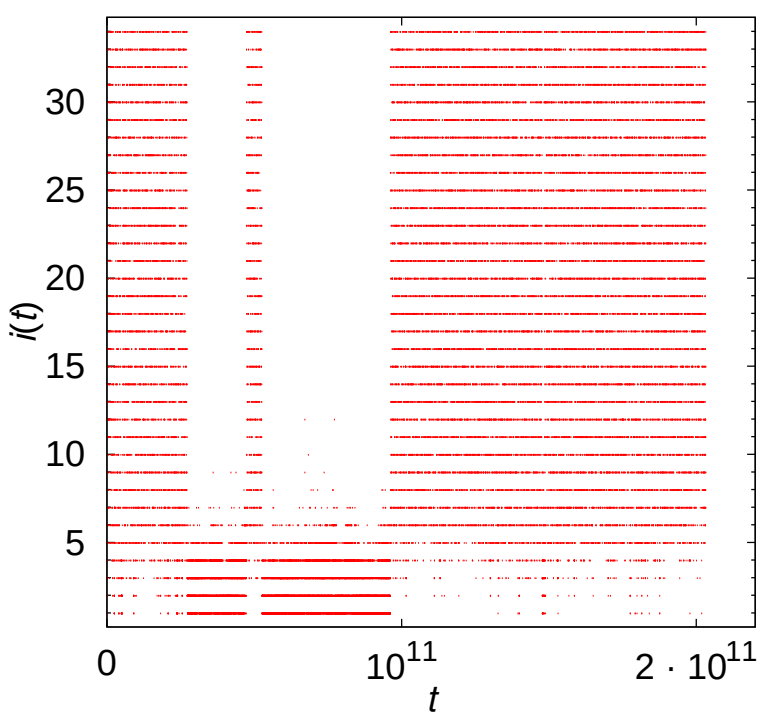

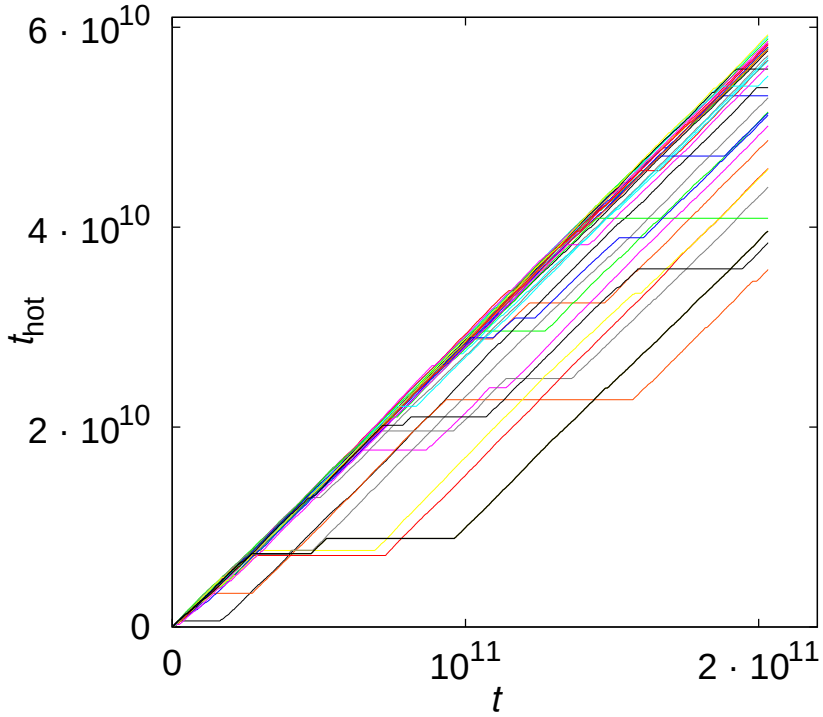

- All the configurations must cover the whole temperature range.
- Samples with long thermalisation times will have long plateaux.
- We can quantify this idea to provide a robust thermalisation test.

Quantifying the temperature random walk (I)

- For each configuration we have a temperature index, indicating its temperature at time t

$$i(t) \in \{1, 2, \ldots, N_T\}, \qquad T_1 < T_2 < \ldots < T_c < \ldots < T_{N_T}$$

