# Mathematics in the Service of Physical Science:

## A mini-symposium to mark the career of Ian Enting

## March 2015

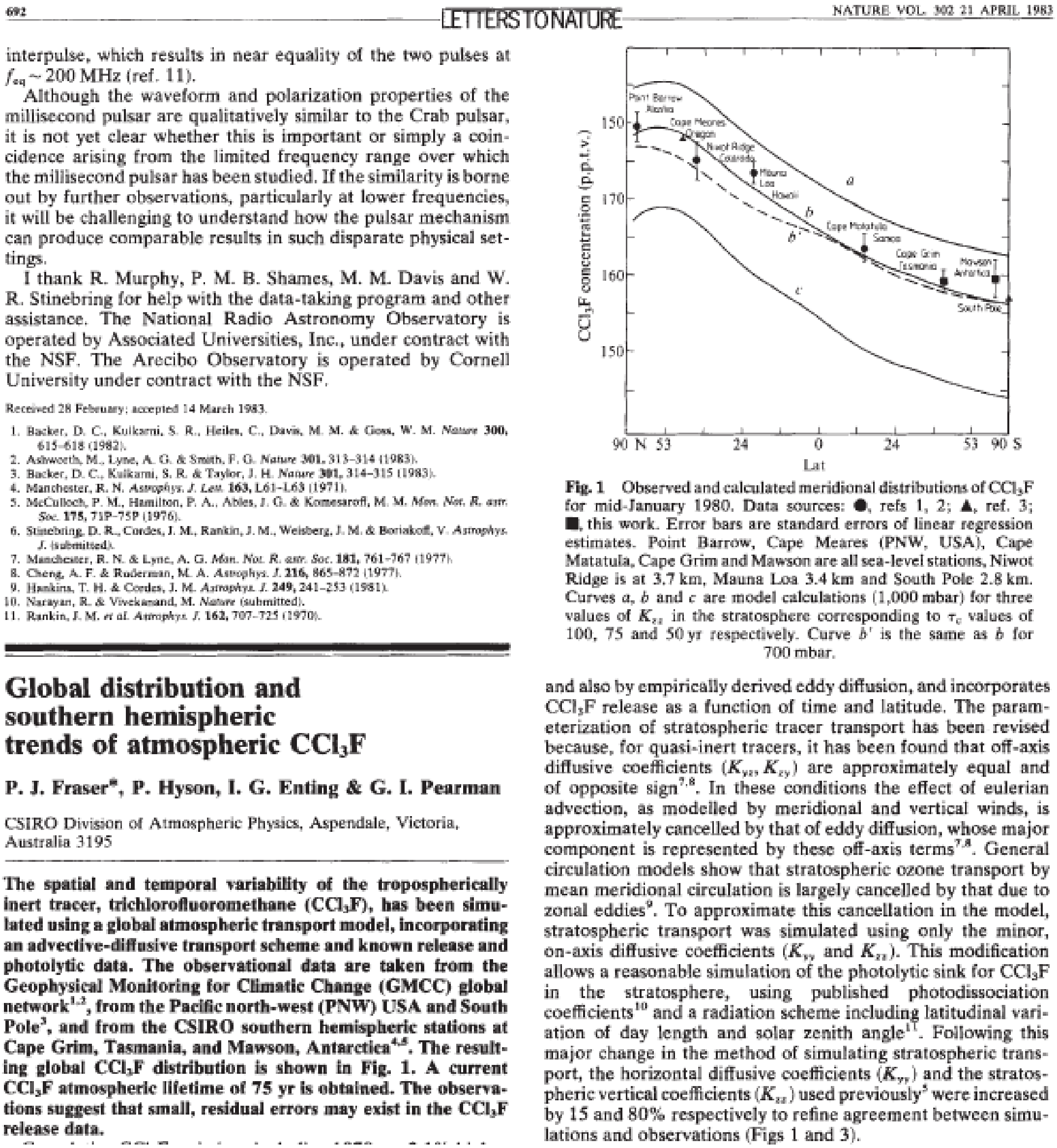
## Historical perspective

David Etheridge

CSIRO Oceans and Atmosphere Flagship

# Nature 1983

## Top-down verification of emissions



interpulse, which results in near equality of the two pulses at $f_{eq} \sim 200$ MHz (ref. 11).

Although the waveform and polarization properties of the millisecond pulsar are qualitatively similar to the Crab pulsar, it is not yet clear whether this is important or simply a coincidence arising from the limited frequency range over which the millisecond pulsar has been studied. If the similarity is borne out by further observations, particularly at lower frequencies, it will be challenging to understand how the pulsar mechanism can produce comparable results in such disparate physical settings.

I thank R. Murphy, P. M. B. Shames, M. M. Davis and W. R. Stinebring for help with the data-taking program and other assistance. The National Radio Astronomy Observatory is operated by Associated Universities, Inc., under contract with the NSF. The Arecibo Observatory is operated by Cornell University under contract with the NSF.

Received 28 February; accepted 14 March 1983.

