Worm algorithm for loop model on the square lattice

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in collaboration with

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outlines

Motivations

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  When crossings of loops are allowed, the low-temperature phase is distinct from nonintersecting loop models. (Jacobsen *et al*, PRL 2003). And the LT branch of the nonintersecting loop model can be mapped onto a tricritical loop model with a different loop weight. (Nienhuis, WG and Blöte, PRE, 2009).

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- An efficient Monte Carlo algorithm of the cluster type is available for 2D nonintersecting loop models (Deng *et al*, PRL 2007). Thus far, no efficient Monte Carlo algorithm to simulate intersecting loop models.

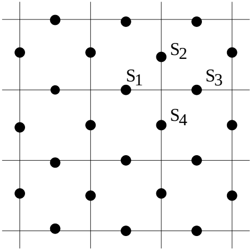
O(n) spin model on the square lattice

Put spins on the middle of the edges of the square lattice

$$\mathcal{Z} = \int \left[\prod_i d\vec{s_i}\right] \prod_V \{1 + u\,(\vec{s_1}\cdot\vec{s_2} + \vec{s_2}\cdot\vec{s_3} + \vec{s_3}\cdot\vec{s_4} + \vec{s_4}\cdot\vec{s_1}) +$$

$$v\,(\vec{s_1}\cdot\vec{s_3} + \vec{s_2}\cdot\vec{s_4}) + w\,[(\vec{s_1}\cdot\vec{s_2})(\vec{s_3}\cdot\vec{s_4}) + (\vec{s_2}\cdot\vec{s_3})(\vec{s_4}\cdot\vec{s_1})] + c(\vec{s_1}\cdot\vec{s_3})(\vec{s_2}\cdot\vec{s_4})\}$$

$\vec{s_i}$: n-component vector spin, the weight is O(n) symmetric



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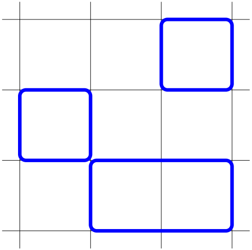
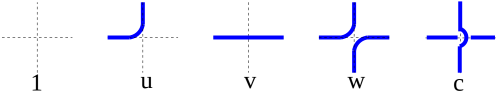


Expansion of $\mathcal{Z}$ in powers of u, v, w, c yields an O(n) (intersecting) loop model

$$\mathcal{Z} = \sum_{\mathcal{A}} n^l \prod_{i\in V} \omega_i = \sum_{\mathcal{A}} n^l u^{N_u} v^{N_v} w^{N_w} c^{N_c}$$

$u^{10}v^2w^1n^3$



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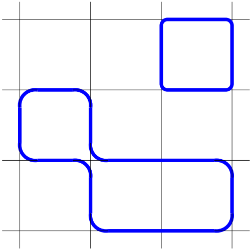

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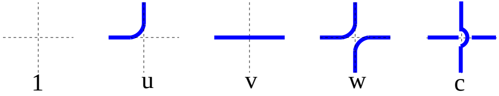

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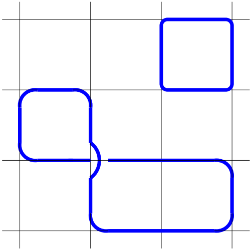

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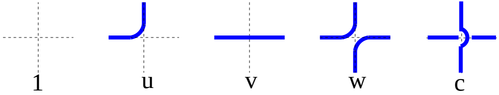
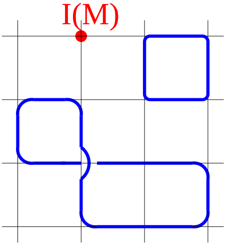


Worm algorithm

Enlarge the set of configurations of the loop model by including two "special" vertices I and M.



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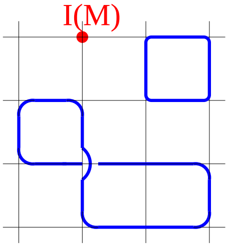

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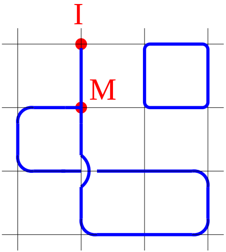