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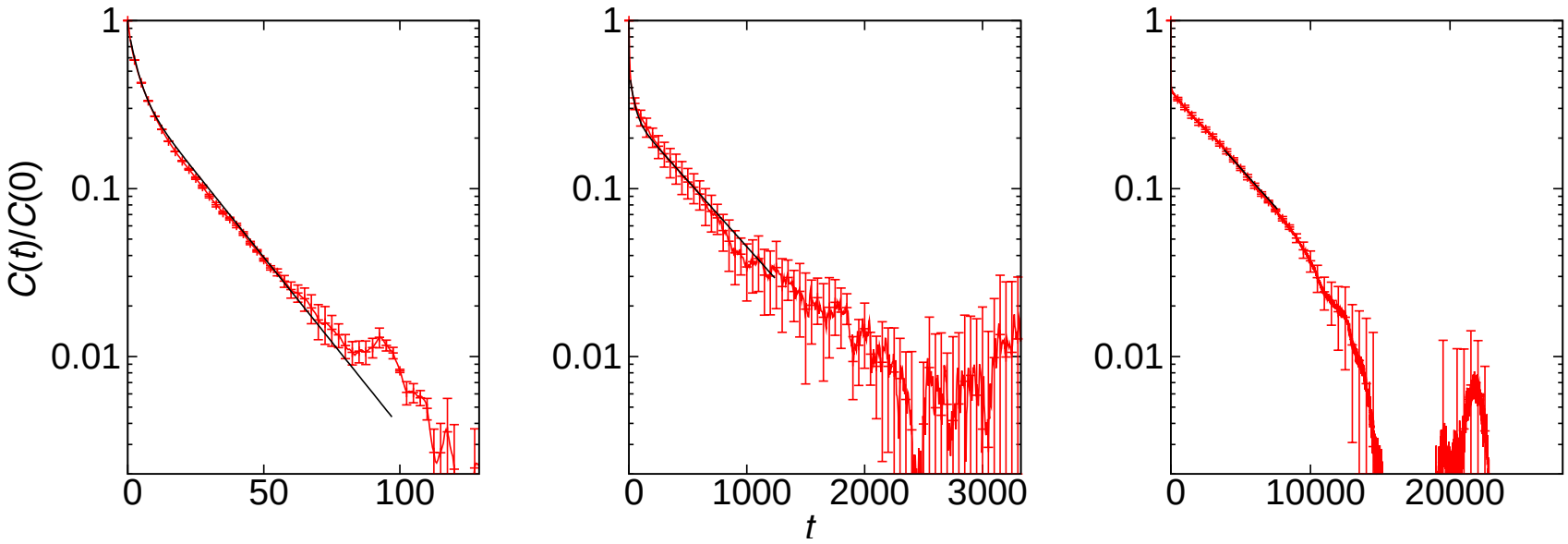
- The integrated autocorrelation time is now easy to compute.
- For critical-point studies $\tau_{\text{int}} \simeq \tau_{\text{exp}} \longrightarrow$ use τ_{int} to assess thermalisation. First used in L.A. Fernandez et al., PRE **80**, 051105 (2009) (A.P. Young's talk).

Quantifying the temperature random walk (II)

- At low temperatures, the correlation functions are more complicated:



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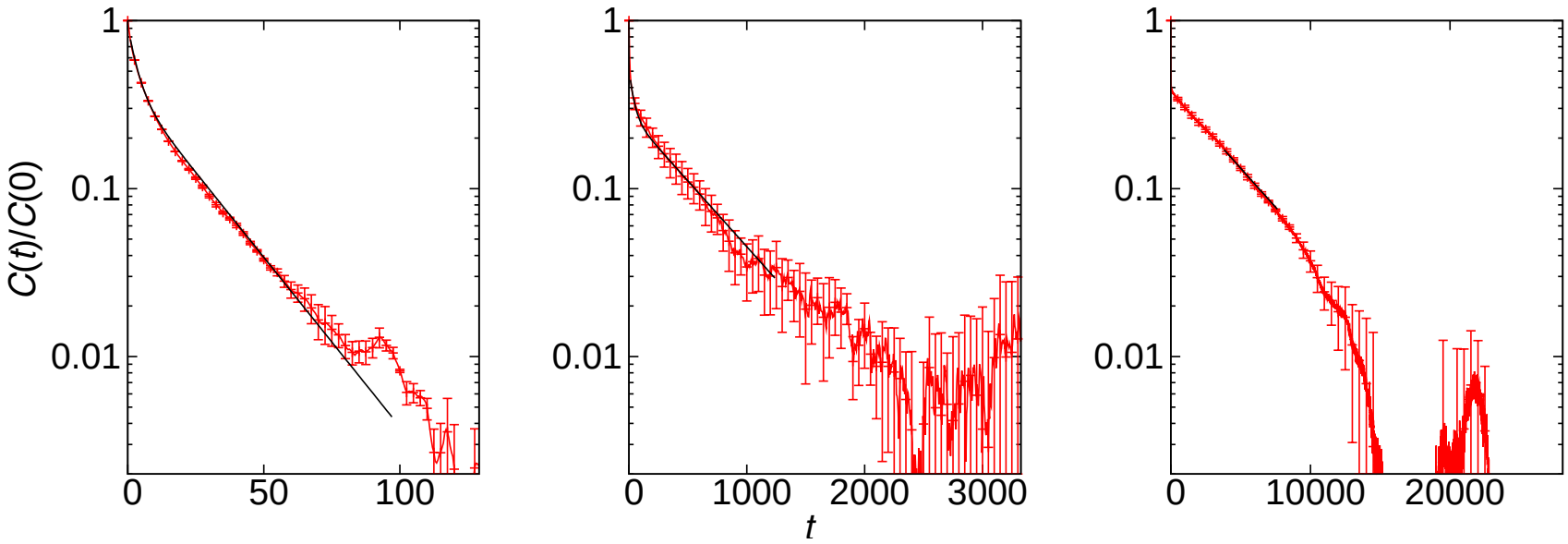