1. Backer, D. C., Kulkarni, S. R., Heiles, C., Davis, M. M. & Goss, W. M. *Nature* **300**, 615–618 (1982).
2. Ashworth, M., Lyne, A. G. & Smith, F. G. *Nature* **301**, 313–314 (1983).
3. Backer, D. C., Kulkarni, S. R. & Taylor, J. H. *Nature* **301**, 314–315 (1983).
4. Manchester, R. N. *Astrophys. J. Lett.* **163**, L61–L63 (1971).
5. McCulloch, P. M., Hamilton, P. A., Ables, J. G. & Komesaroff, M. M. *Mon. Not. R. astr. Soc.* **175**, 71P–75P (1976).
6. Stinebring, D. R., Cordes, J. M., Rankin, J. M., Weisberg, J. M. & Boriakoff, V. *Astrophys. J.* (submitted).
7. Manchester, R. N. & Lyne, A. G. *Mon. Not. R. astr. Soc.* **181**, 761–767 (1977).
8. Cheng, A. F. & Ruderman, M. A. *Astrophys. J.* **216**, 865–872 (1977).
9. Hankins, T. H. & Cordes, J. M. *Astrophys. J.* **249**, 241–253 (1981).
10. Narayan, R. & Vivekanand, M. *Nature* (submitted).
11. Rankin, J. M. *et al. Astrophys. J.* **162**, 707–725 (1970).

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### Global distribution and southern hemispheric trends of atmospheric CCl$_3$F

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**The spatial and temporal variability of the tropospherically inert tracer, trichlorofluoromethane (CCl$_3$F), has been simulated using a global atmospheric transport model, incorporating an advective-diffusive transport scheme and known release and photolytic data. The observational data are taken from the Geophysical Monitoring for Climatic Change (GMCC) global network[1,2], from the Pacific north-west (PNW) USA and South Pole[3], and from the CSIRO southern hemispheric stations at Cape Grim, Tasmania, and Mawson, Antarctica[4,5]. The resulting global CCl$_3$F distribution is shown in Fig. 1. A current CCl$_3$F atmospheric lifetime of 75 yr is obtained. The observations suggest that small, residual errors may exist in the CCl$_3$F release data.**

and also by empirically derived eddy diffusion, and incorporates CCl$_3$F release as a function of time and latitude. The parameterization of stratospheric tracer transport has been revised because, for quasi-inert tracers, it has been found that off-axis diffusive coefficients ($K_{yz}$, $K_{zy}$) are approximately equal and of opposite sign[7,8]. In these conditions the effect of eulerian advection, as modelled by meridional and vertical winds, is approximately cancelled by that of eddy diffusion, whose major component is represented by these off-axis terms[7,8]. General circulation models show that stratospheric ozone transport by mean meridional circulation is largely cancelled by that due to zonal eddies[9]. To approximate this cancellation in the model, stratospheric transport was simulated using only the minor, on-axis diffusive coefficients ($K_{yy}$ and $K_{zz}$). This modification allows a reasonable simulation of the photolytic sink for CCl$_3$F in the stratosphere, using published photodissociation coefficients[10] and a radiation scheme including latitudinal variation of day length and solar zenith angle[11]. Following this major change in the method of simulating stratospheric transport, the horizontal diffusive coefficients ($K_{yy}$) and the stratospheric vertical coefficients ($K_{zz}$) used previously[5] were increased by 15 and 80% respectively to refine agreement between simulations and observations (Figs 1 and 3).

**Fig. 1** Observed and calculated meridional distributions of CCl$_3$F for mid-January 1980. Data sources: ●, refs 1, 2; ▲, ref. 3; ■, this work. Error bars are standard errors of linear regression estimates. Point Barrow, Cape Meares (PNW, USA), Cape Matatula, Cape Grim and Mawson are all sea-level stations, Niwot Ridge is at 3.7 km, Mauna Loa 3.4 km and South Pole 2.8 km. Curves *a*, *b* and *c* are model calculations (1,000 mbar) for three values of $K_{zz}$ in the stratosphere corresponding to $\tau_c$ values of 100, 75 and 50 yr respectively. Curve *b'* is the same as *b* for 700 mbar.