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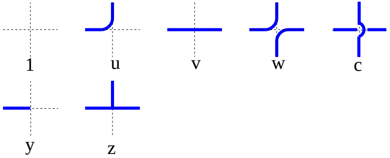
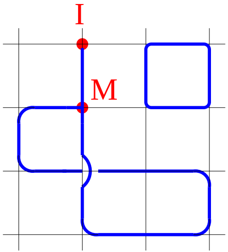
Let $\mathcal{S}$ be a state in the enlarged space

$$\phi_{\mathcal{S}} = n^{l} \cdot \prod_{i \in V} \omega_i \ ,$$

ω_i can be 1, u, v, w, c, and y, z





$$\mathcal{Z}_{\mathrm{w}} = \sum_{\mathcal{S}} \phi_{\mathcal{S}} \ .$$

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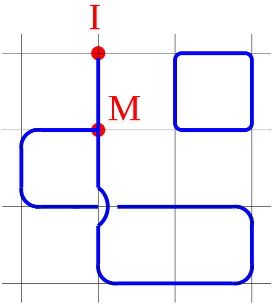
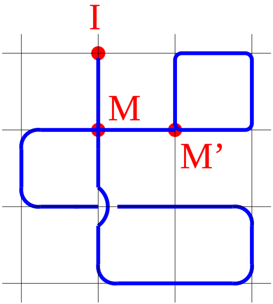
Acceptance probability

step 2, current state $\mathcal{S}$



Acceptance probability

step 2, a test move



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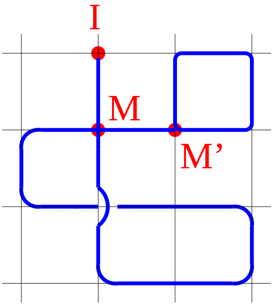
A given bond configuration may correspond with different loop configurations.



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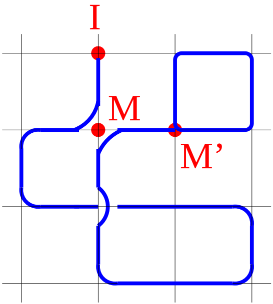

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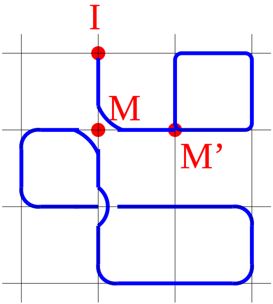
step 3, select a state $\mathcal{S}'$

Acceptance probability



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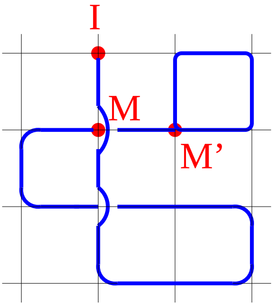

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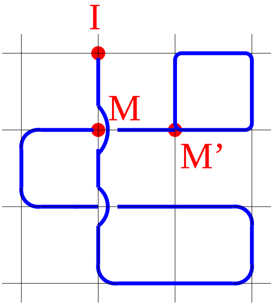

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$$P_a(\mathcal{S} \to \mathcal{S}') = \min\left(1, \frac{p_p(\mathcal{S}|\mathcal{S}')}{p_p(\mathcal{S}'|\mathcal{S})} \cdot \frac{\phi_{\mathcal{S}'}}{\phi_{\mathcal{S}}}\right) ,$$

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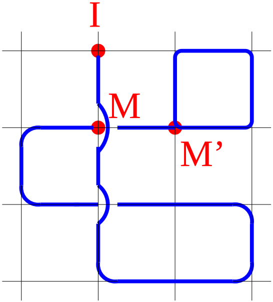

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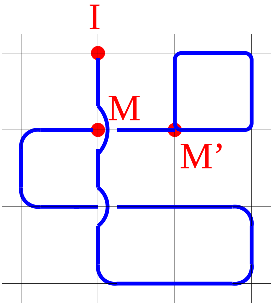
$p_p(\mathcal{S}'|\mathcal{S})$ the proposal probability from $\mathcal{S}$ to $\mathcal{S}'$:

$$p_p(\mathcal{S}'|\mathcal{S}) = \frac{1}{8} \cdot \frac{1}{d_M(\mathcal{S}')} \cdot \frac{1}{d_{M'}(\mathcal{S}')}, \quad d_M(\mathcal{S}') = 3, or, 1$$

$1/8$ accounts for the random choices of I (M) and of one out of four neighbors.

Acceptance probability



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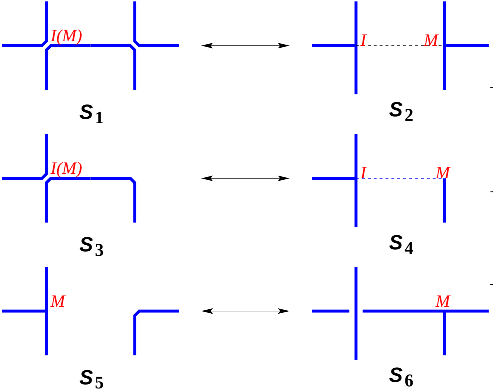
$$= \min\left(1, \frac{d_M(\mathcal{S}')d_{M'}(\mathcal{S}')\omega_M(\mathcal{S}')\omega_{M'}(\mathcal{S}')n^{\Delta l}}{d_M(\mathcal{S})d_{M'}(\mathcal{S})\omega_M(\mathcal{S})\omega_{M'}(\mathcal{S})}\right) \ .$$

where $\Delta l = l(\mathcal{S}') - l(\mathcal{S})$
is the change of the number of loops