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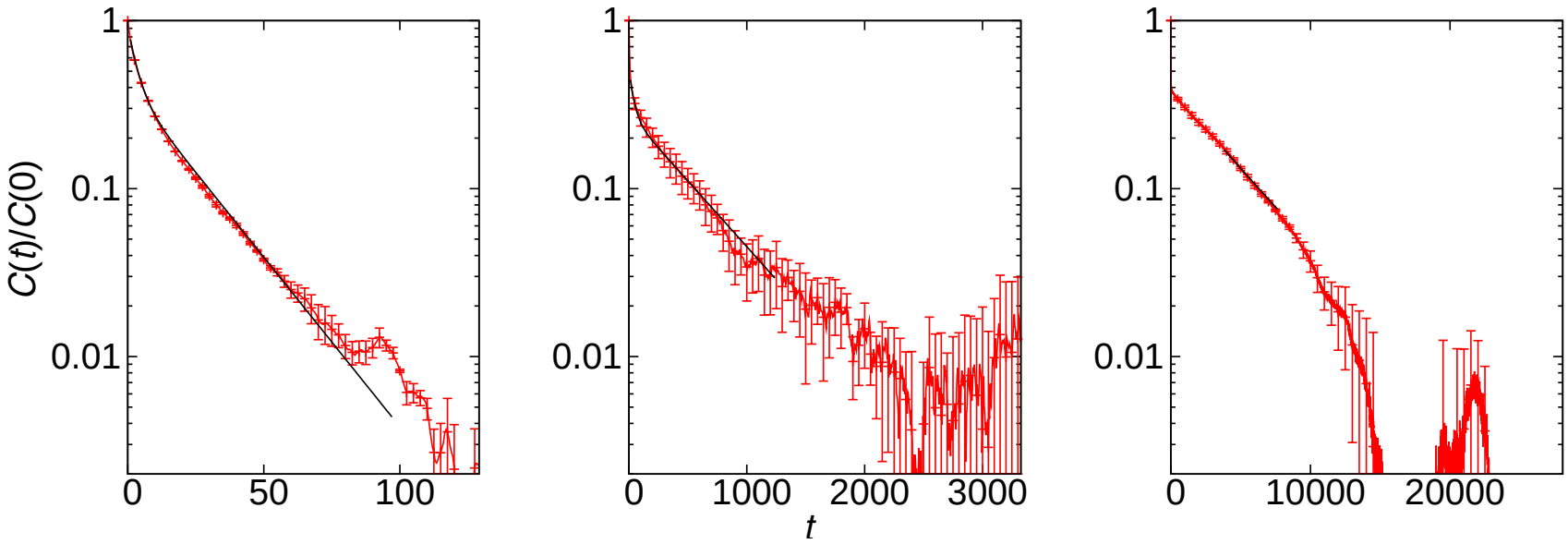

- We parameterise $C(t) \simeq A e^{-t/\tau_{\text{exp}}} + A_1 e^{-t/\tau_1}$ and perform a double fit.

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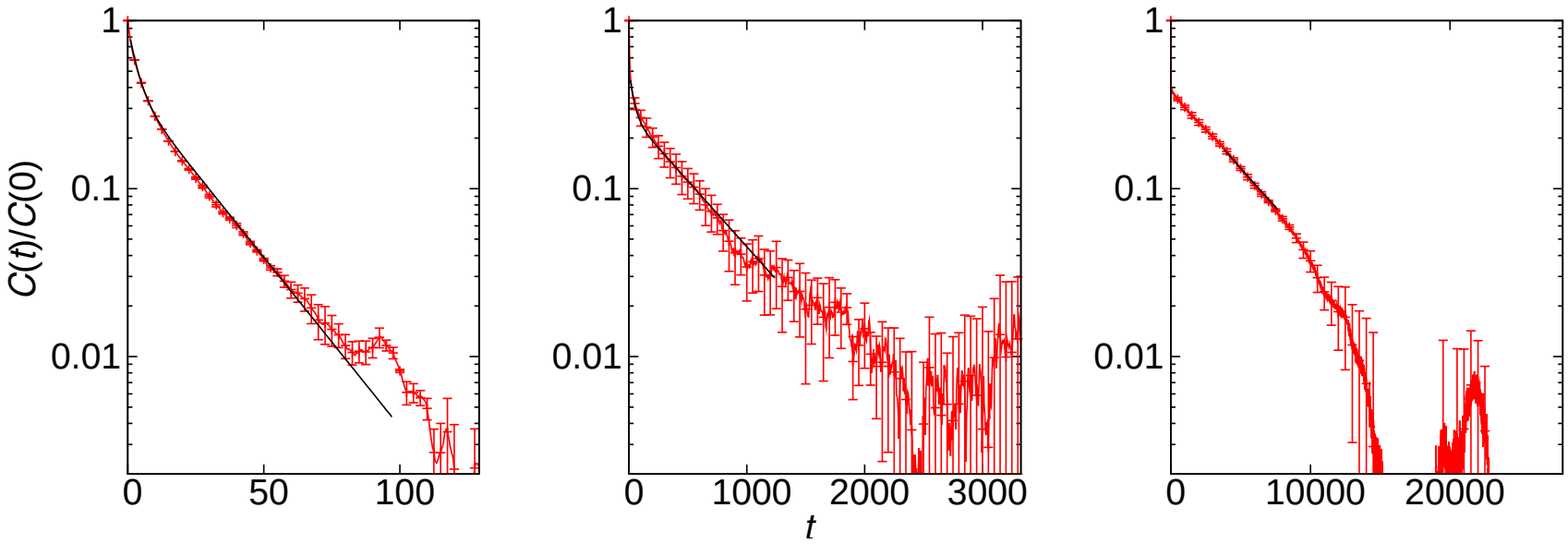