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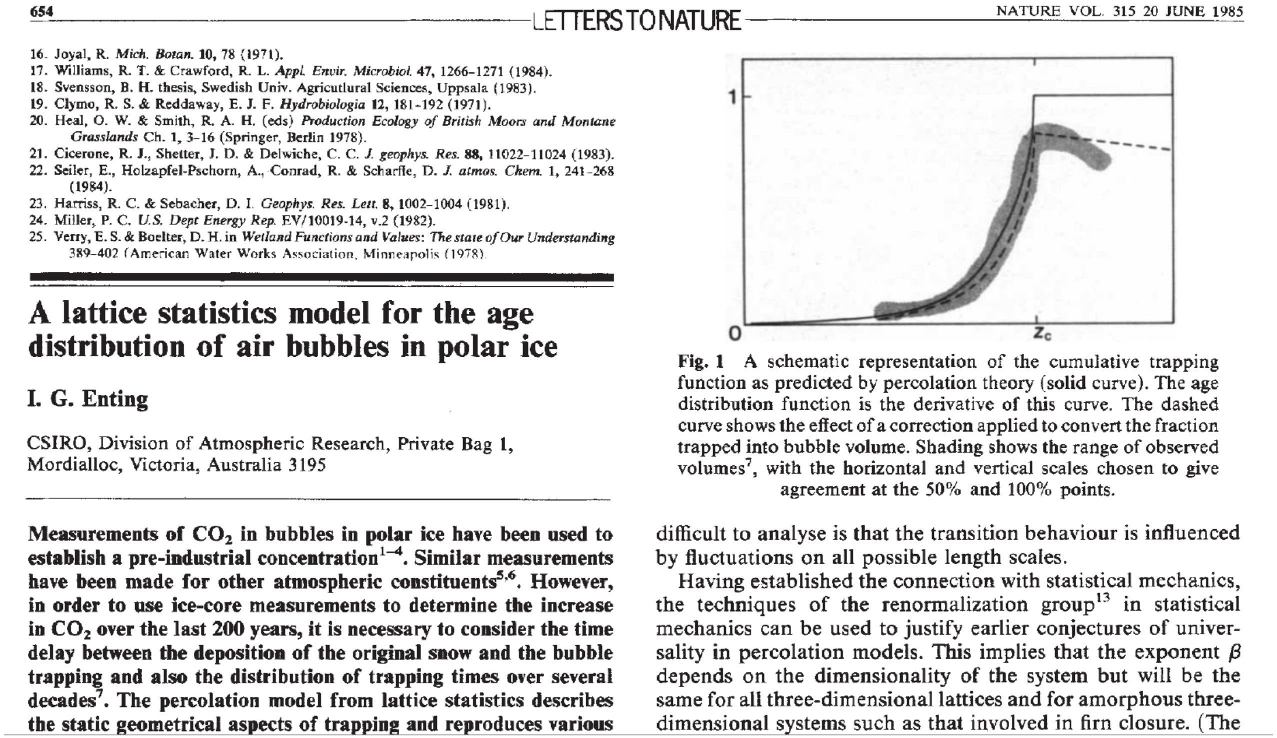
The emissions of CCl$_3$F derived from atmospheric data were significantly different than the bottom-up emissions.... this result was later confirmed when the bottom-up emissions were revised. The bottom-up emissions assumed a tight turn around in line with the rapid pre-Montreal Protocol phase out of CFC-11 use as an aerosol propellant (largely driven by US EPA).

Lovelock reviewed the paper for Nature and it was accepted unchanged - he described it as a brilliant use of atmospheric observations - the brilliance was Ian's not mine - I hardly understood what inverse studies meant.

Paul Fraser March 2015

Jim Lovelock and Paul Fraser with the GC-ECD, Aspendale 2007



16. Joyal, R. *Mich. Botan.* **10,** 78 (1971).
17. Williams, R. T. & Crawford, R. L. *Appl. Envir. Microbiol.* **47,** 1266–1271 (1984).
18. Svensson, B. H. thesis, Swedish Univ. Agricutlural Sciences, Uppsala (1983).
19. Clymo, R. S. & Reddaway, E. J. F. *Hydrobiologia* **12,** 181–192 (1971).
20. Heal, O. W. & Smith, R. A. H. (eds) *Production Ecology of British Moors and Montane Grasslands* Ch. 1, 3–16 (Springer, Berlin 1978).
21. Cicerone, R. J., Shetter, J. D. & Delwiche, C. C. *J. geophys. Res.* **88,** 11022–11024 (1983).
22. Seiler, E., Holzapfel-Pschorn, A., Conrad, R. & Scharfle, D. *J. atmos. Chem.* 1, 241–268 (1984).
23. Harriss, R. C. & Sebacher, D. I. *Geophys. Res. Lett.* **8,** 1002–1004 (1981).
24. Miller, P. C. *U.S. Dept Energy Rep.* EV/10019-14, v.2 (1982).
25. Verry, E. S. & Boelter, D. H. in *Wetland Functions and Values: The state of Our Understanding* 389–402 (American Water Works Association, Minneapolis (1978).

# A lattice statistics model for the age distribution of air bubbles in polar ice

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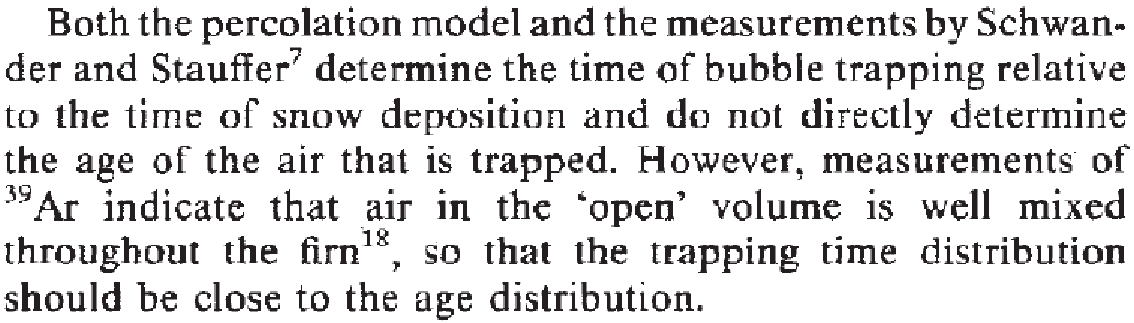
**Measurements of $CO_2$ in bubbles in polar ice have been used to establish a pre-industrial concentration[1–4]. Similar measurements have been made for other atmospheric constituents[5,6]. However, in order to use ice-core measurements to determine the increase in $CO_2$ over the last 200 years, it is necessary to consider the time delay between the deposition of the original snow and the bubble trapping and also the distribution of trapping times over several decades[7]. The percolation model from lattice statistics describes the static geometrical aspects of trapping and reproduces various**