Examples



$$P_a(\mathcal{S}_1 \to \mathcal{S}_2) = \min\left(1, \frac{z^2 n^{l_2}}{9w^2 n^{l_1}}\right)$$

$$P_a(\mathcal{S}_3 \to \mathcal{S}_4) = \min\left(1, \frac{zyn^{l_4}}{3wun^{l_3}}\right)$$

$$P_a(\mathcal{S}_5 \to \mathcal{S}_6) = \min\left(1, \frac{3czn^{l_6}}{uzn^{l_5}}\right)$$

$n \neq 1$ case

In the calculation of the acceptance probability P_a, one has to count the change Δl of the loop number. This is a nonlocal procedure.

- Sweeny algorithm
- Coloring technique

Worm algorithm with the coloring technique ($n \geq 1$)

1. Randomly choose a vertex $k \in V$, move $I = M$ to k, and do the following: independently for each loop, color all its occupied *bonds* to be "active" (green) with probability $1/n$ and to be "inactive" (red) with probability $1 - 1/n$; all empty edges are assigned "active" (green).

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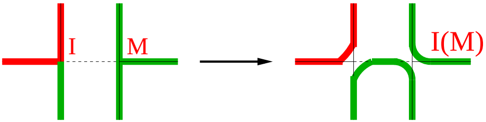

$\mathcal{S}$ $\qquad\qquad$ $\mathcal{S}'$

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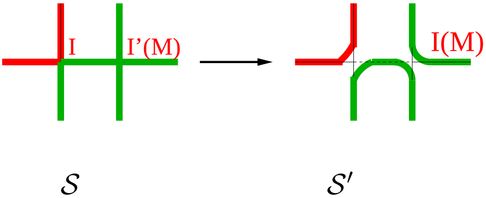

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- Test the worm algorithm by studying the critical properties of the model
- Check the efficiency of the algorithm

Exactly known critical exponents of the O(n) loop model

At the critical branch of the model

- thermal exponent: $y_t = \frac{4g-4}{g}$
- Magnetic exponent: $y_h = 1 + \frac{1}{2g} + \frac{3g}{8}$
- Hull exponent: $y_H = 1 + \frac{1}{2g}$
  which describes the decay of the probability that two bonds are sitting at the same loop, is also the fractal dimension d_l of the loops.

g is the Coulomb-gas coupling: $n = -2\cos(\pi g), \quad 1 \leq g \leq 3/2$

Determination of the critical points and y_t

We determine the critical point in two subspace:

- without crossing bonds, $u = v = x, w = x^2, c = 0$
- with crossing bonds, $u = v = x, w = c = x^2$

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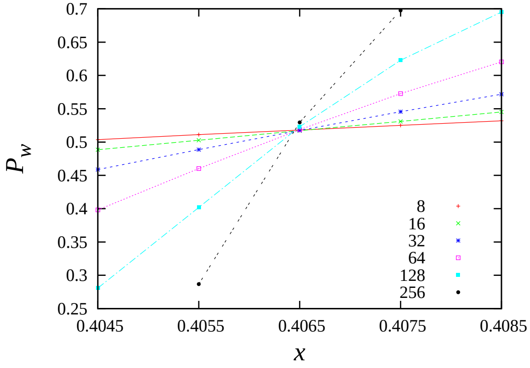

without crossing bonds, $n = 1$

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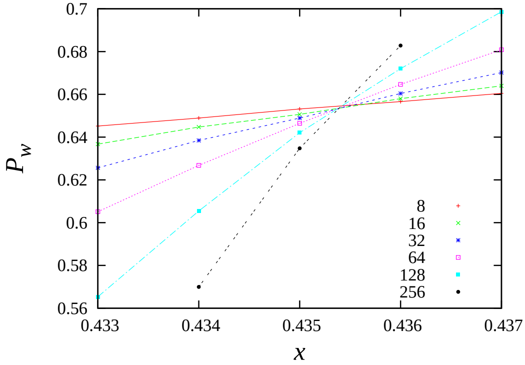


with crossing bonds, $n = 1.5$

Determination of other exponents

Simulate the model at the estimated critical point.