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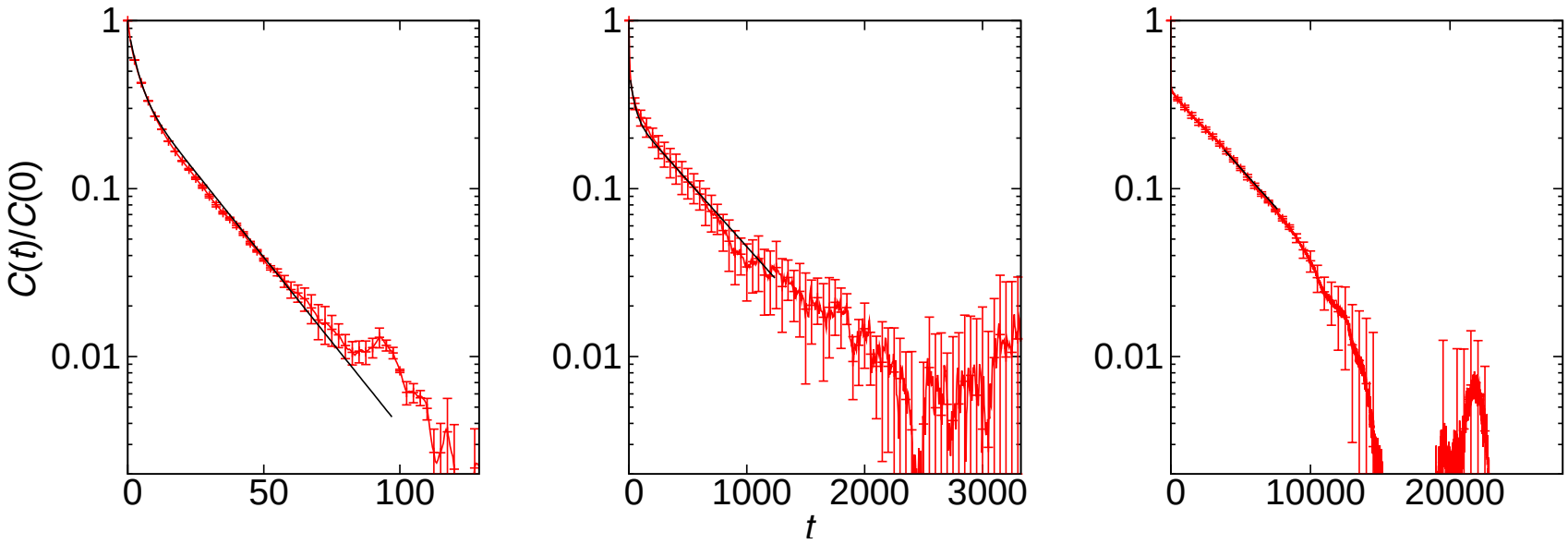

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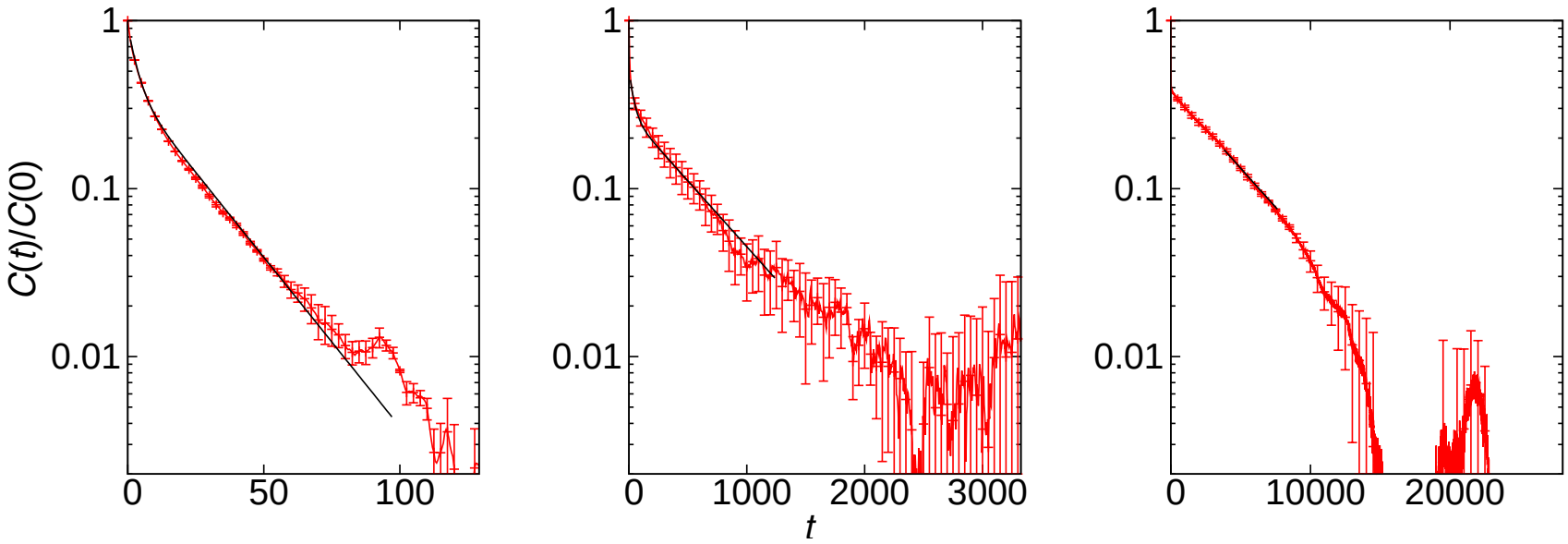

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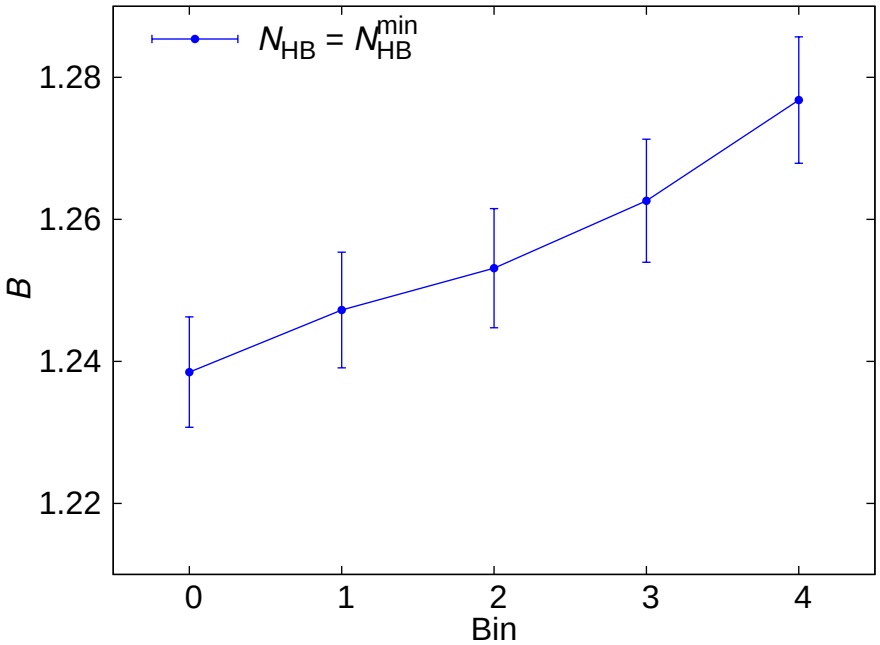