**Fig. 1** A schematic representation of the cumulative trapping function as predicted by percolation theory (solid curve). The age distribution function is the derivative of this curve. The dashed curve shows the effect of a correction applied to convert the fraction trapped into bubble volume. Shading shows the range of observed volumes[7], with the horizontal and vertical scales chosen to give agreement at the 50% and 100% points.

difficult to analyse is that the transition behaviour is influenced by fluctuations on all possible length scales.

Having established the connection with statistical mechanics, the techniques of the renormalization group[13] in statistical mechanics can be used to justify earlier conjectures of universality in percolation models. This implies that the exponent $\beta$ depends on the dimensionality of the system but will be the same for all three-dimensional lattices and for amorphous three-dimensional systems such as that involved in firn closure. (The



Both the percolation model and the measurements by Schwander and Stauffer[7] determine the time of bubble trapping relative to the time of snow deposition and do not directly determine the age of the air that is trapped. However, measurements of $^{39}Ar$ indicate that air in the 'open' volume is well mixed throughout the firn[18], so that the trapping time distribution should be close to the age distribution.

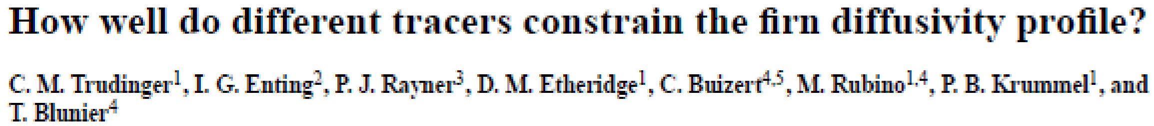
Pearman, Etheridge,
de Silva, Fraser, 1986

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 102, NO. D6, PAGES 6747–6763, MARCH 27, 1997

# Modeling air movement and bubble trapping in firn

C.M. Trudinger,[1] I.G. Enting,[1] D.M. Etheridge, R.J. Francey,[1]
V.A. Levchenko, and L.P. Steele[1]
Division of Atmospheric Research, CSIRO, Aspendale, Victoria, Australia

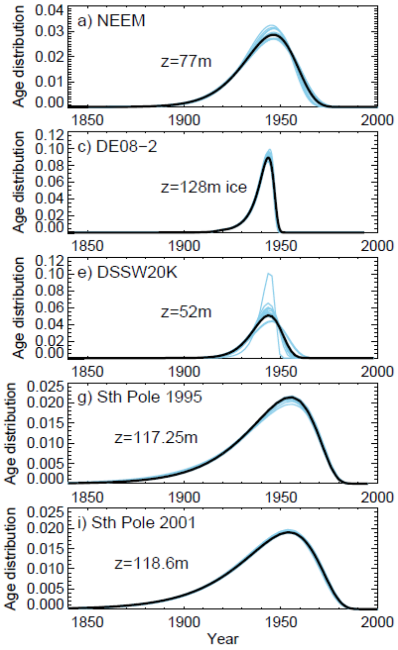
D. Raynaud and L. Arnaud
Laboratoire de Glaciologie et Géophysique de l'Environnement, Centre National de la Recherche Scientifique, Saint Martin d'Hères, France



# How well do different tracers constrain the firn diffusivity profile?

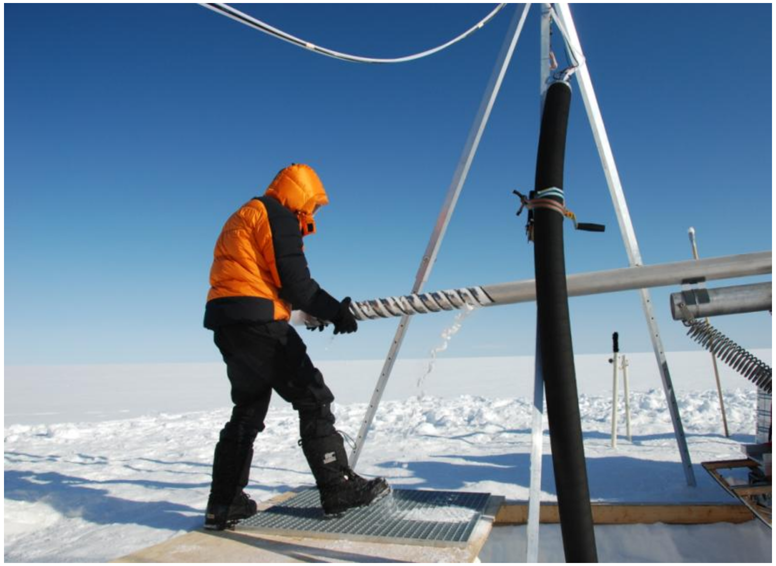
C. M. Trudinger[1], I. G. Enting[2], P. J. Rayner[3], D. M. Etheridge[1], C. Buizert[4,5], M. Rubino[1,4], P. B. Krummel[1], and T. Blunier[4]