$n_b =$ the average density of occupied bonds

$n_w =$ the average fraction of edges covered by the wrapping loop

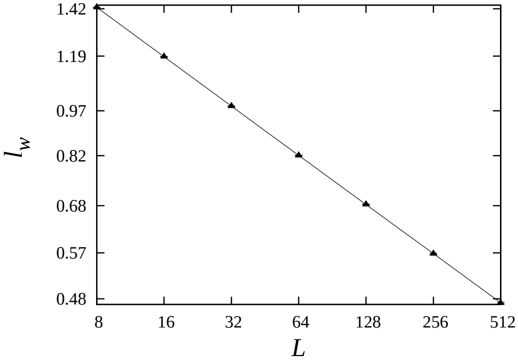
$S_2 =$ the average of the sum of squares of loop lengths per site

$l_w =$ the average length of worm steps per site

Determination of other exponents: y_h

$n = 1.5$, in the subspace with crossing bonds

$$l_w \propto L^{2y_h - 4}$$



$$y_h = 1.8679(6)$$

Determination of other exponents: y_H

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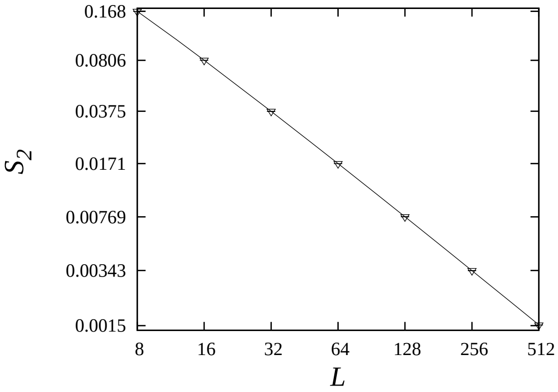
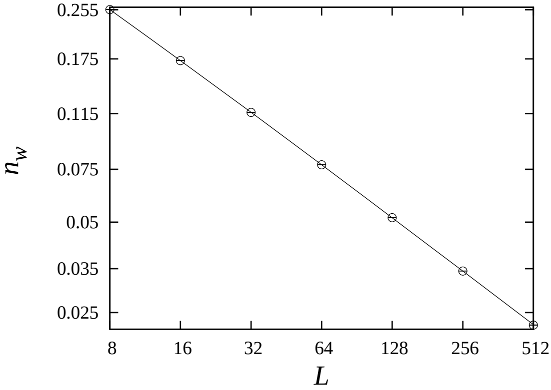


$$y_H = 1.405(2)$$

Determination of other exponents: y_H

$n = 1.5$, in the subspace with crossing bonds

$$n_w \propto L^{y_H - 2}$$



$$y_H = d_l = 1.405(2)$$

Numerical results

Simulation results (S) in the subspace $u = v = x, w = x^2, c = 0$.

n		x_c	y_t	y_h	y_H	$P_w^{(0)}$
1	E		1	1.875	1.375	
	T	0.40644(1)				
	S	0.40644(1)	1.002(3)	1.8749(3)	1.374(1)	0.516(1)
1.5	E		0.748109	1.86776	1.40649	
	T	0.43535(2)				
	S	0.43535(1)	0.747(5)	1.8675(5)	1.4067(6)	0.6530(4)

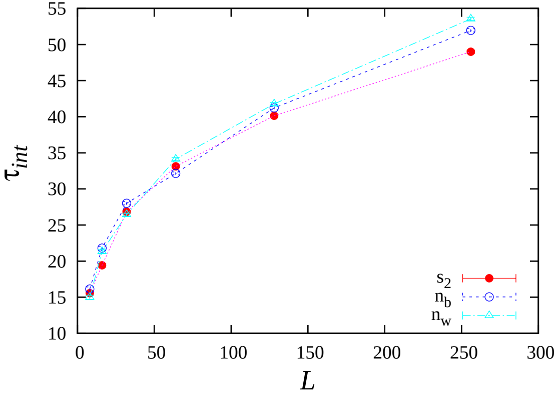
Numerical results

Simulation results (S) in the subspace $u = v = x, w = c = x^2$.

n		x_c	y_t	y_h	y_H	$P_w^{(0)}$
1	E		1	1.875	1.375	
	T	0.398048(2)				
	S	0.398050(5)	1.001(3)	1.8749(3)	1.3755(6)	0.516(1)
1.5	E		0.748109	1.86776	1.40649	
	T	0.423622(2)				
	S	0.42366(5)	0.744(5)	1.8679(6)	1.405 (2)	0.654(1)

Dynamic behavior of the algorithm

Integrated autocorrelation times τ_{int} versus lattice size L ($n = 1.5$) in the subspace $u = v = x, w = c = x^2$. (The unit of time is normalized to 'visit per site').



$z(S_2) \approx 0.2, z(n_b) \approx 0.3, z(n_w) \approx 0.3$

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