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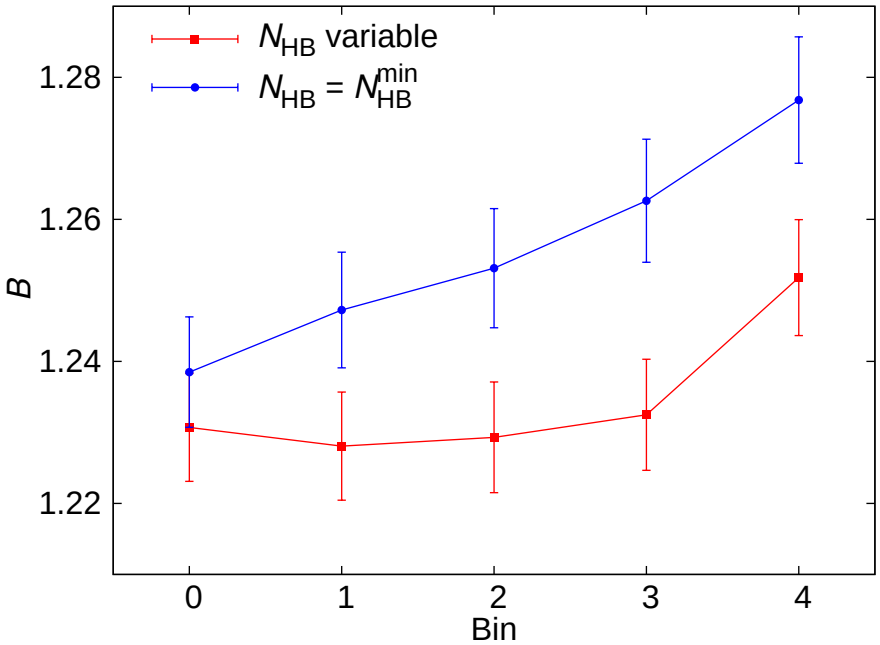

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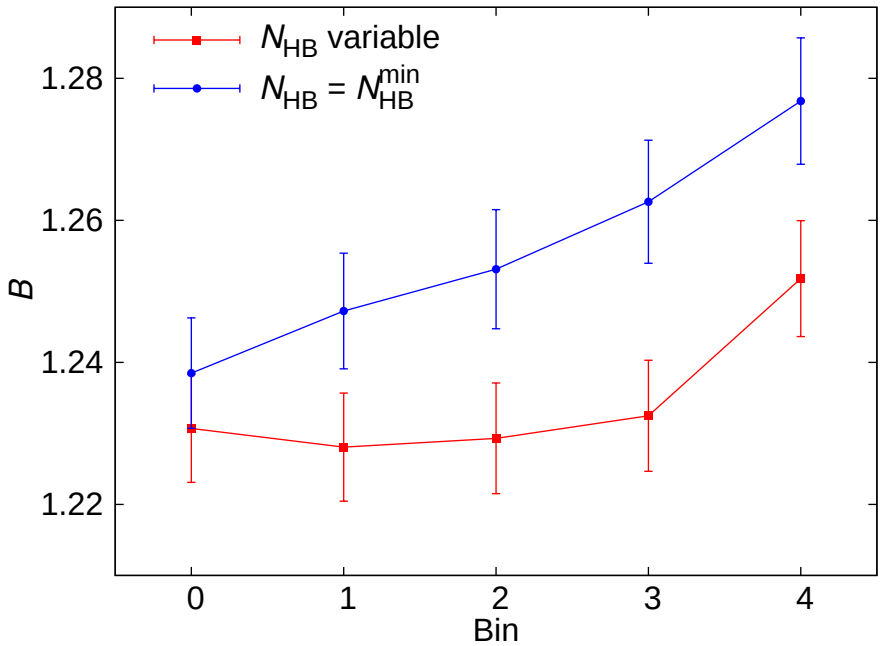