ACP 2013

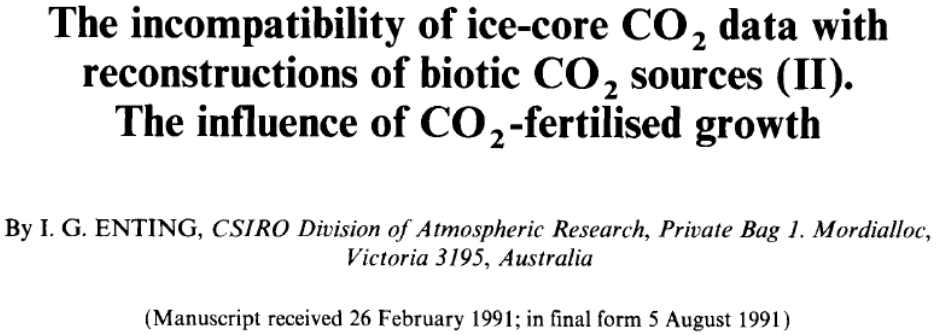


Firn air modelling- 18 papers and 4 more in prep

Jean-Marc Barnola, DE08, Law Dome, 1993- porosity



Vas Petrenko, NEEM, Greenland, 2009- firn air

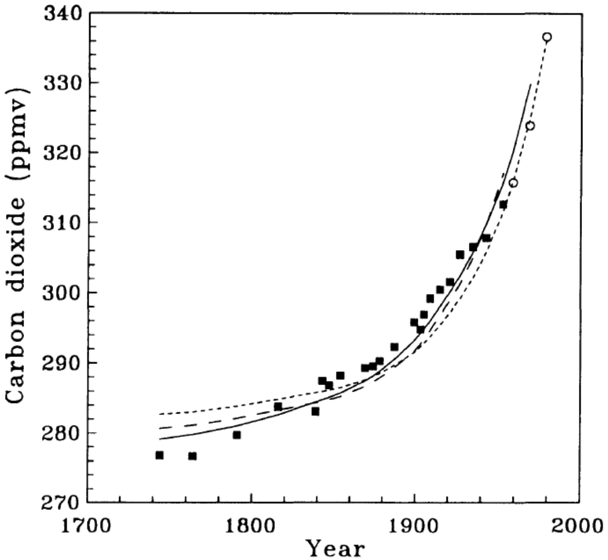


# The incompatibility of ice-core $CO_2$ data with reconstructions of biotic $CO_2$ sources (II). The influence of $CO_2$-fertilised growth

By I. G. ENTING, *CSIRO Division of Atmospheric Research, Private Bag 1. Mordialloc, Victoria 3195, Australia*

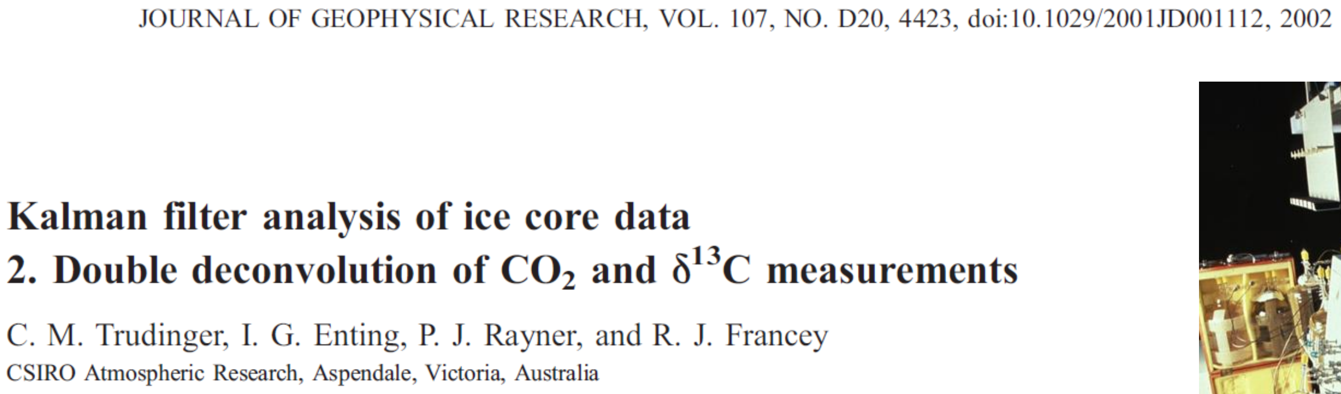
(Manuscript received 26 February 1991; in final form 5 August 1991)

Tellus, 1992



...Given the importance of such an effect which would close a feedback loop between $CO_2$ and climate, the acquisition of high quality ice-core $CO_2$ data covering the little ice age would seem to be a very high priority. Isotopic data could provide a basis for helping distinguish biotic changes from oceanic changes...