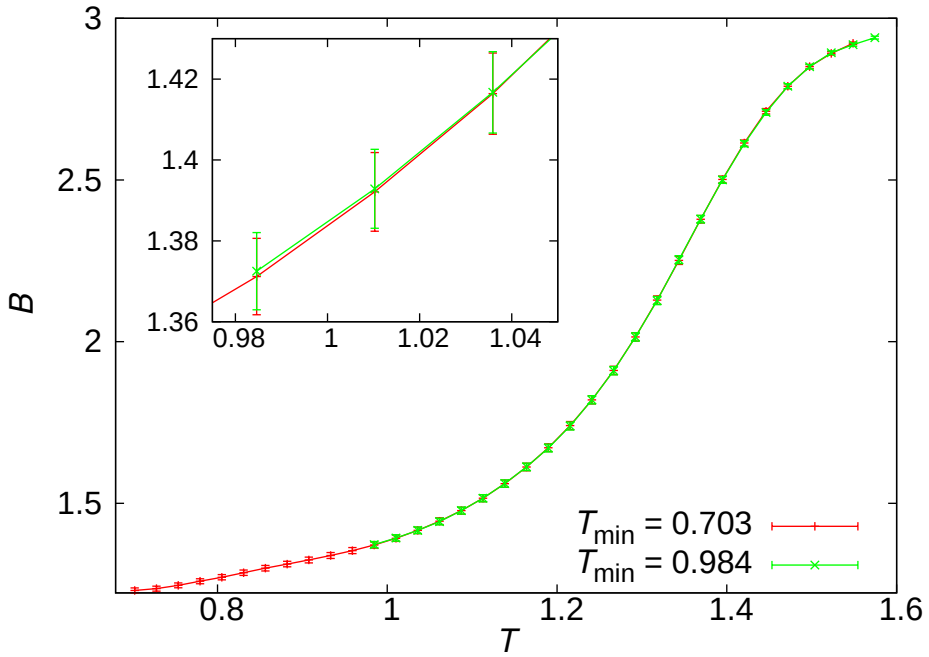

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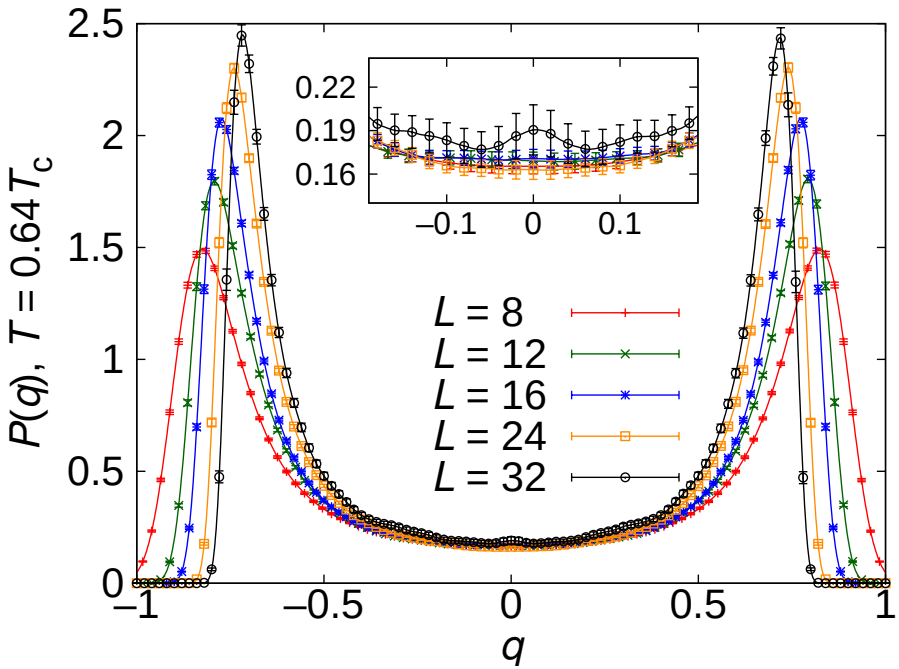

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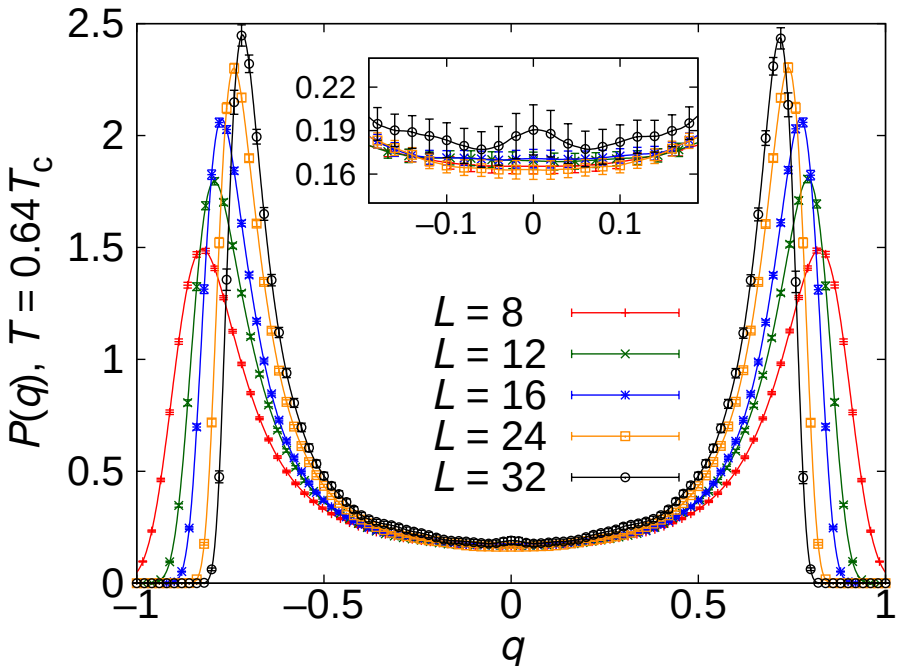