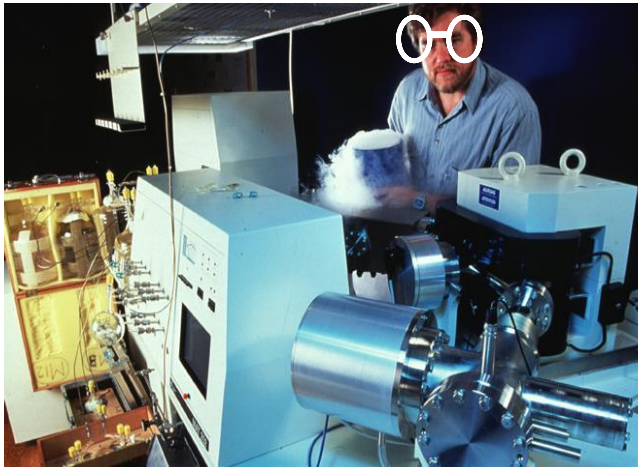
Challenge to the ice core measurement community.......................



JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 107, NO. D20, 4423, doi:10.1029/2001JD001112, 2002

**Kalman filter analysis of ice core data**
**2. Double deconvolution of $CO_2$ and $\delta^{13}C$ measurements**

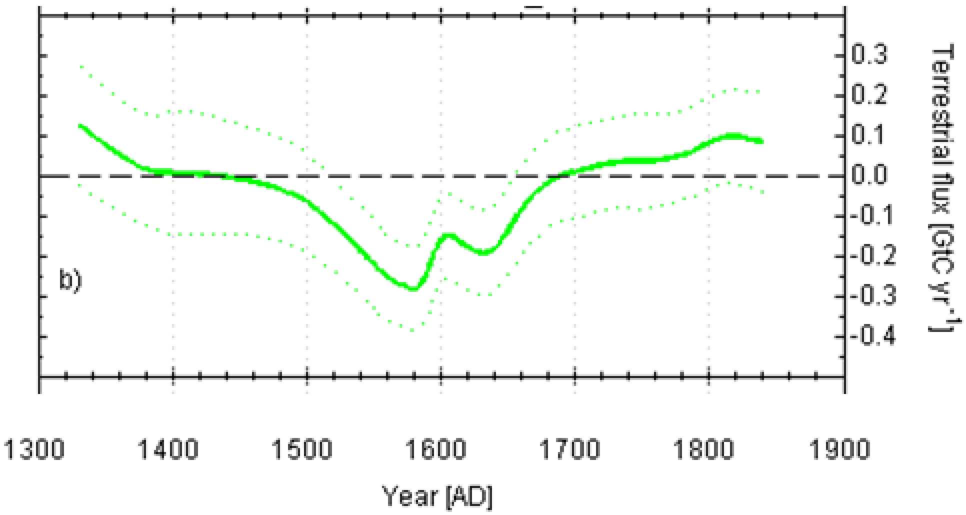
C. M. Trudinger, I. G. Enting, P. J. Rayner, and R. J. Francey
CSIRO Atmospheric Research, Aspendale, Victoria, Australia



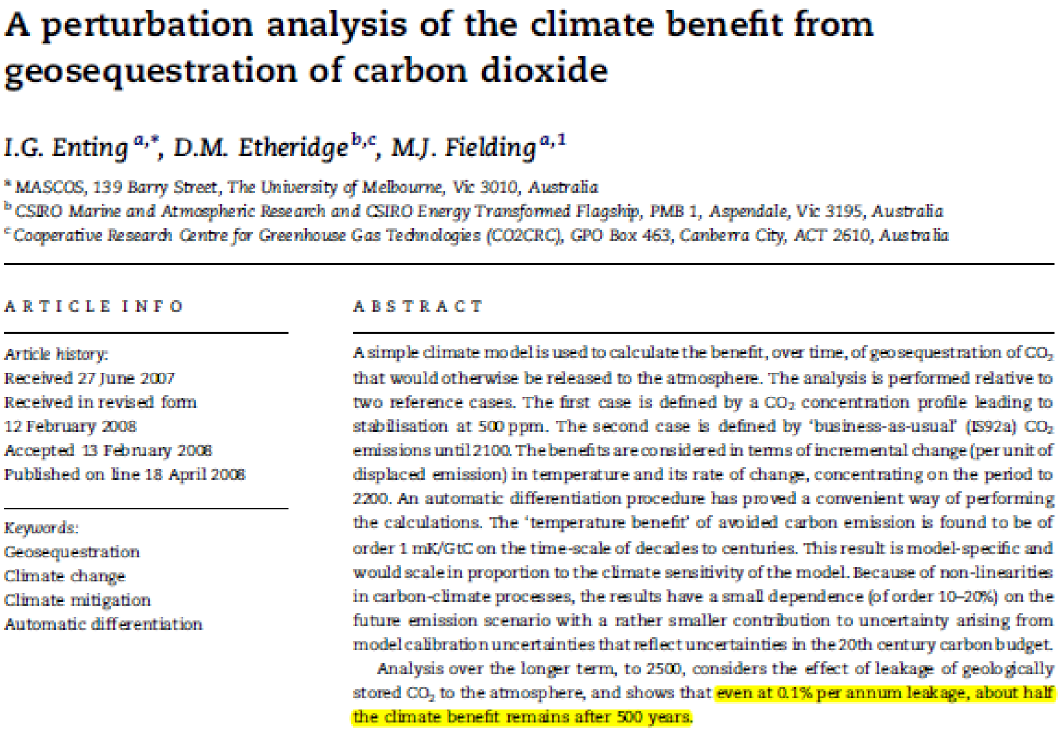
Colin Allison, GASLAB, CO2 isotopes
(safety glasses fitted later)

**Atmospheric $CO_2$ and $^{13}C$-$CO_2$ reconstruction of the Little Ice Age from Antarctic ice cores**

Rubino, M.[1,*] Etheridge, D.M.[1] Trudinger, C.M.[1] Allison, C.E.[1] Rayner P.J.[2]
Mulvaney, R.[3] Steele, L.P.[1] Langenfelds, R.L.[1] Sturges, W.T. [4] Curran, M.[5]
***in preparation***



Andrew Smith, ANSTO AMS 14C



# A perturbation analysis of the climate benefit from geosequestration of carbon dioxide

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ARTICLE INFO

*Article history:*
Received 27 June 2007
Received in revised form
12 February 2008
Accepted 13 February 2008
Published on line 18 April 2008

*Keywords:*
Geosequestration
Climate change
Climate mitigation
Automatic differentiation

ABSTRACT

A simple climate model is used to calculate the benefit, over time, of geosequestration of $CO_2$ that would otherwise be released to the atmosphere. The analysis is performed relative to two reference cases. The first case is defined by a $CO_2$ concentration profile leading to stabilisation at 500 ppm. The second case is defined by 'business-as-usual' (IS92a) $CO_2$ emissions until 2100. The benefits are considered in terms of incremental change (per unit of displaced emission) in temperature and its rate of change, concentrating on the period to 2200. An automatic differentiation procedure has proved a convenient way of performing the calculations. The 'temperature benefit' of avoided carbon emission is found to be of order 1 mK/GtC on the time-scale of decades to centuries. This result is model-specific and would scale in proportion to the climate sensitivity of the model. Because of non-linearities in carbon-climate processes, the results have a small dependence (of order 10–20%) on the future emission scenario with a rather smaller contribution to uncertainty arising from model calibration uncertainties that reflect uncertainties in the 20th century carbon budget.

Analysis over the longer term, to 2500, considers the effect of leakage of geologically stored $CO_2$ to the atmosphere, and shows that even at 0.1% per annum leakage, about half the climate benefit remains after 500 years.