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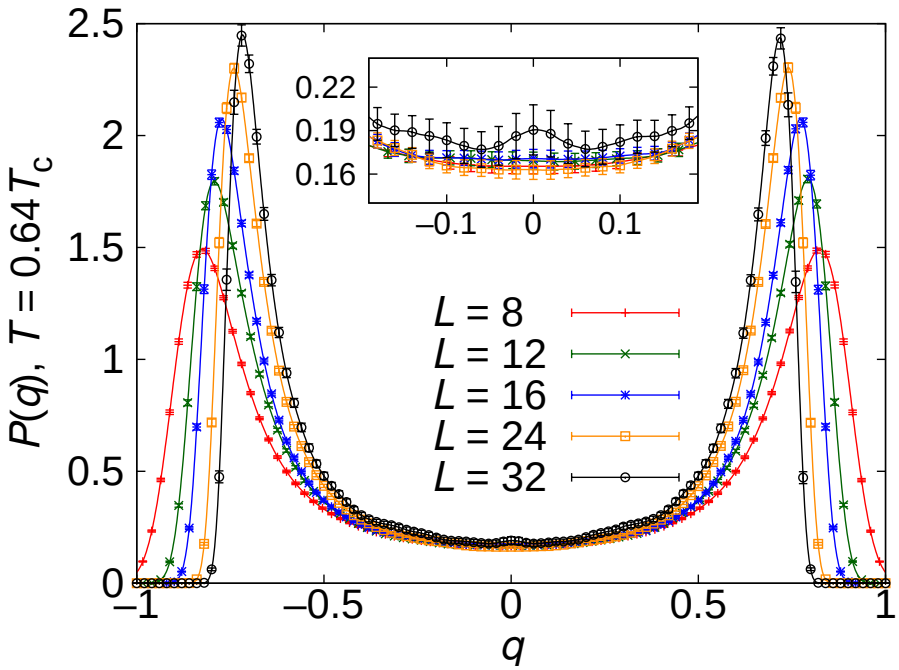

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- In real space, one has to perform a subtraction that complicates the analysis.

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$$\hat{C}_4(\boldsymbol{k}|q^2 < q^2_{\mathrm{EA}}) \propto k^{\theta(q)-D} + \ldots \qquad \hat{C}_4(\boldsymbol{k}|q^2 > q^2_{\mathrm{EA}}) \propto \frac{1}{k^2 + \xi_q^{-2}}$$

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- From very general RG arguments we can derive a scaling law:

$$\theta(q_{\rm EA}) = 2/\hat{\nu}.$$

The computation of q_{EA} (I)

- According to Finite-Size Scaling,

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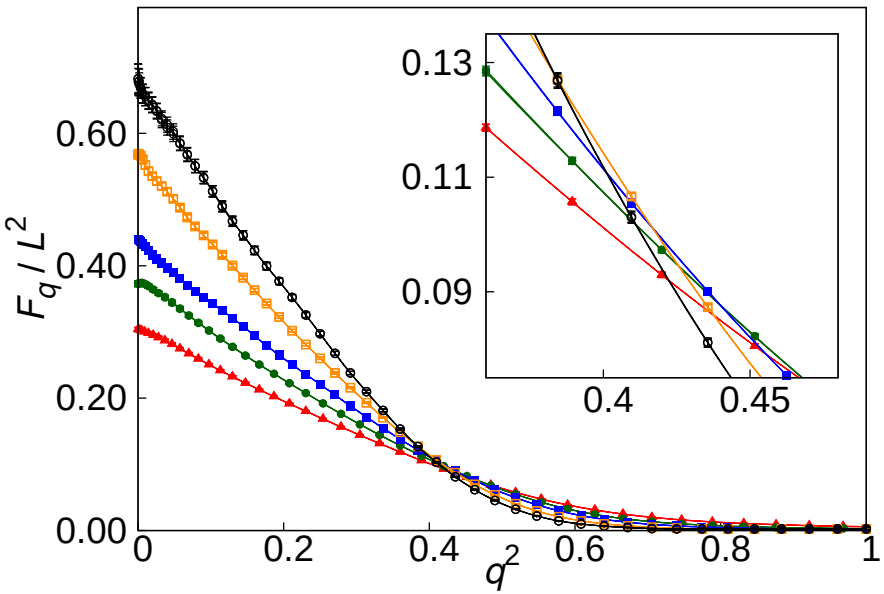