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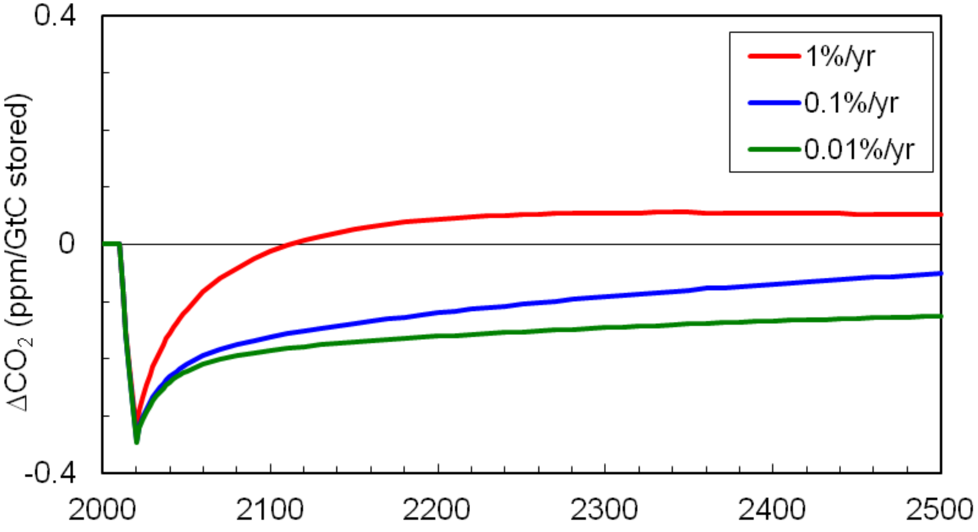
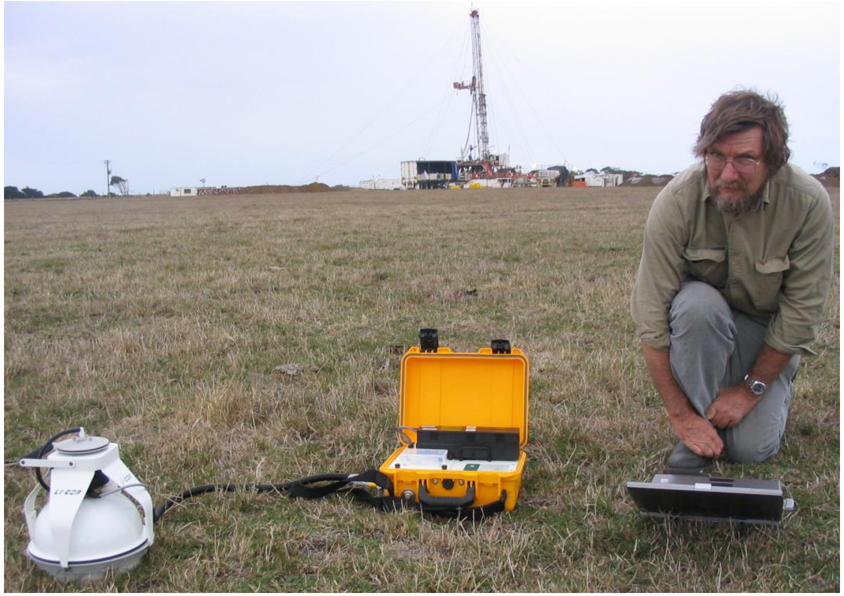
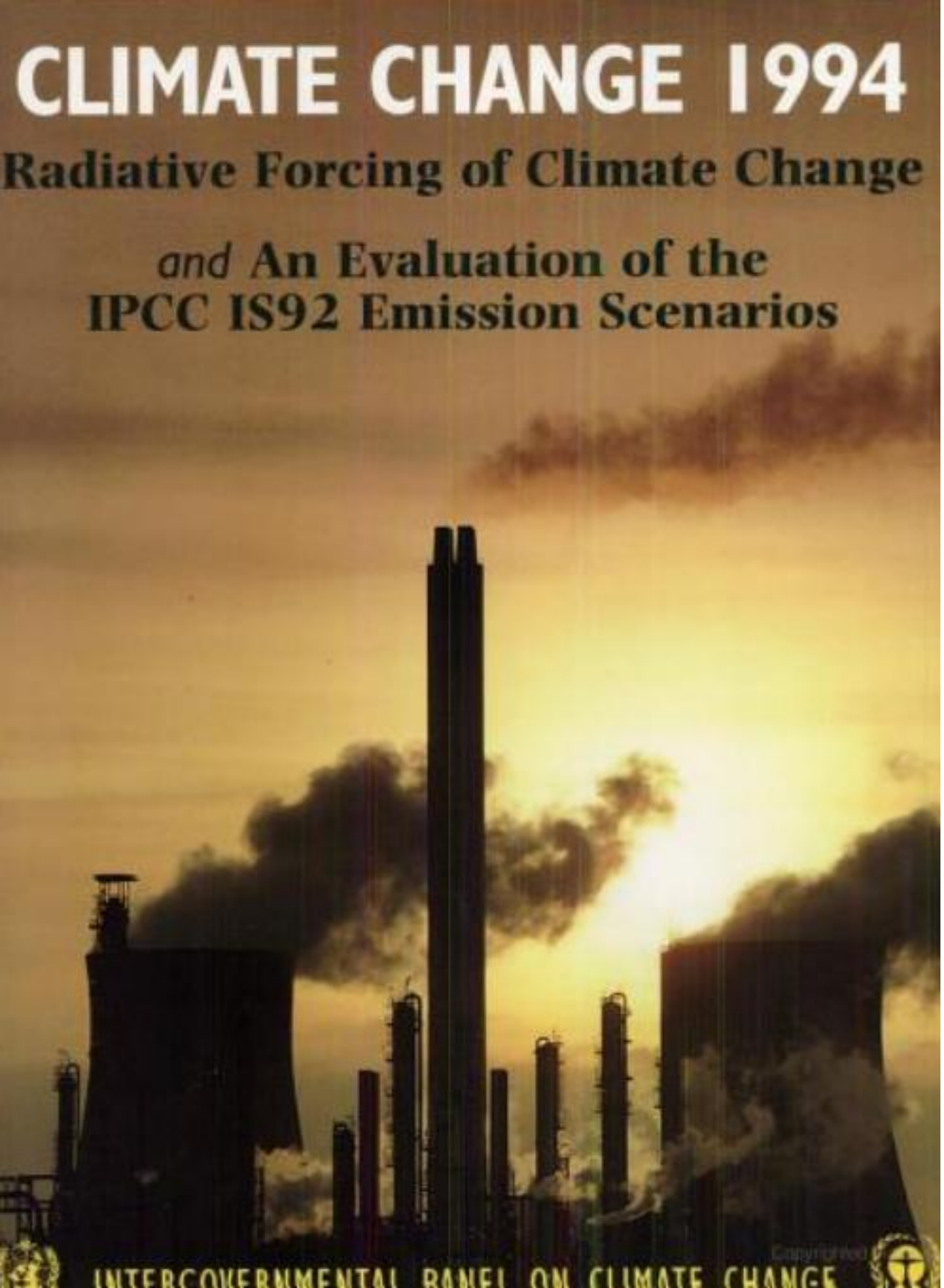
Greenhouse Gas Control





Mick Meyer, Soil CO2 flux, CO2CRC Otway



# CLIMATE CHANGE 1994

## Radiative Forcing of Climate Change

## *and* An Evaluation of the IPCC IS92 Emission Scenarios

INTERGOVERNMENTAL PANEL ON CLIMATE CHANGE

# 1

## $CO_2$ and the Carbon Cycle

D. SCHIMEL, I.G. ENTING, M. HEIMANN, T.M.L. WIGLEY,
D. RAYNAUD, D. ALVES, U. SIEGENTHALER

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“Twisted” at 2700 m in Aurora Basin camp, Antarctica, 2013