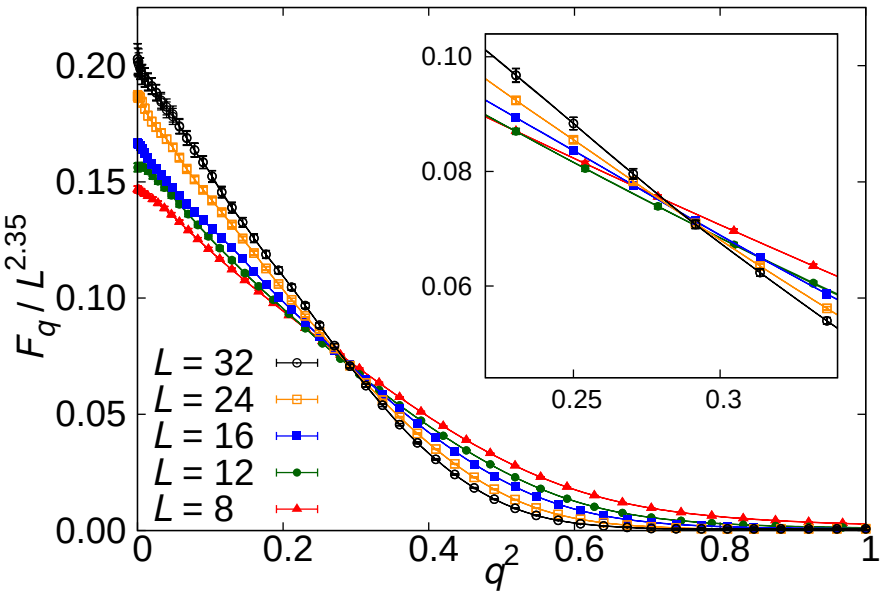

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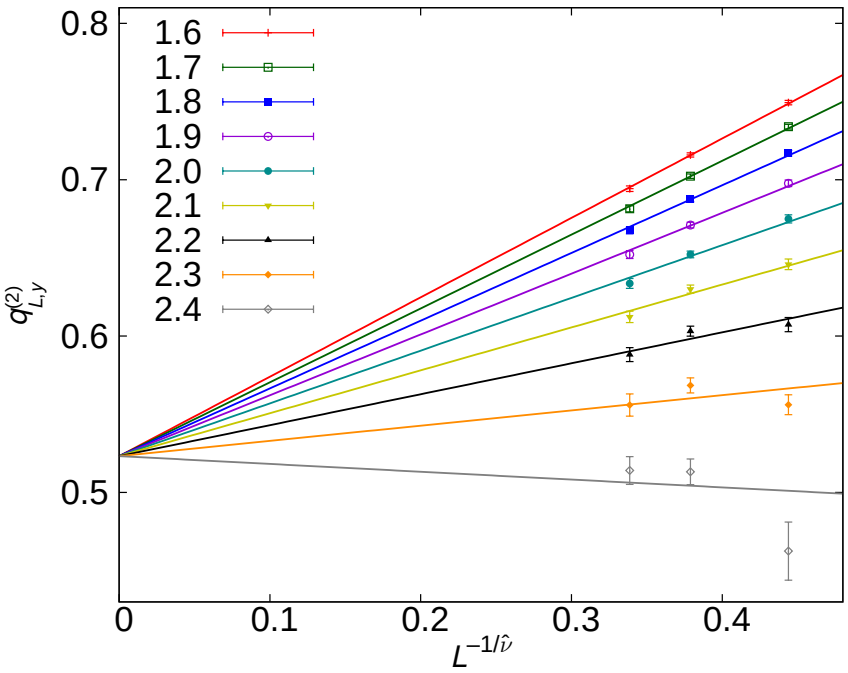

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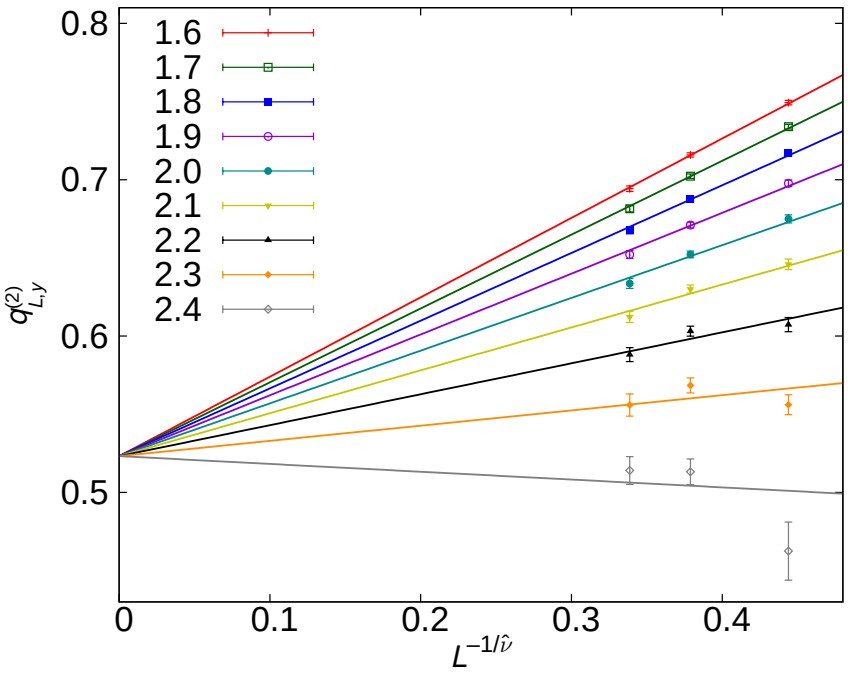

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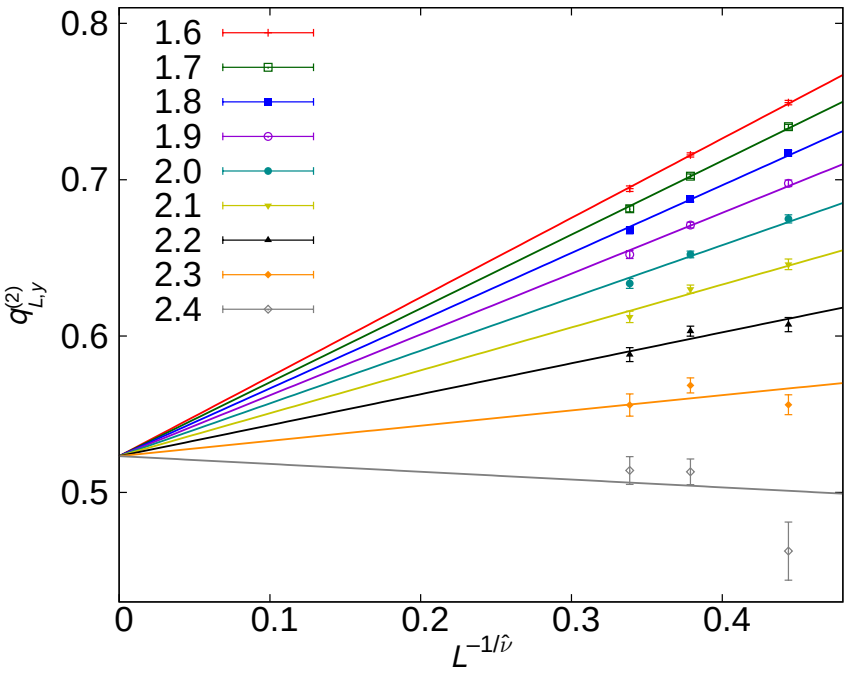

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- Other physical results:
  - We have established a time-length dictionary, relating non-equilibrium and equilibrium.
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